

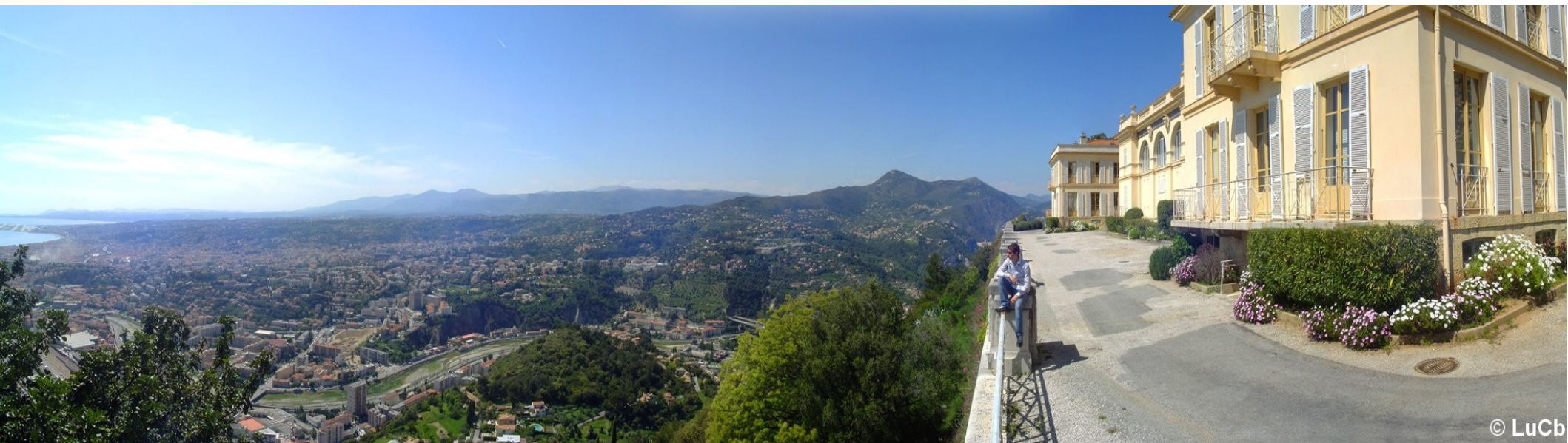
# Periodic box simulations and tools

*Yannick Ponty*

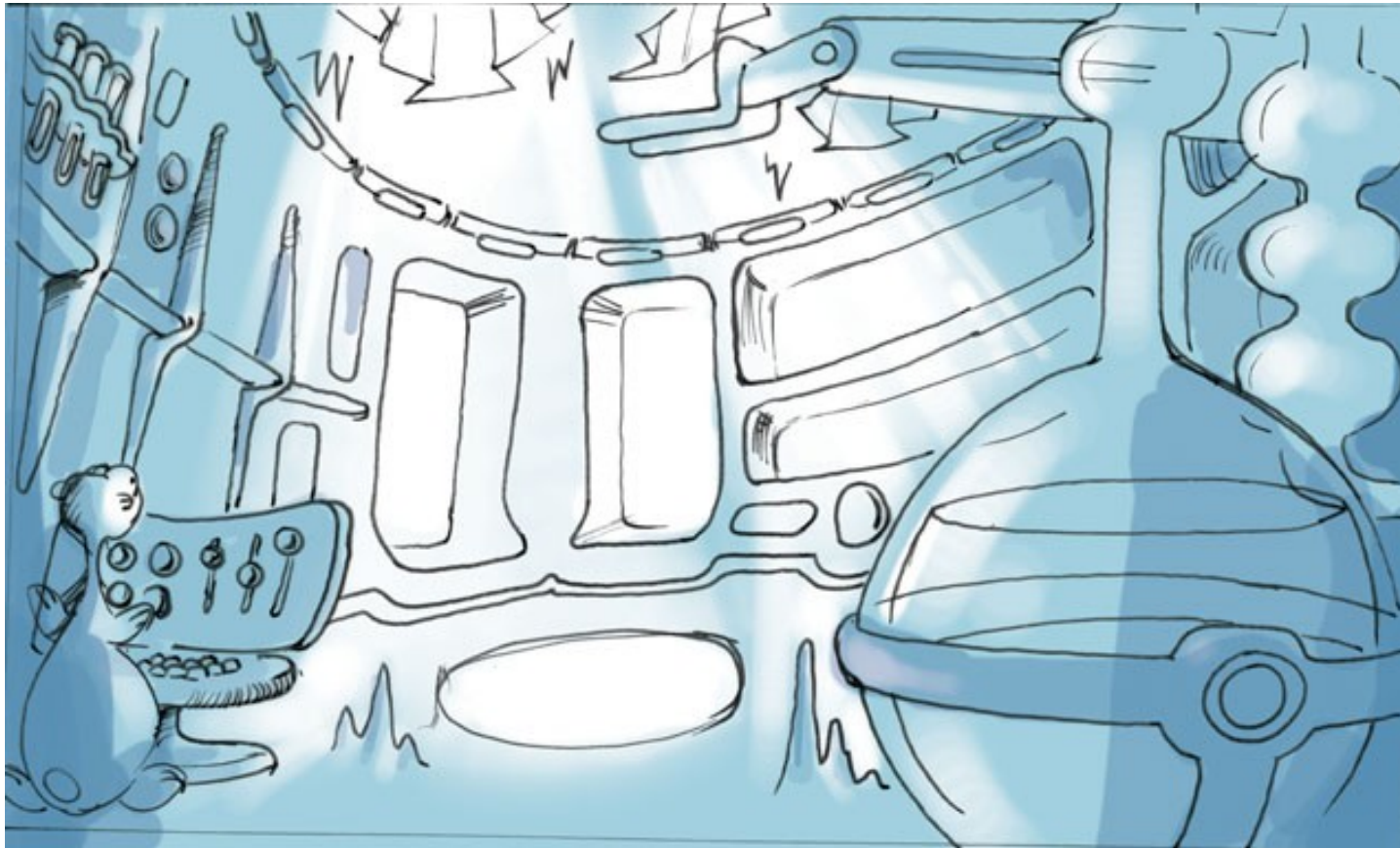
Laboratoire Lagrange CNRS  
Observatoire de la Côte d'Azur  
University of the Côte d'Azur (UCA)

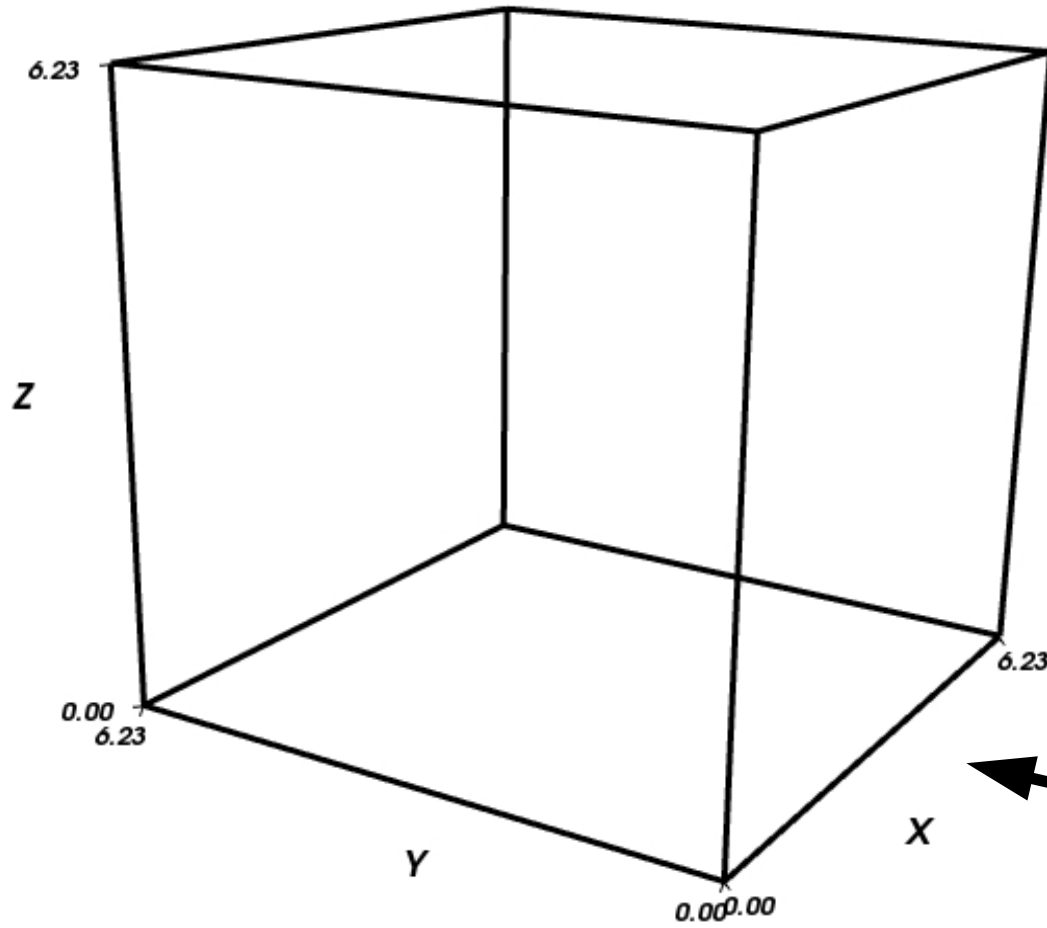


Observatoire  
de la CÔTE d'AZUR

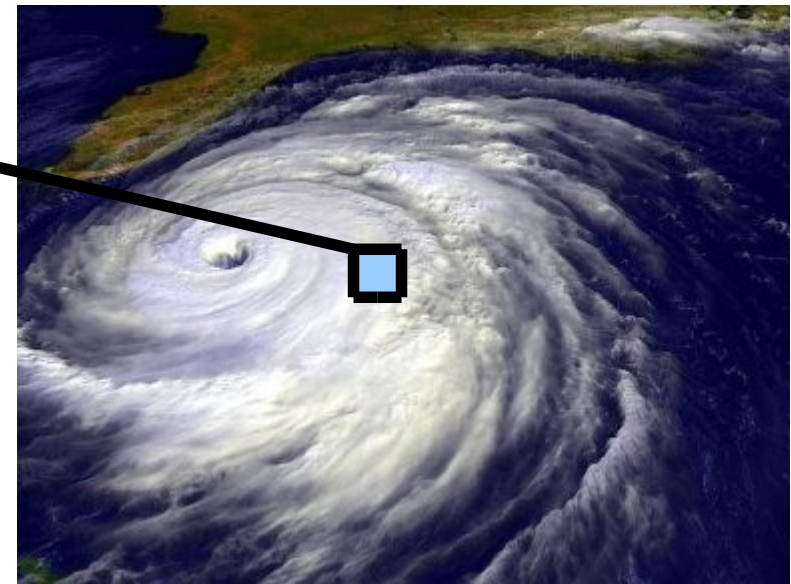
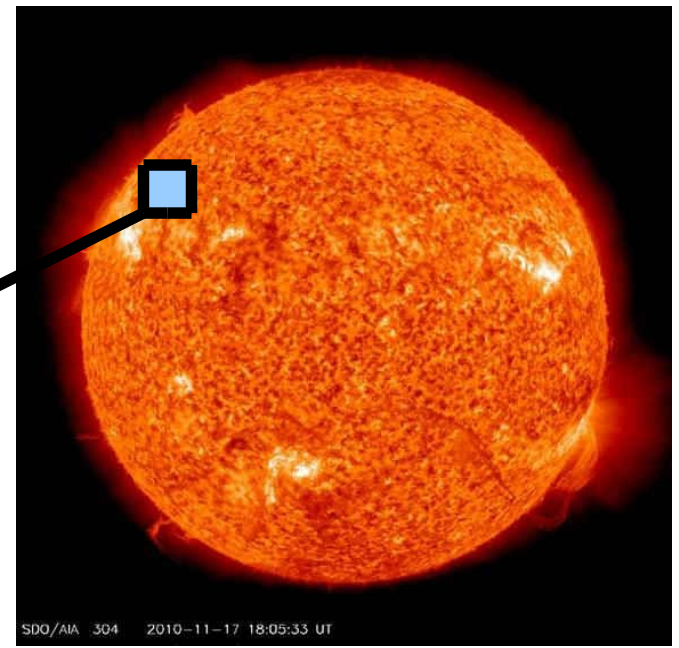


**Do we need complex geometry ?**





$(2\pi)^3$  *periodic box*



PRIX PREMIERE DU PUBLIC GRAND PRIX PRIX DE LA CRITIQUE  
FESTIVAL FANTASTIC'ARTS GERARDMER 99



Ne cherchez pas une raison.  
Cherchez une Issue.

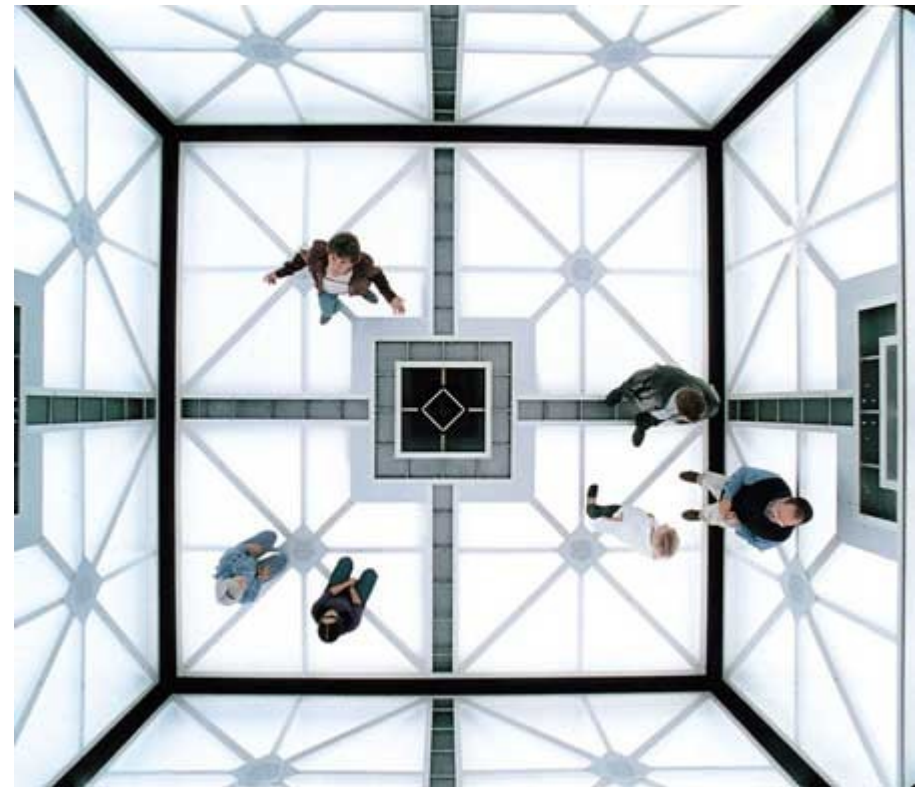
# CUBE

LA SOLUTION EST EN VOUS

TRIMARK PICTURES IS THE FEATURE FILM PROJECT PRESENTING AN ORIGINAL CUBE LIBRARY/EMC NICOLE SIDER MIKY SHAGAGI DAVID HEWLETT ANDREW MILLER JULIAN RICHUS WARRIE BOSTON MAURICE DEAN WINT DIRECTOR OF PHOTOGRAPHY DEREK BURNS COSTUME DESIGNER JASNA STERANOVIC EDITOR JOHN SANDERS MUSIC BY MARK KORMAN EXECUTIVE PRODUCERS Z.C.B. DIGITAL PICTURES PRODUCED BY TRIMARK PICTURES PRODUCED BY CAUDAN STUDIOS WRITTEN BY ANDRE BLUJIC DIRECTED BY VINCENTO NATALI EXECUTIVE PRODUCERS DANIEL BRUNTON PRODUCED FOR MEMBA DEB ET BETTY GIBB PRODUCED BY VINCENTO NATALI  
PROJET DE LA FILMATION DE TELUKUT CANADA CUBED FILM DEVELOPMENT CORPORATION LA FONDATION NAFFELD GREENSON STADIM CANADA



[www.netrolling.com](http://www.netrolling.com)



**Let's go inside the**



**CUBE**

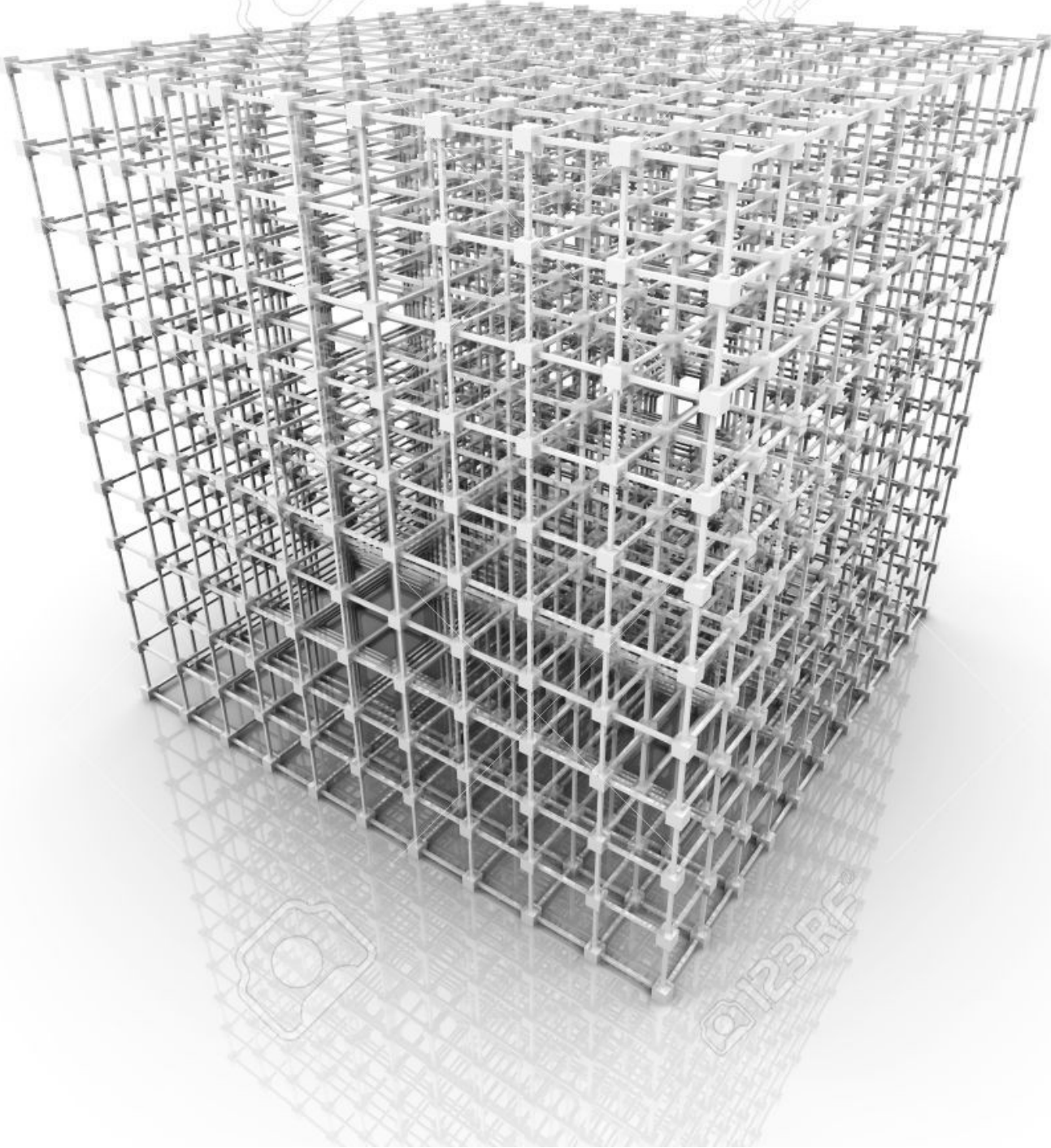
## Outline

### Numerical method

**(How make a cube yourself : The unrevealed story ! )**

- 1) The periodic box a numerical experiment  
*Fundamental equation, Basic non dimensional number*  
*Pressure, projector , Numerical schema*  
*Parallelization procedure, Different forcing*  
*tracers*
- 2) Numerical quantities output and experimental observable  
*Probe, time signal*  
*Mean flow*  
*3D visualization*
- 3) LES, energy transfer
- 4) Penalization method

# Regular Computing grid



# incompressible MHD equations in a periodic box

$$\partial_t \vec{u} + \vec{u} \cdot \nabla \vec{u} = -\nabla P + \nu \Delta \vec{u} + \vec{j} \times \vec{b} + \vec{F}$$

$$\partial_t \vec{b} = \nabla \times (\vec{u} \times \vec{b}) + \eta \Delta \vec{b}$$

$$\nabla \cdot \vec{u} = 0 \quad \nabla \cdot \vec{b} = 0$$

$$\vec{u}(x, y, z, t) = \vec{u}(x + 2\pi i, y + 2\pi j, z + 2\pi k, t)$$

$$\vec{b}(x, y, z, t) = \vec{b}(x + 2\pi i, y + 2\pi j, z + 2\pi k, t)$$

$$i, j, k = 0, 1, 2, \dots, \infty$$



# Pseudo-spectral method

Real space

$$\vec{u}(\vec{x}, t)$$



$$\hat{u}(\vec{k}, t)$$

spectral space

**Fast Fourier Transform**

$$\partial_x \vec{u}(\vec{x}, t)$$



$$i k_x \hat{u}(\vec{k}, t)$$

$$\vec{w}(\vec{x}, t) = \nabla \times \vec{u}(\vec{x}, t)$$



$$i \vec{k} \times \hat{u}(\vec{k}, t)$$

$$\vec{h}(\vec{x}, t) = \vec{u}(\vec{x}, t) \times \vec{w}(\vec{x}, t)$$



$$\hat{h}(\vec{k}, t)$$

# Numerical convergence

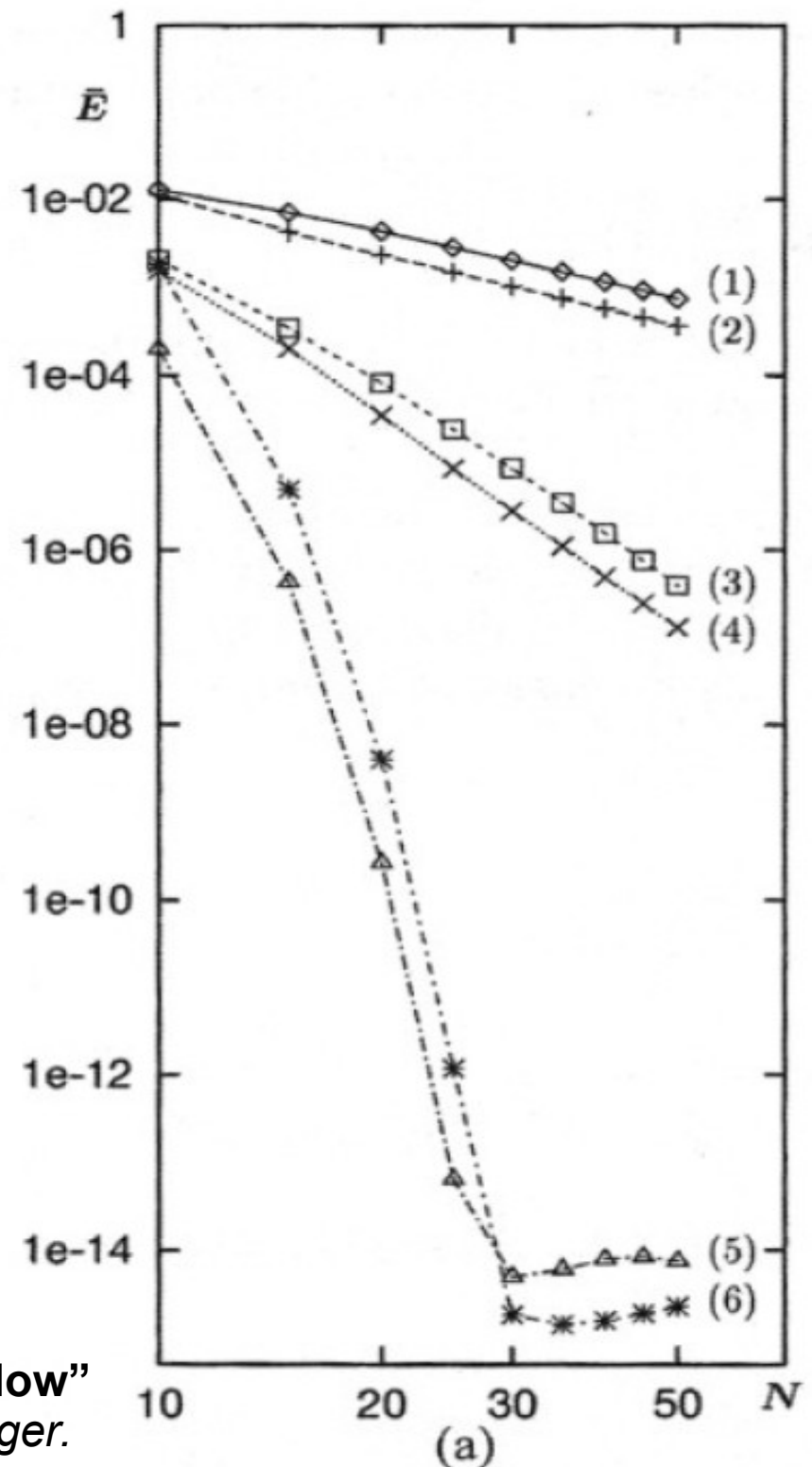
## Helmotz equation

$$-\nu u'' + u = f, \quad -1 < x < 1, \quad \nu = 10^{-2}$$

$$u(-1) = g_-, \quad u(1) = g_+,$$

$$u(x) = 1 - \frac{\sinh[(x+1)/\sqrt{\nu}]}{\sinh(2/\sqrt{\nu})}$$

- (1) Second-order finite-difference method with uniform mesh ( $\Delta x = 2/N$ ).
- (2) Second-order finite-difference method with Gauss-Lobatto mesh (3.75).
- (3) Sixth-order Hermitian method with uniform mesh ( $\Delta x = 2/N$ ).
- (4) Sixth-order Hermitian method with Gauss-Lobatto mesh (3.75).
- (5) Chebyshev collocation method [Gauss-Lobatto mesh (3.75)].
- (6) Chebyshev tau method (polynomial degree =  $N$ ).



## Numerical method :

$$\partial_t \vec{u} + \vec{u} \nabla \vec{u} = -\nabla P + \nu \Delta \vec{u} + \vec{j} \times \vec{b} + \vec{F}$$

$$\partial_t \vec{u} - \nu \Delta \vec{u} = \vec{u} \times \vec{\omega} - \nabla \left( P + \frac{u^2}{2} \right) + \vec{j} \times \vec{b} + \vec{F}$$

$$\vec{\omega} = \nabla \times \vec{u}$$

## Elimination of the pressure :

**Projector  $\Pi$  in the solenoidal space function :**

In spectral space the projector tensor :  $\Pi_{ij} = \delta_{ij} - \frac{k_i k_j}{k^2}$

$$\partial_t \vec{u} - \nu \Delta \vec{u} = \Pi \left[ \vec{u} \times \vec{\omega} + \vec{j} \times \vec{b} + \vec{F} \right]$$

**Numerical method :**  $\partial_t \hat{u} - \nu k^2 \hat{u} = \Pi [\widehat{\vec{u}} \times \widehat{\vec{\omega}} + \widehat{j} \times \widehat{\vec{b}} + \widehat{F}]$

$$\partial_t \hat{b} - \eta k^2 \hat{b} = \widehat{\nabla} \times (\widehat{\vec{u}} \times \widehat{\vec{b}})$$

The diffusion term is implemented implicitly with the exponential method.

$$\partial_t U_k(t) = -\nu k^2 U_k(t) + G_k(t)$$

$$\partial_t (e^{\nu k^2 t} U_k) = e^{\nu k^2 t} G_k(t)$$

For the other time step the forward Euler - Adams-Basford schemes is implemented :

$$\frac{e^{\nu k^2 (t+\Delta t)} U_k(t+\Delta t) - e^{\nu k^2 t} U_k(t)}{\Delta t} = \frac{3}{2} G_k(t) e^{\nu k^2 t} - \frac{1}{2} G_k(t-\Delta t) e^{\nu k^2 (t-\Delta t)}$$

$$U_k(t+\Delta t) = U_k(t) e^{-\nu k^2 \Delta t} + e^{-\nu k^2 \Delta t} \Delta t \left[ \frac{3}{2} G_k(t) - \frac{1}{2} G_k(t-\Delta t) e^{-\nu k^2 \Delta t} \right]$$

# Aliasing removal :

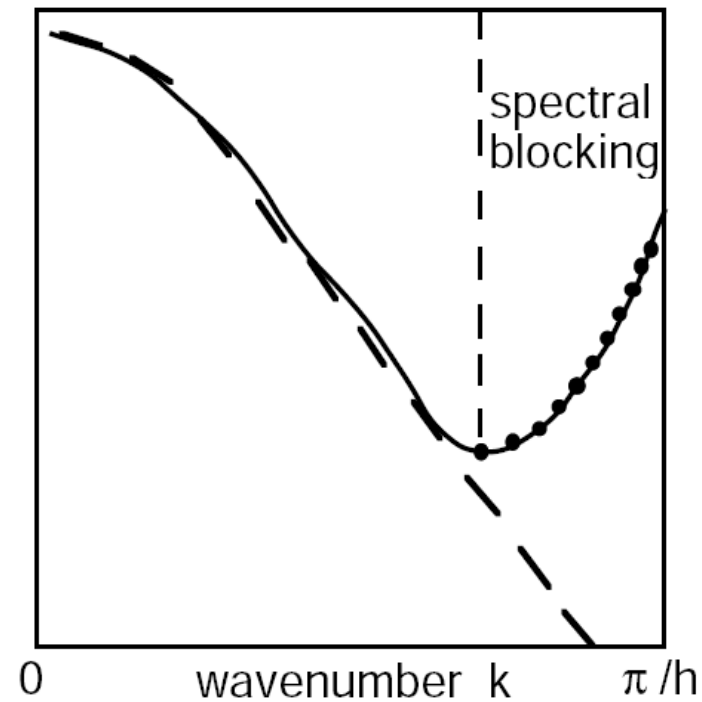
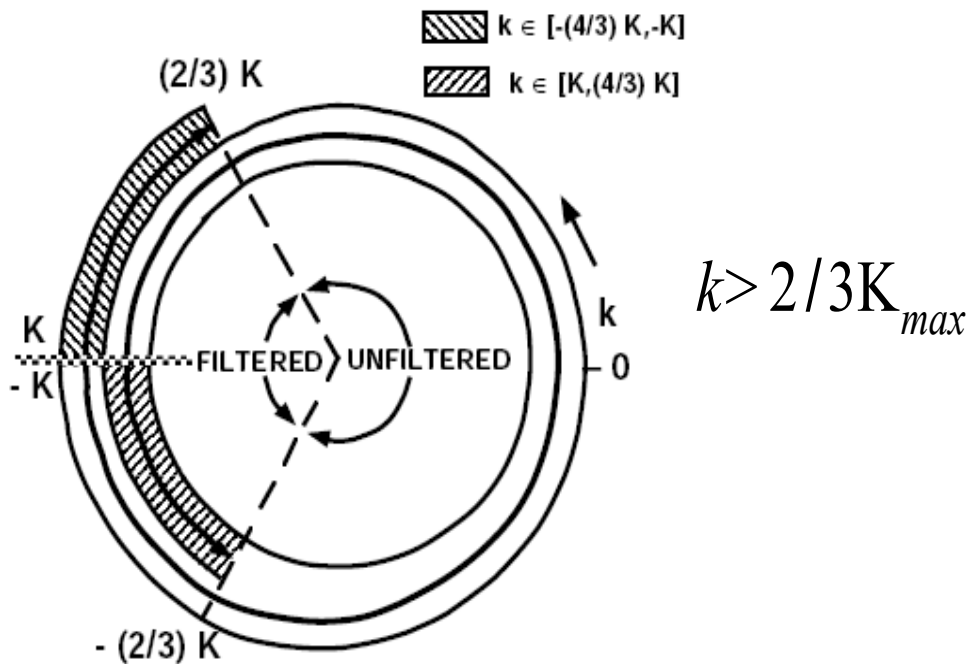
Aliasing can cause numerical instability in the time integration of *nonlinear* equations. For example, a typical quadratically nonlinear term is

$$u u_x = \left( \sum_{p=-K}^K a_p e^{ipx} \right) \left( \sum_{q=-K}^K i q a_q e^{iqx} \right) \quad (11.10)$$

$$= \sum_{k=-2K}^{2K} b_k e^{ikx} \quad (11.11)$$

where the  $b_k$  are given by a sum over products of the  $a_k$ . The nonlinear interaction has generated high zonal wavenumbers which will be aliased into wavenumbers on the range  $k \in [-K, K]$ , creating a wholly unphysical cascade of energy from high wavenumbers to low.

## DEALIASING AND THE ORSZAG TWO-THIRDS RULE



**Forcing :** 
$$\partial_t \vec{u} + \vec{u} \nabla \vec{u} = -\nabla P + \nu \Delta \vec{u} + \vec{F}$$

Energy Injection :

**Constant force (Torque)** 
$$\vec{F} = \vec{F}(x, y, z) \quad \varepsilon_i = \vec{F} \cdot \vec{u}$$

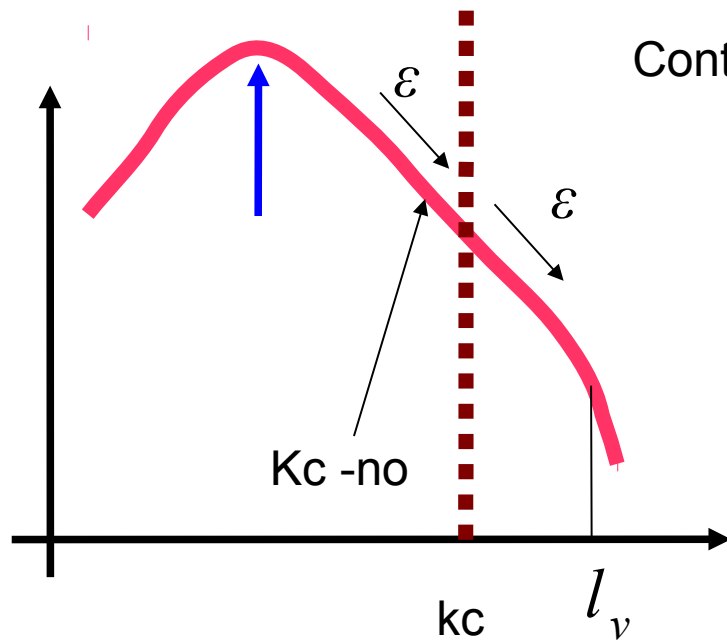
Add + F a each time step

**Constant velocity :** 
$$\vec{u}(\vec{k}_i) = \vec{u}_F \quad \varepsilon_i = \text{fft}^{-1} \left( \hat{u}(k_i) - \hat{u}_F(k_i) \right) \cdot \vec{u}$$

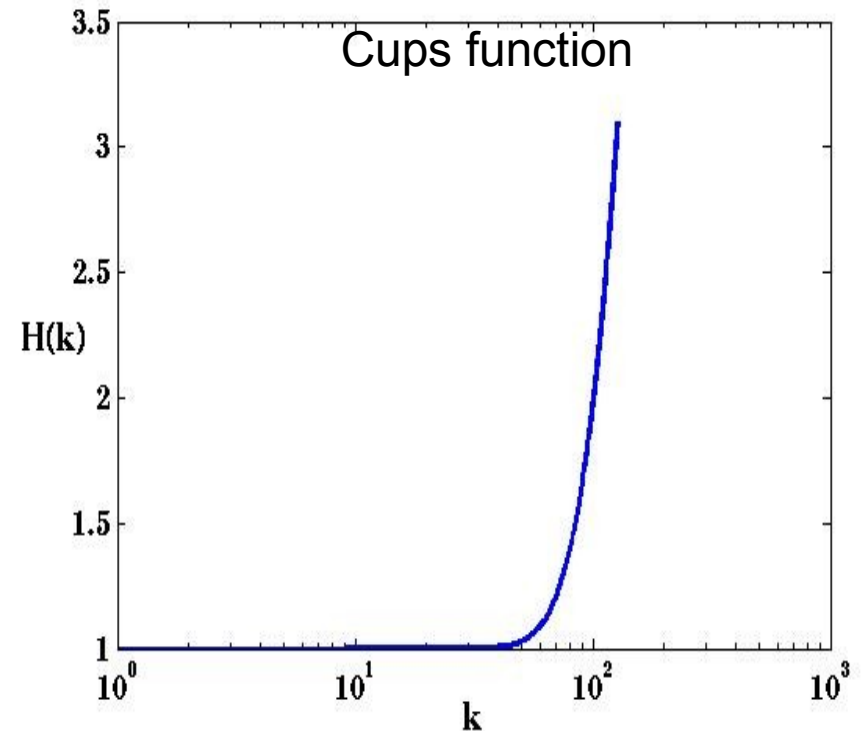
Keep some spectral modes constants at each time step

**Constant energy**

# Large eddy simulation (LES) : J.P. Chollet & M. Lesieur J. Atmos. Sci 38 (1981)



$$v(k, t) = 0.441 C \frac{3}{2} \sqrt{\frac{E(K_c - n_o, t)}{K_c - n_o}} H(k)$$



Ref: "Turbulence in Fluids" M. Lesieur *Kluwer*

"Large Eddy Simulation for incompressible flow" P. Sagaut *Springer*

# Non dimensional Parameters

Reynolds number :

$$Rv = \frac{LU}{\nu}$$

eddy turn over time

Magnetic Reynolds number :

$$Rm = \frac{LU}{\eta} = \frac{L/U}{L^2/\eta}$$

magnetic diffusion time

Magnetic Prandtl number:

$$Pm = \frac{\nu}{\eta} = \frac{Rm}{Rv} = \frac{\tau_\eta}{\tau_\nu}$$

magnetic diffusion time  
viscous time scale

Liquid Metal :

$$Pm \sim 10^{-5} \quad \longrightarrow \quad Rv \sim 10^7$$

$$Rm \sim 100$$



How to choose those quantities :  $\mathbf{U}$  ,  $\mathbf{L}$  ?

$$\partial_t \vec{u} + \vec{u} \nabla \vec{u} = -\nabla P + \nu \Delta \vec{u} + \vec{j} \times \vec{b} + \vec{F}$$
$$\nabla \vec{v} = 0$$

$U \sim 1$  ,  $F \sim 1$       Order one

$$Rv = \frac{LU}{\nu}$$

**A posteriori :**

$$U_{rms} = \sqrt{2 E \nu}$$

$u_{max}$

**L = size of the box**

$$L_i = \frac{\sum E(k) / k}{\sum E(k)}$$

Grashof number

$$Gr = \frac{FL^3}{\nu^2}$$

## Turbulence and mesh point :

64<sup>3</sup> TG1 R ~ 200  
 128<sup>3</sup> TG1 R ~ 460  
 256<sup>3</sup> TG1 R ~ 900  
 512<sup>3</sup> ABC1 R ~ 3500

$$R_v = \frac{L_i U_{rms}}{\nu}$$

Y. Ponty et al 2007 (Dynamo, Mhd)

Run	$N$	f	$k_F$	$\nu$	$Re$
I	256	TG	2	$2 \times 10^{-3}$	675
II	512	TG	2	$1.5 \times 10^{-3}$	875
III	1024	TG	2	$3 \times 10^{-4}$	3950
IV	256	ABC	10	$2.5 \times 10^{-3}$	275
V	256	ABC	3	$2 \times 10^{-3}$	820
VI	512	ABC	3	$6.2 \times 10^{-4}$	2520
VII	1024	ABC	3	$2.5 \times 10^{-4}$	6200
VIII	256	RND	1	$1.5 \times 10^{-3}$	2030

P. Mininni et al 2006 (Hydro)

$K_{max} \eta \sim 1$

$K_{max} \eta \sim 2$

512<sup>3</sup> R ~ 2040  
 1024<sup>3</sup> R ~ 7700  
 2048<sup>3</sup> R ~ 18 000  
 4096<sup>3</sup> R ~ 45 000

512<sup>3</sup> R ~ 1150  
 1024<sup>3</sup> R ~ 2666  
 2048<sup>3</sup> R ~ 6100  
**4096<sup>3</sup> R ~ 15 000**

Kaneda et al *PoF* 15 2003 (hydro)  
 ( Earth Simulator, japan )

# Let's stir of cup of tea



Radius of the cup  $R = 0.05 \text{ m}$  ( 5 cm)

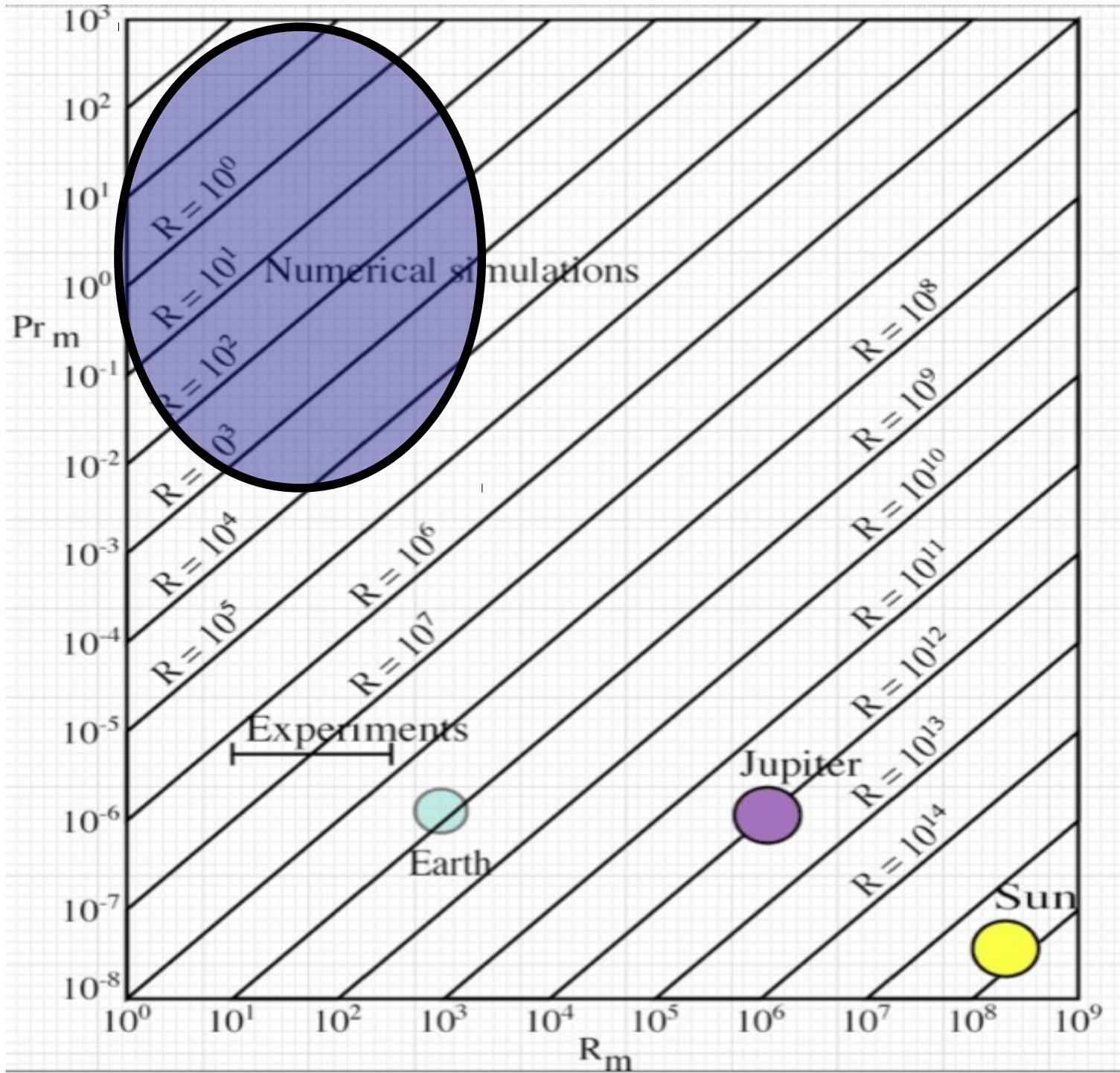
Motion of the spoon : 1 turn by second  
 $V = 2 \pi * R / 1 \text{ s} = 0.01570 \text{ m/s}$

$$R_v = \frac{2 \pi R^2}{\nu \tau}$$

Kinematic viscosity of water :

5 °C	$1.52 \cdot 10^{-6} \text{ m}^2/\text{s}$	$R \sim 10\ 000$	
20 °C	$1.0 \cdot 10^{-6} \text{ m}^2/\text{s}$	$R \sim 15\ 000$	$4096^3 R \sim 15\ 000$
30 °C	$0.804 \cdot 10^{-6} \text{ m}^2/\text{s}$	$R \sim 19\ 000$	

Proposal : Exascale computing to solve  
**HOT** or a **larger** cup of tea turbulence



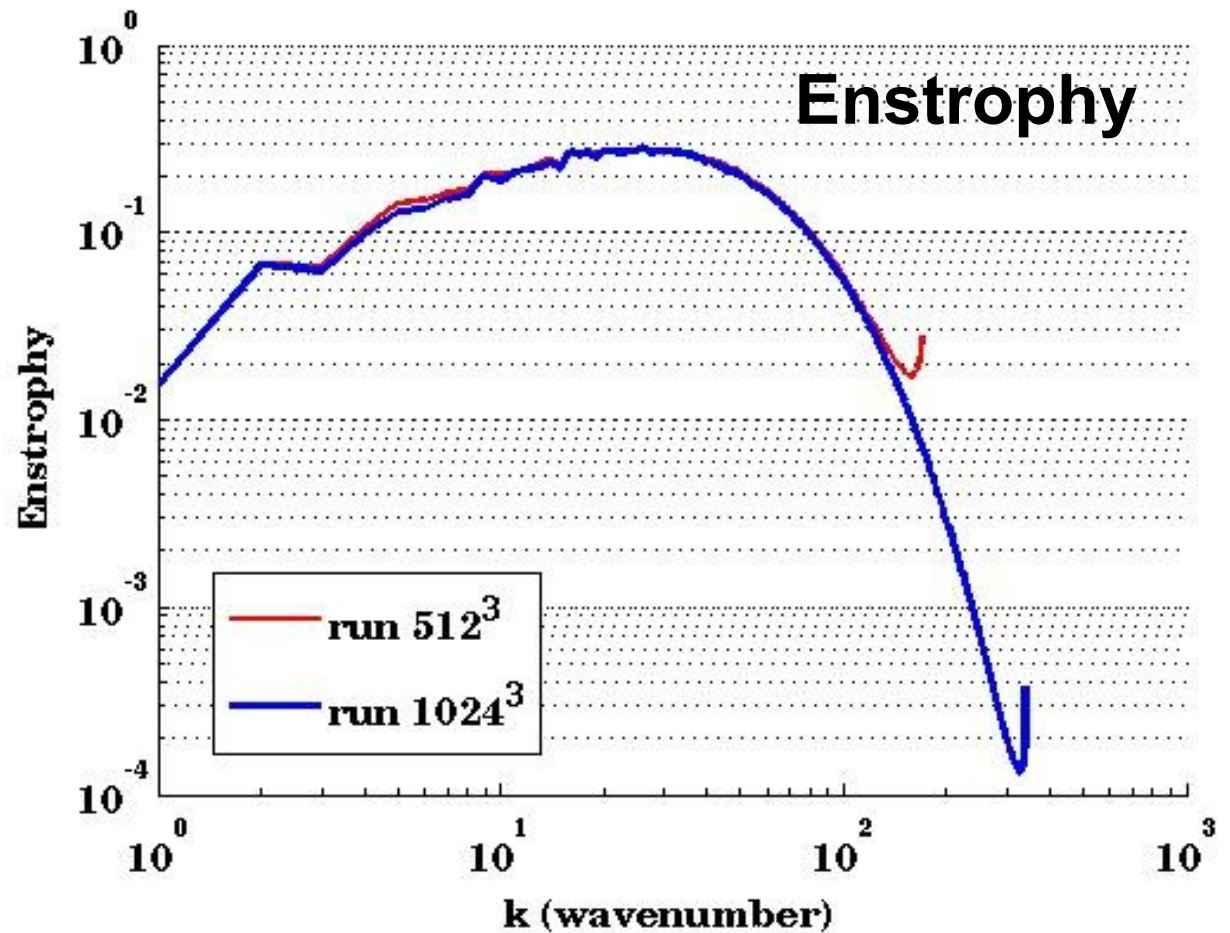
# Grid Resolution !!!

$$\partial_t E = I - 2\nu\Omega$$

$$\eta_{hydro} \sim \left(\frac{\nu^3}{\langle \epsilon \rangle}\right)^{1/4} = \left(\frac{2\Omega}{\nu^2}\right)^{-1/4}$$

$$k_{max-dealiasing} \cdot \eta_{hydro} \sim 1 \text{ ou } \sim 2.$$

$$2/3 k_{max} \left(\frac{2\Omega}{\nu^2}\right)^{-1/4} \sim 2$$



**— 512<sup>3</sup>**  $2/3 k_{max} \eta_{hydro} \sim 1.21$

**— 1024<sup>3</sup>**  $2/3 k_{max} \eta_{hydro} \sim 2.41.$

# DNS of Von Karman Turbulent

$$\begin{array}{l} L_0 \sim 1 m \\ R_e \sim 10^6 \end{array} \longrightarrow \frac{L_0}{\eta} \sim R_e^{3/4} \sim 32\,000$$

Need at least  $65536^3 = 2^{16}$

Need 20 scalar fields in double precision

$$65536^3 * 20 * 8 \text{ octets} = 40 \text{ peta octets}$$

**40 Po / 10,649,600 cores ~ 3.6 Go by core !**

**1<sup>st</sup> rank TOP500 : Sunway TaihuLight CHINA**

**1310720 GB = 1,3 Po > 40 Po !!!**



## Computer time and mesh size : (double precision)

64<sup>3</sup> dt=0.002 1 time step : 0.72 s mono-proc

128<sup>3</sup> dt=0.001 1 time step : 7.4 s mono-proc

256<sup>3</sup> dt=0.0005 1 time step : 58.54 s mono-proc

512<sup>3</sup> dt=0.0003 1 time step : 464 s mono-proc

CFL condition :  $dt < \frac{1}{U_{max} K_{max}}$  -> Factor 16 at least

100 over time: 64<sup>3</sup> -> 10 h mono proc

128<sup>3</sup>-> 205 h mono proc = 8 d 13 h

256<sup>3</sup>-> 3252 h mono proc = 135 d

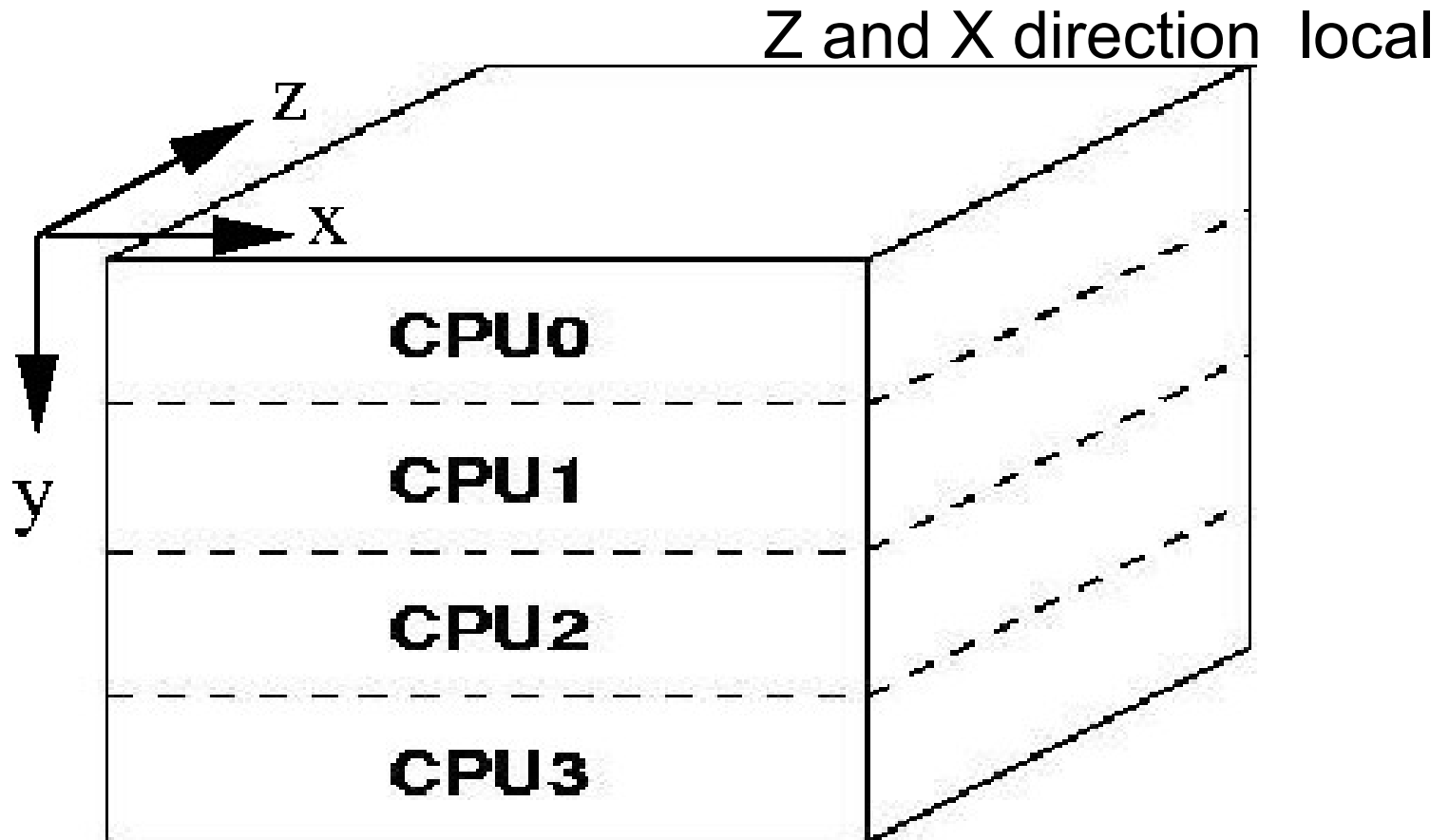
512<sup>3</sup> -> ~ 5 y mono proc = 167 h ~ 7 d with 256 proc

# We need PARALLELIZATION !

# Parallelization :

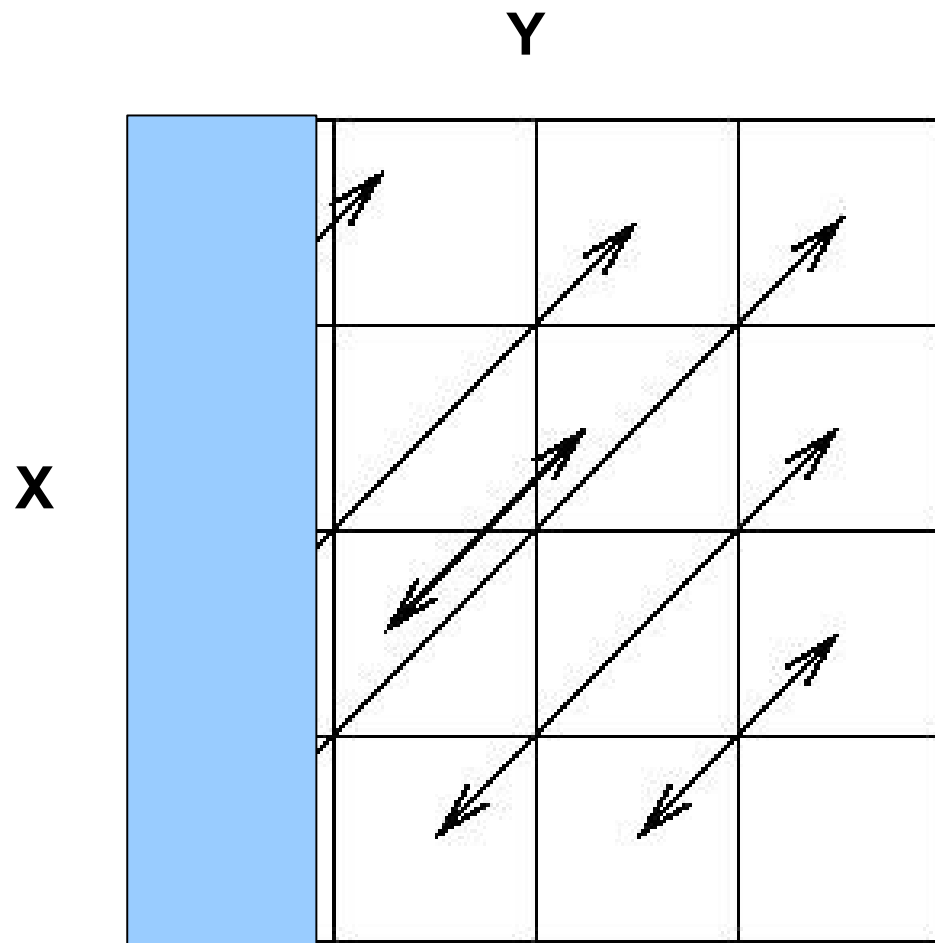
**MPI : Message Passing Interface**

**MPI  
MPI2**

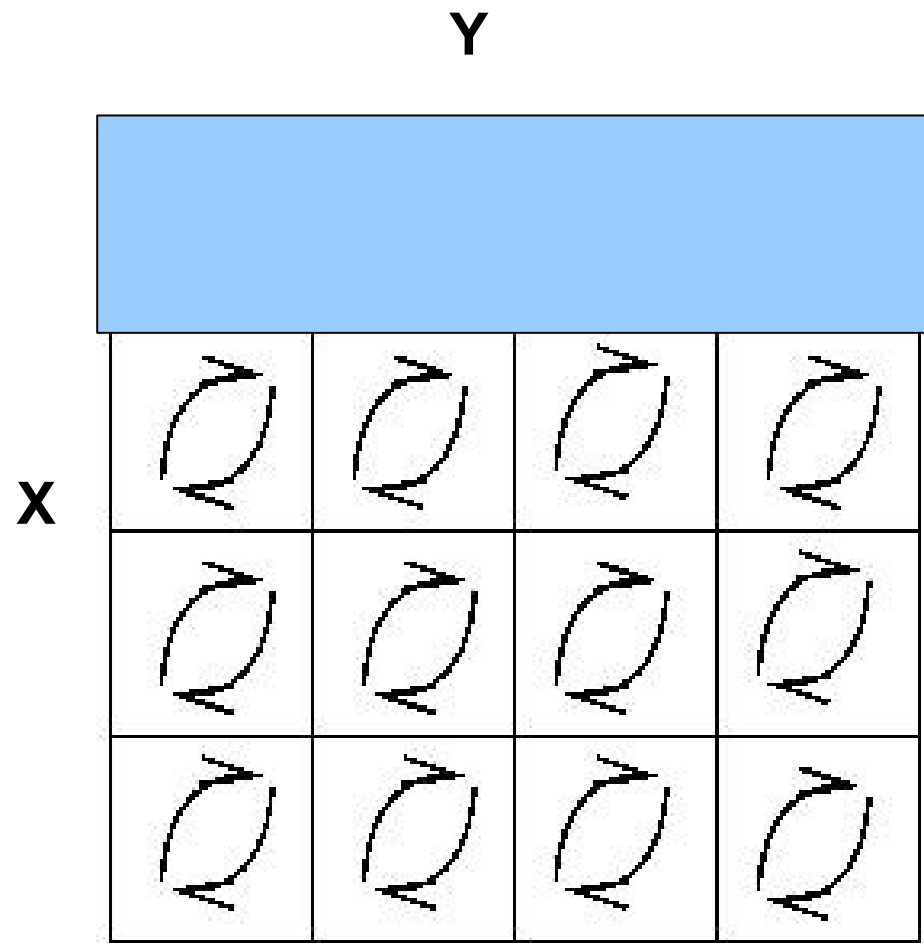




# Transposition :



Global transpose of blocks

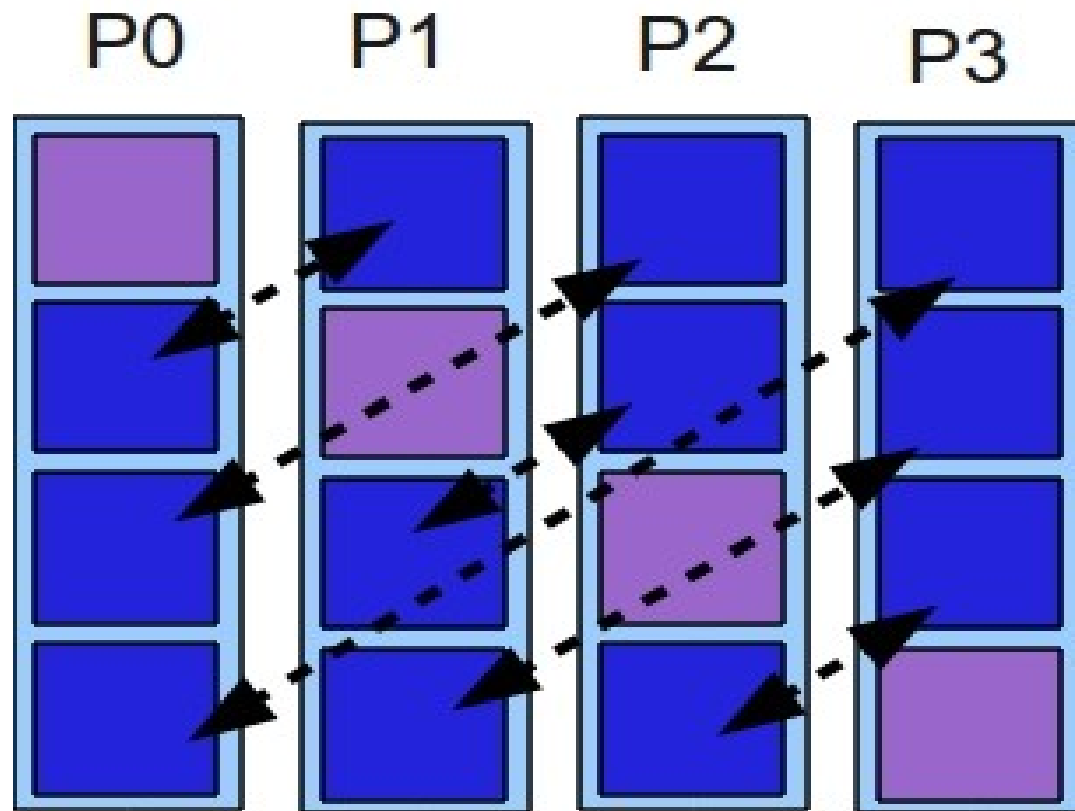


Local transposes within blocks

Recommend : [www.fftw.org](http://www.fftw.org)

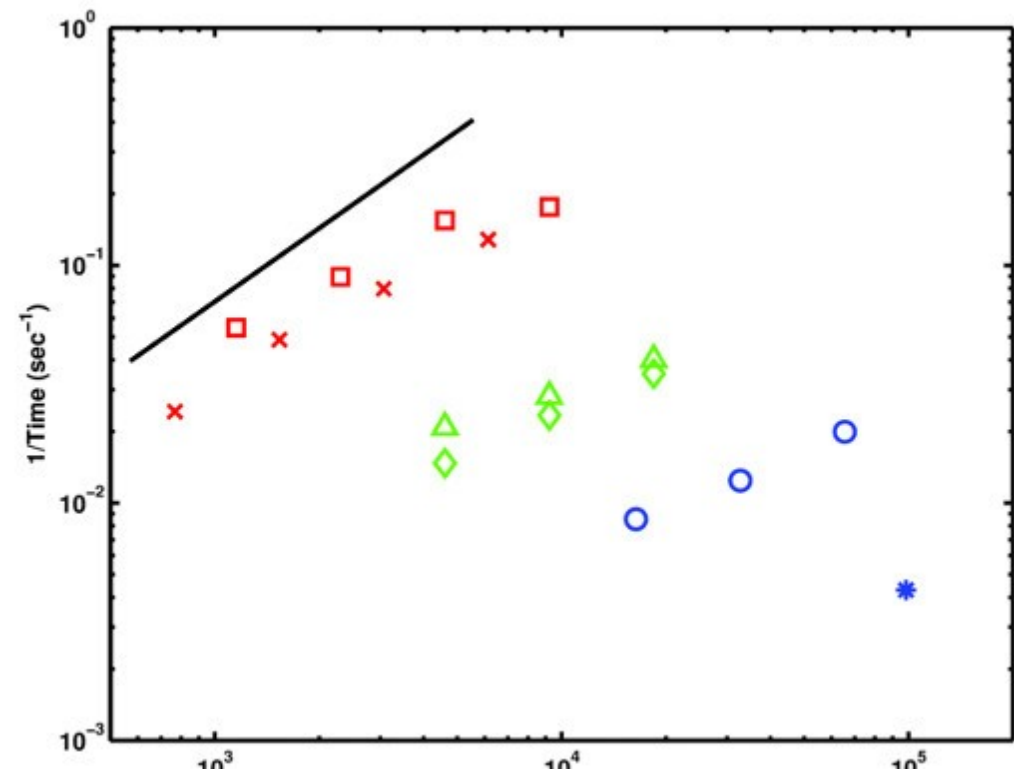
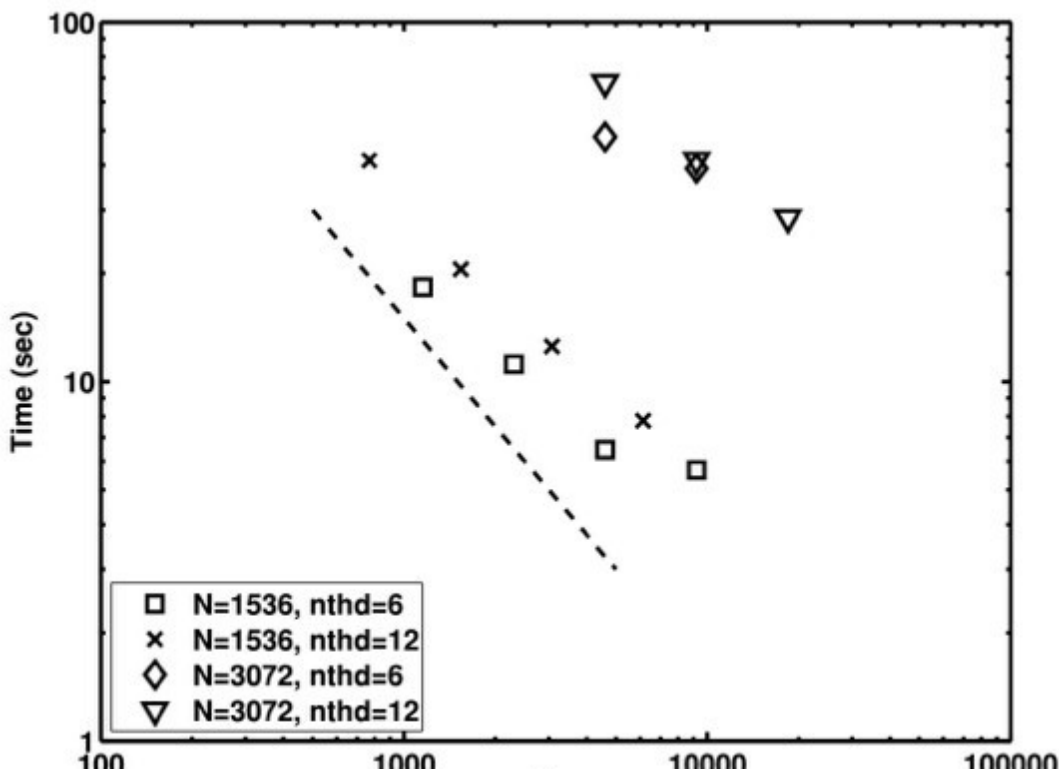
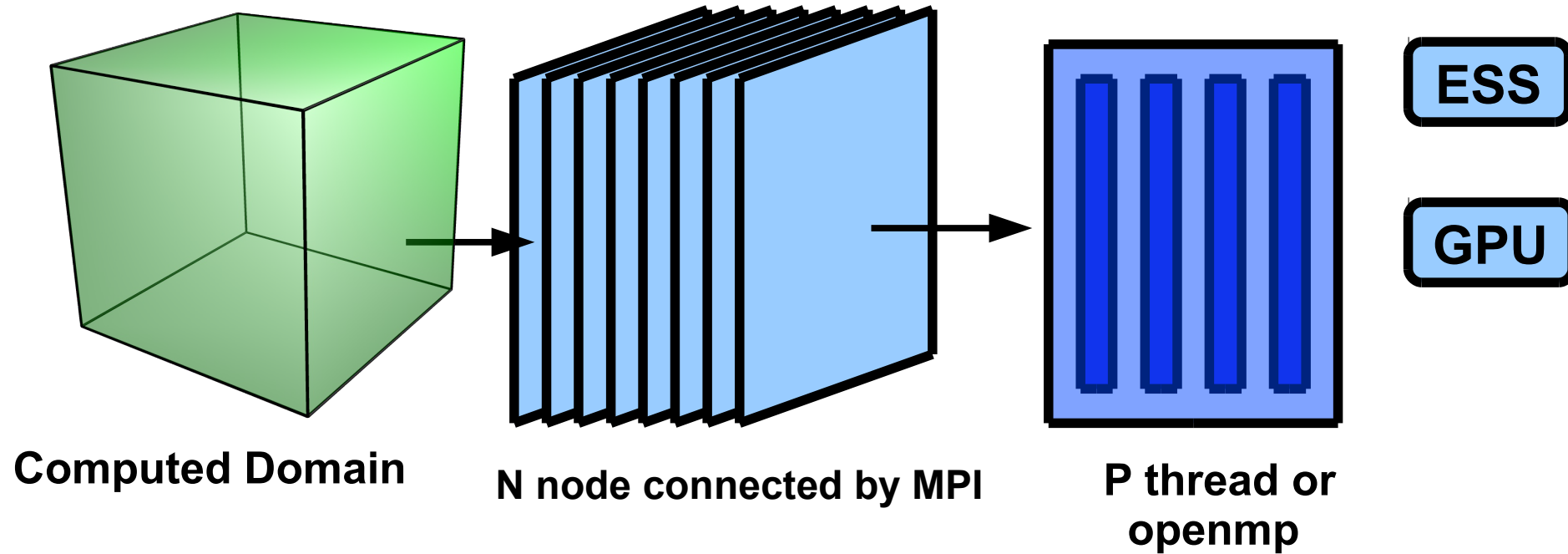
# FFT 3D → Transposition of the cube !

**maximum moving data algorithm**

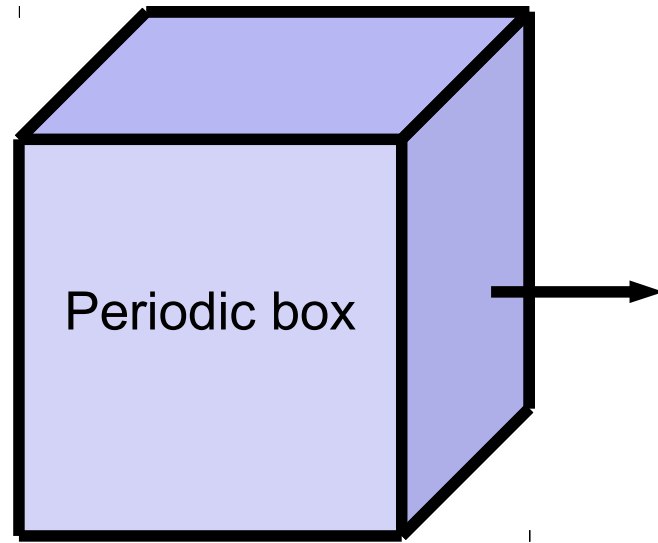


# Slab decomposition

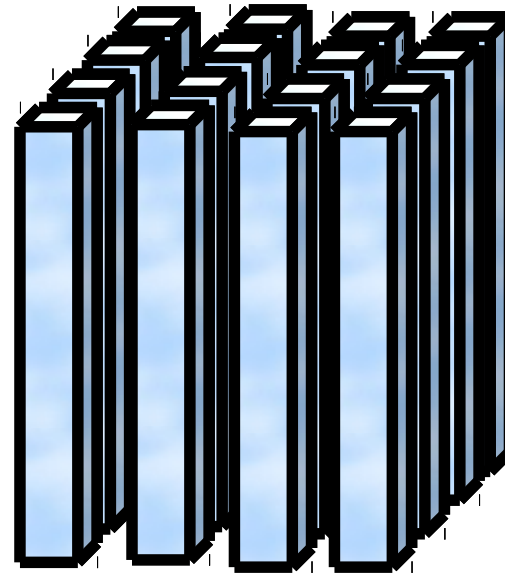
# vectorization



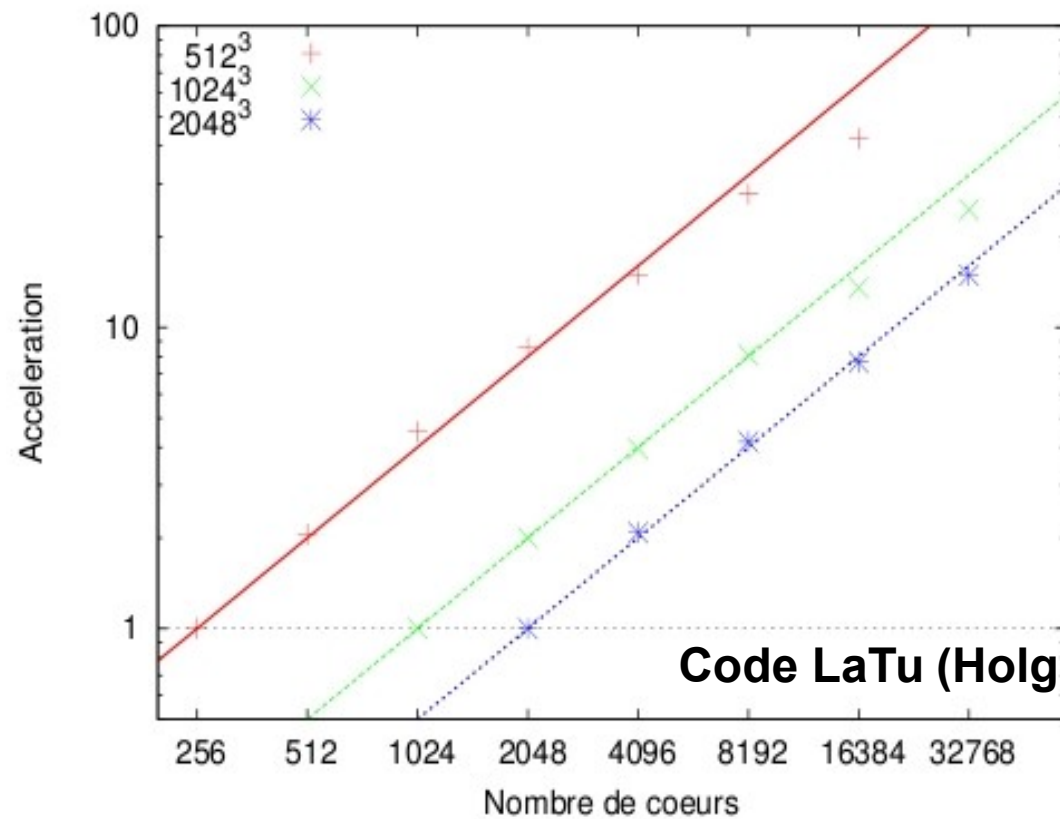
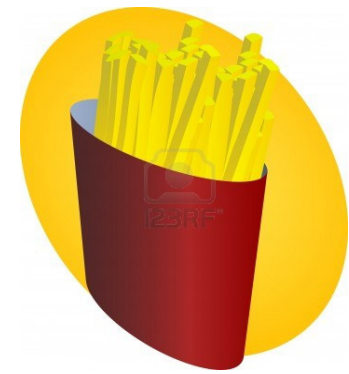
# French Fries configuration



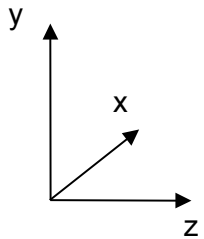
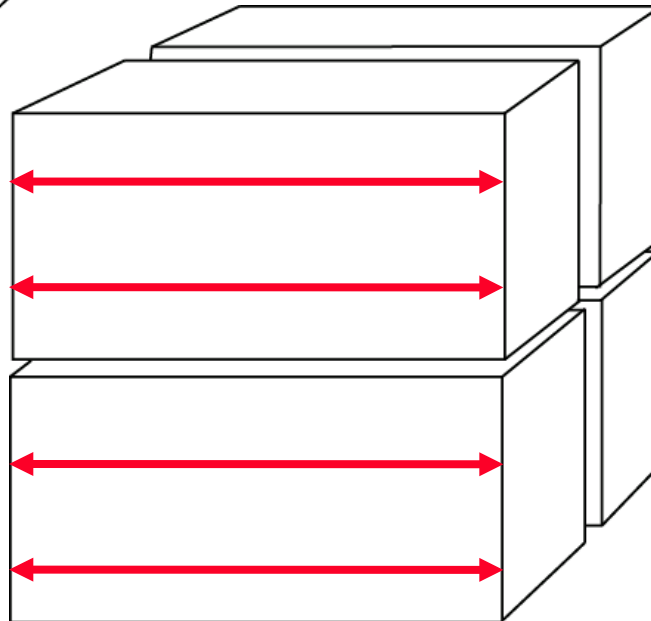
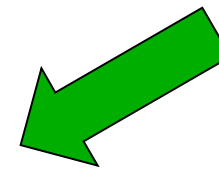
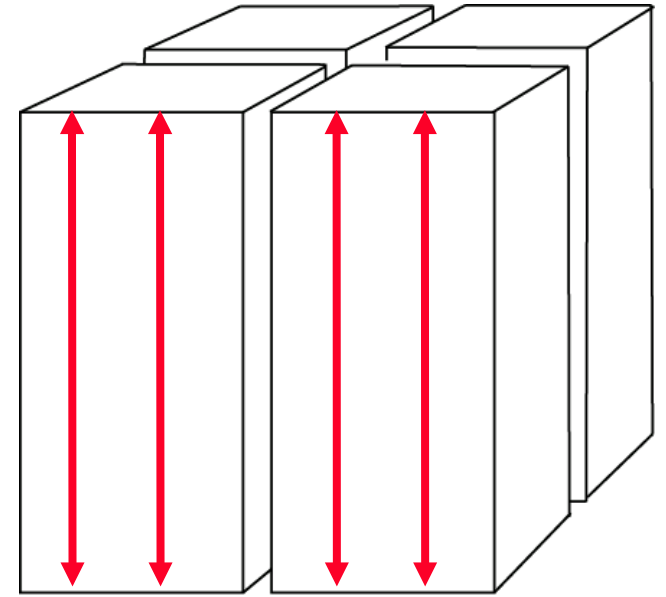
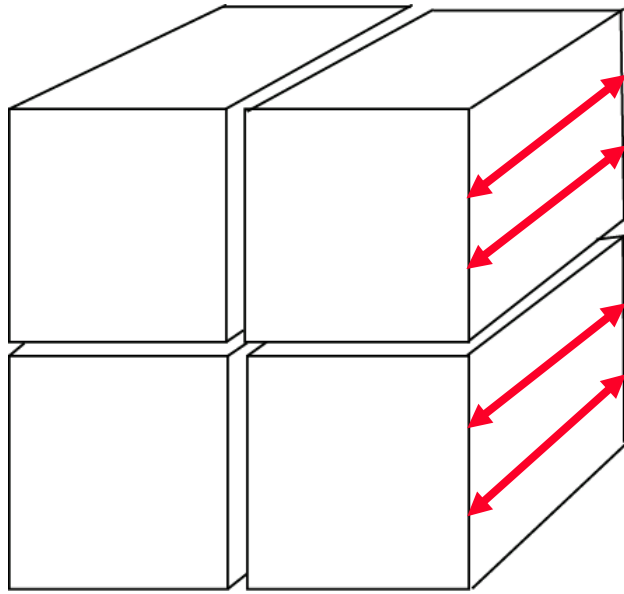
Computed Domain



Pencil strategy



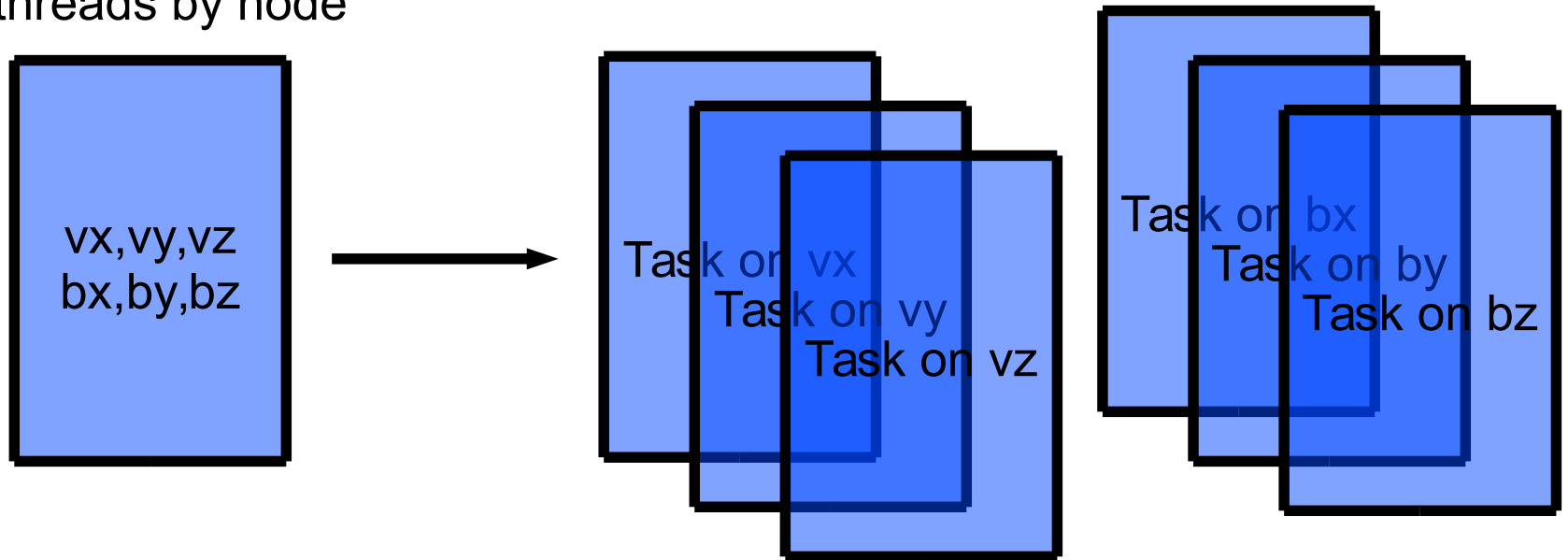
# 2D decomposition



**p3dfft library**

## MPI and Multi-Threading task strategies

6 threads by node

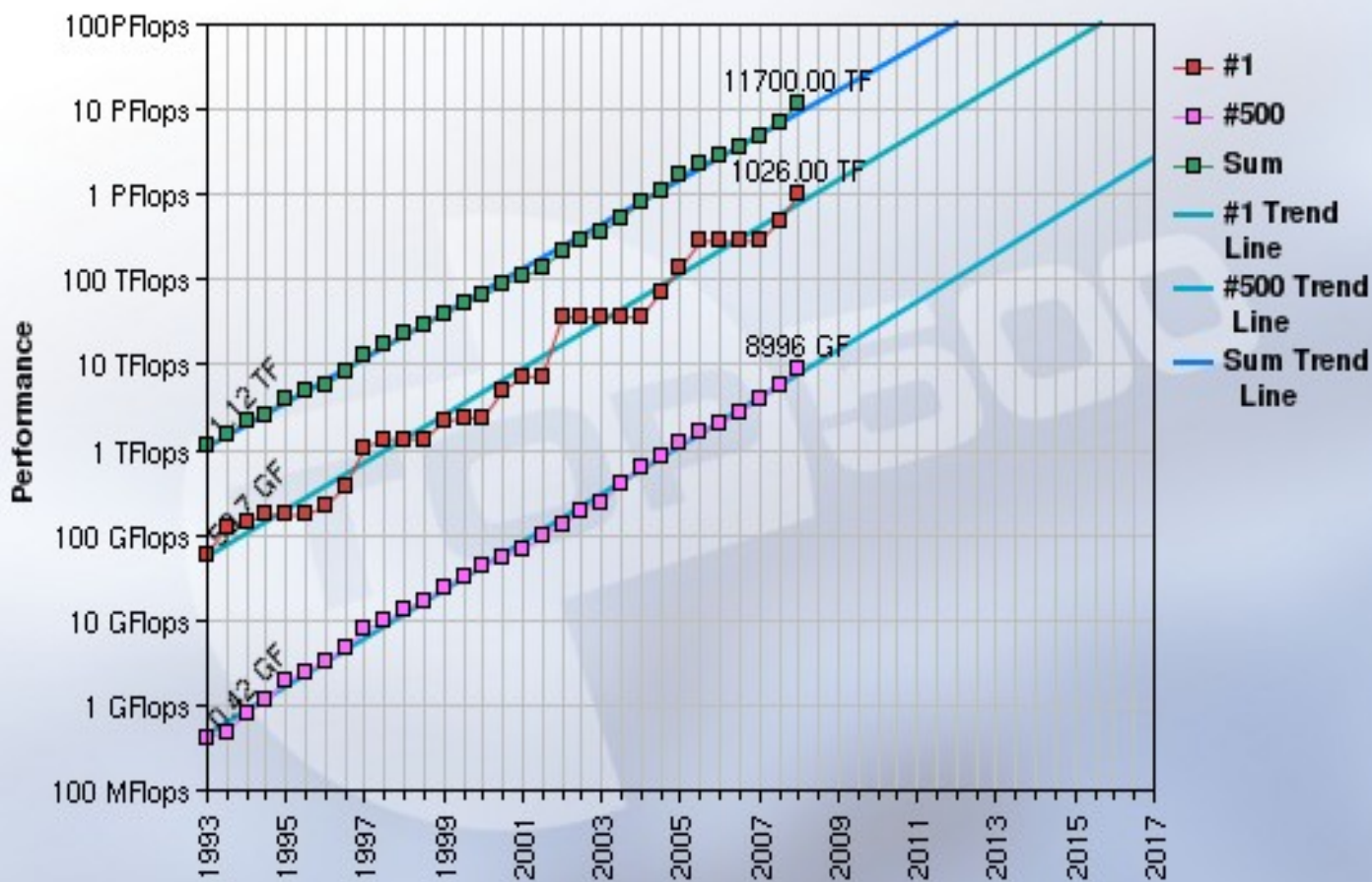


Strategy 2 → cut in 6 task, 1 task by thread

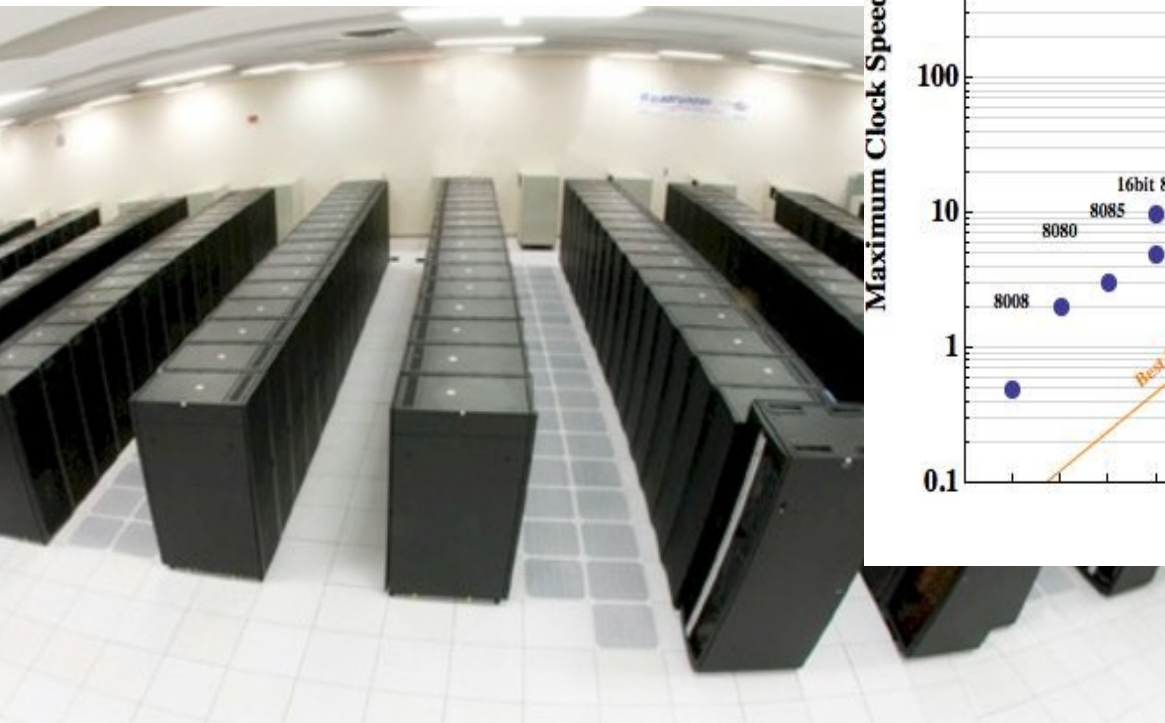
**Overlap : Computation and communication in the transposition !**



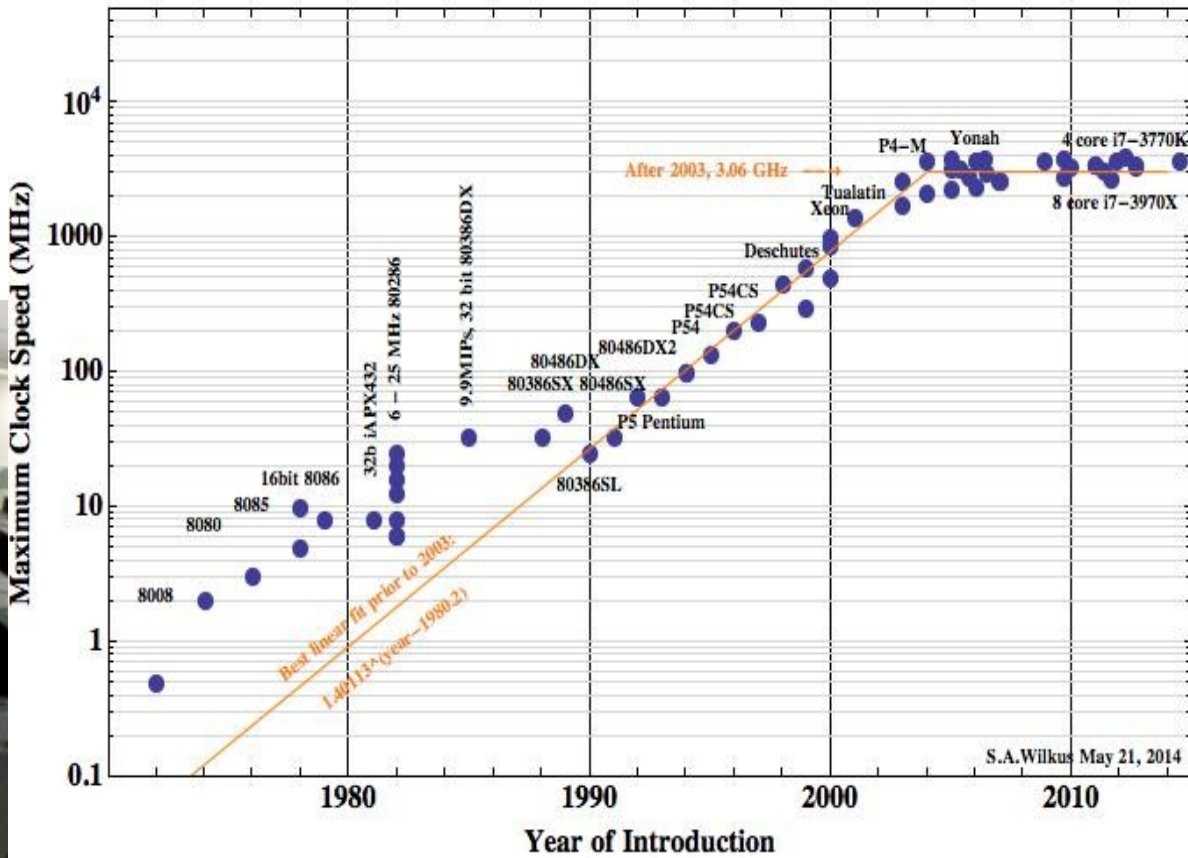
### Projected Performance Development



# Why many core ?

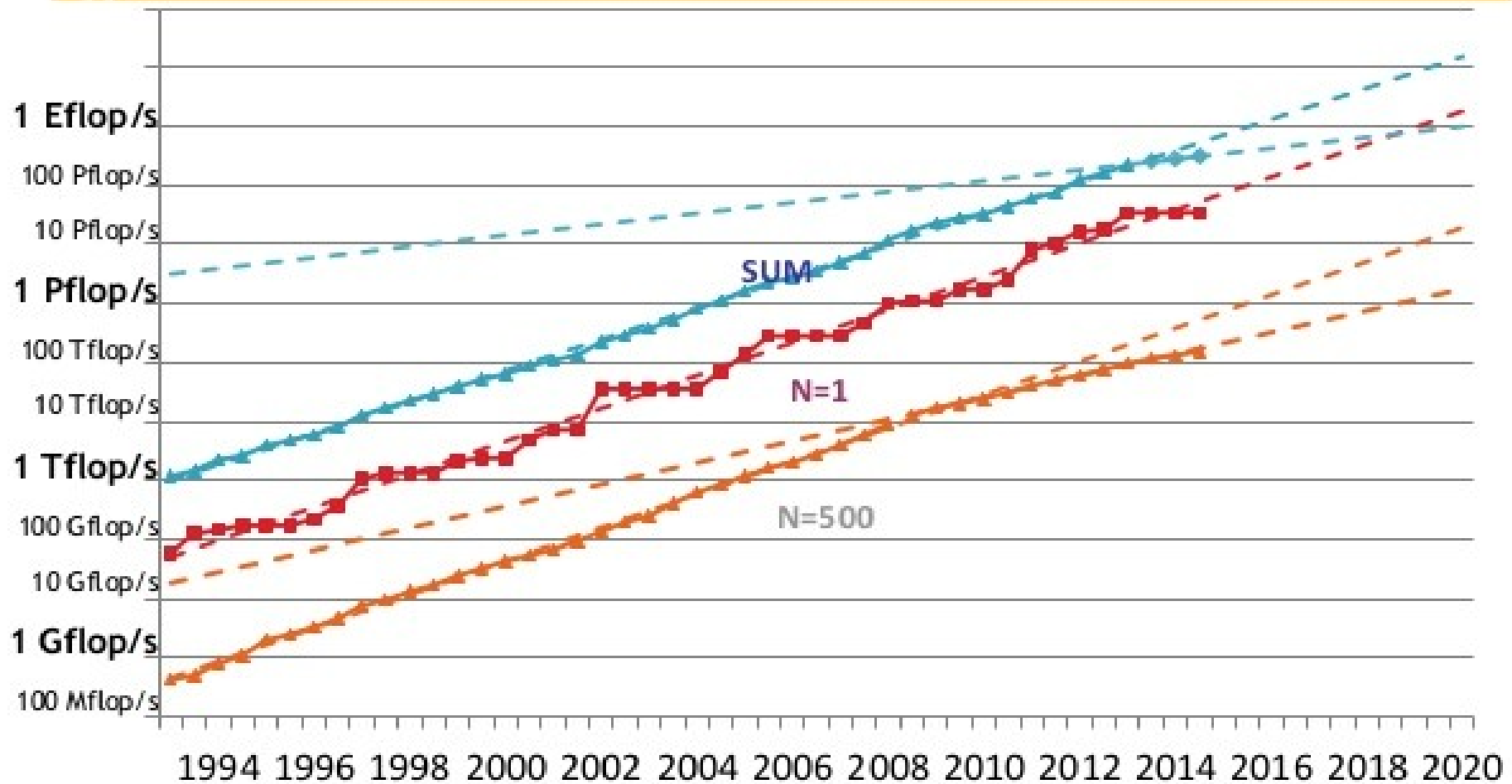


**μProcessor Clock Speed Trends**

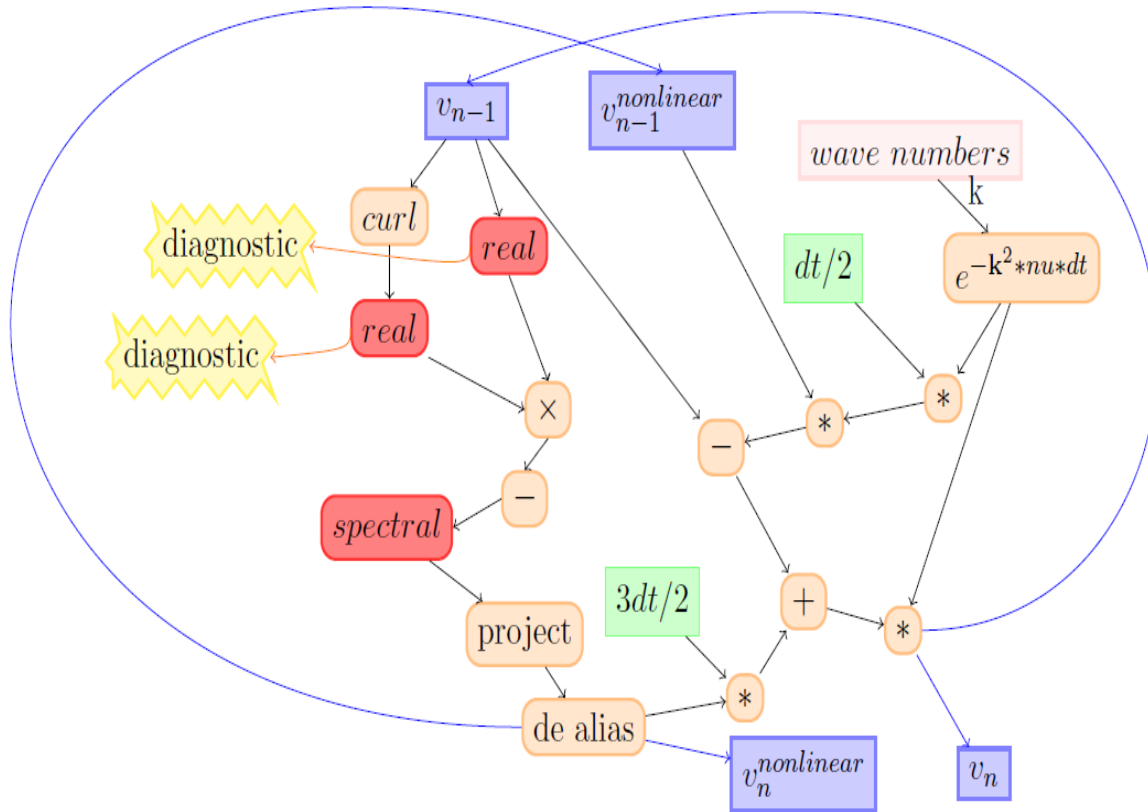




# PROJECTED PERFORMANCE DEVELOPMENT



# Dependence Tree



Code C++

High level library  
« for physicist »

Low level library  
Parallelisation  
MPI, thread, GPU

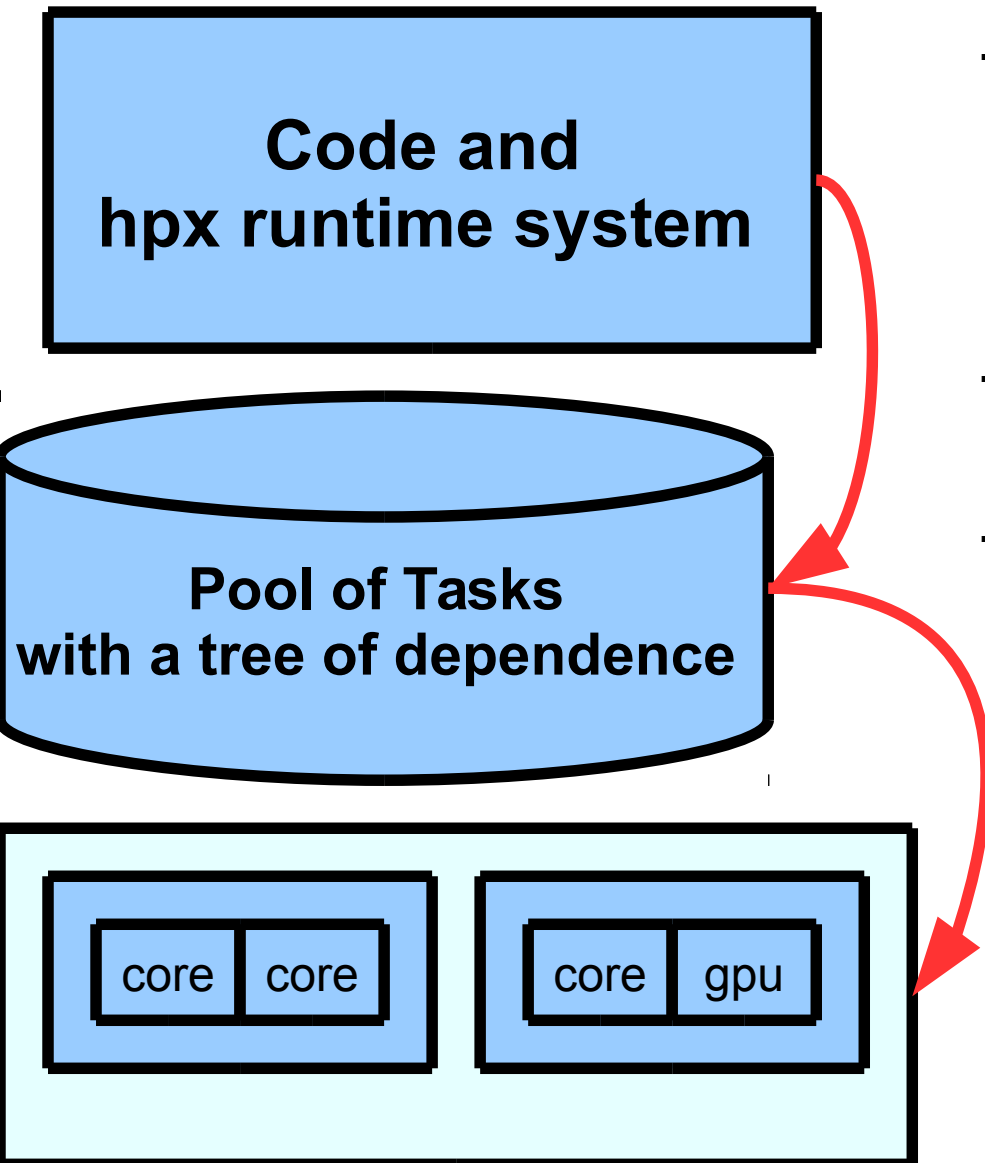
Tasks management

Figure 8: Basic dependence tree for a simplified simulation loop.

**Overlap : computation and communication !**

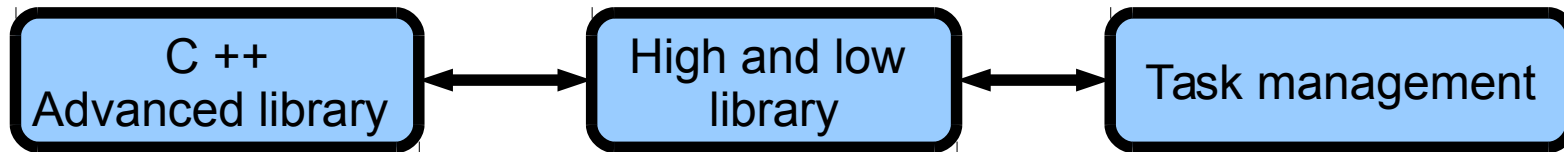
# (High Performance ParalleX)

c++ runtime system for parallel and distributed applications of any scale.  
It strives to provide a unified programming model which transparently utilizes the available resources to achieve unprecedented levels of scalability.



- Embrace Fine-grained Parallelism instead of Heavyweight Threads
- Rediscover Constrained Based Synchronization to replace Global Barriers
- Prefer Moving Work to the Data over Moving Data to the Work
- Favor Message Driven Computation over Message Passing

# Perspectives in parallelization



**Overlapping computation, communication,  
advanced diagnostics, IO, multiphysics**

**Challenge : Resolution of course**

**But also long run for thousand of eddy turn of time !**

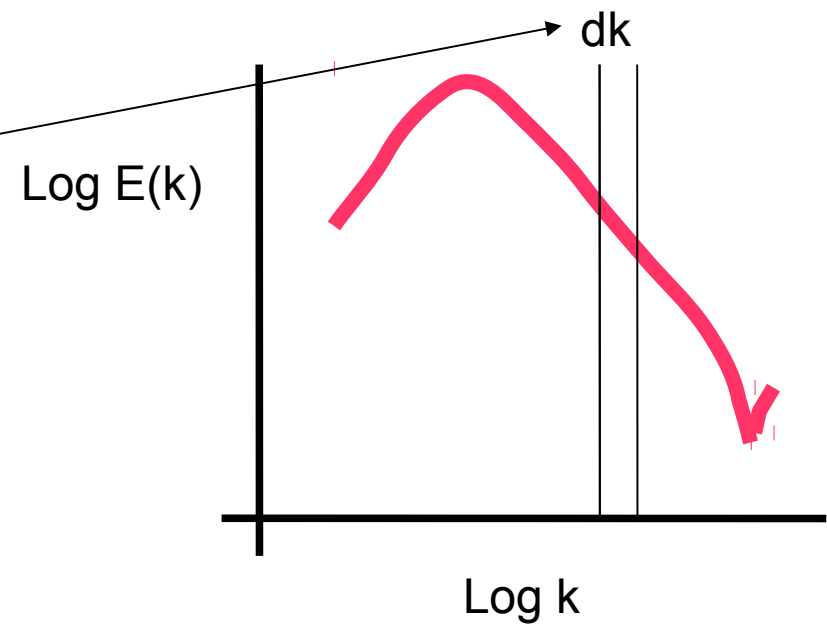
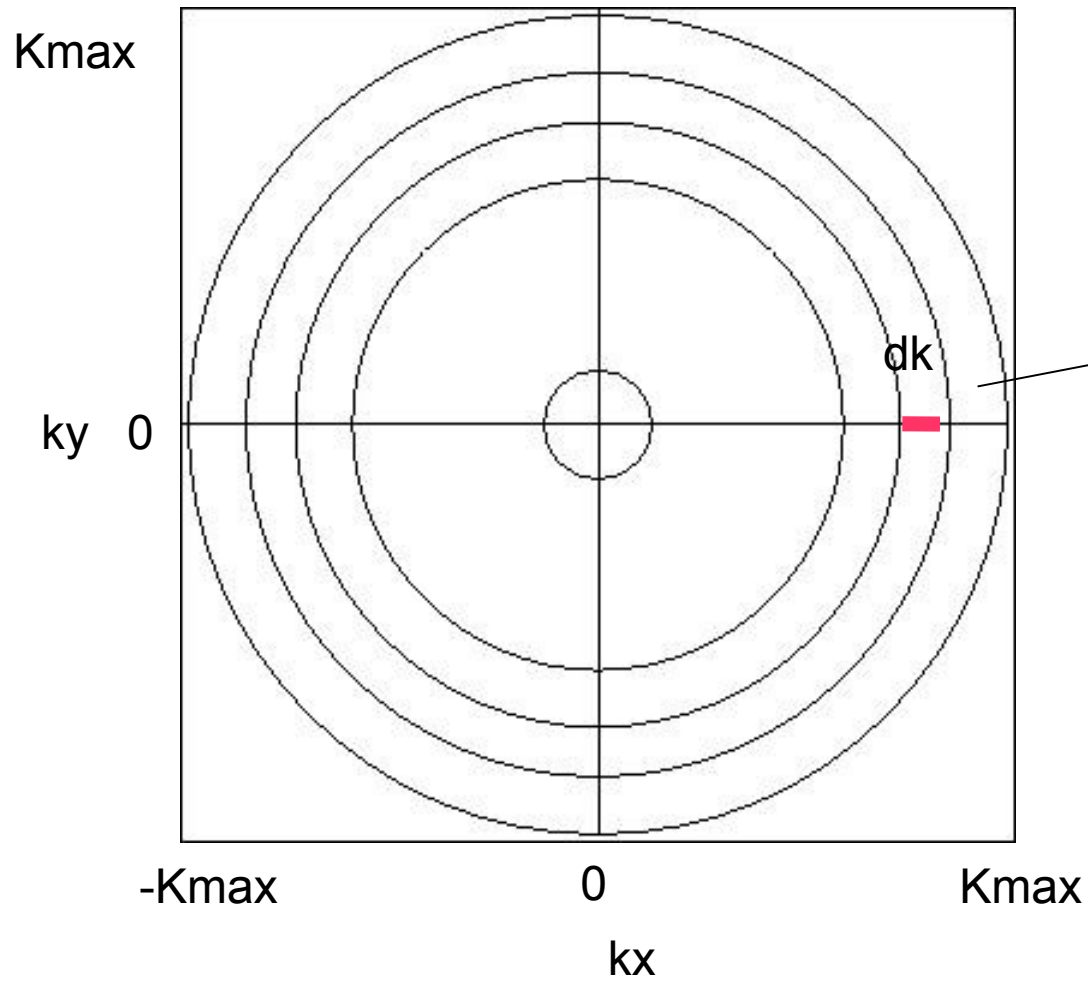
**Do Spectral methods are really for Exascale computer ?**

# Diagnostics in the

A glowing blue cube with intricate circuit patterns on its faces, set against a dark background with faint grid lines and light rays.

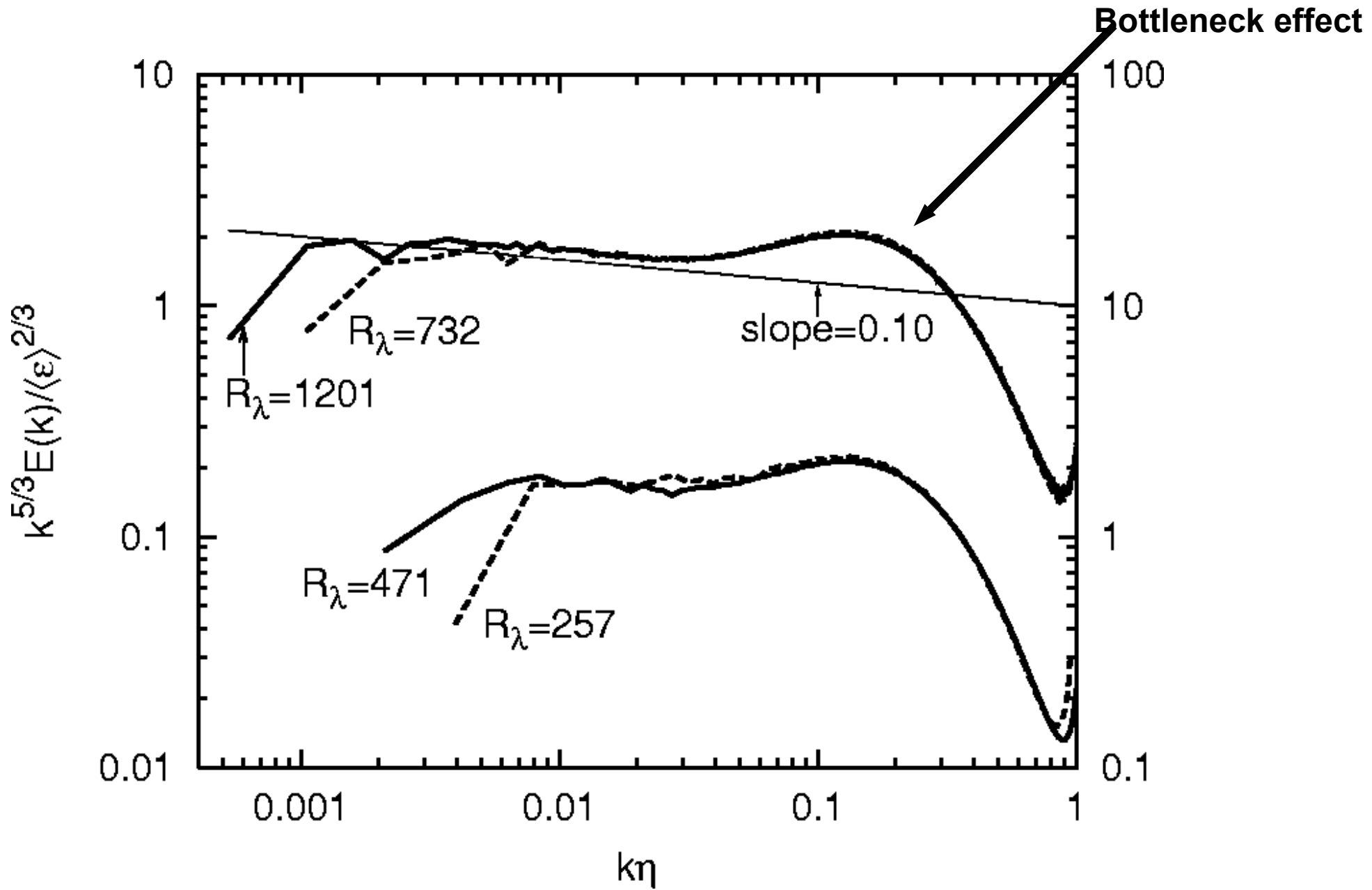
# CUBE

# 1 D spectra from 3D velocity field :

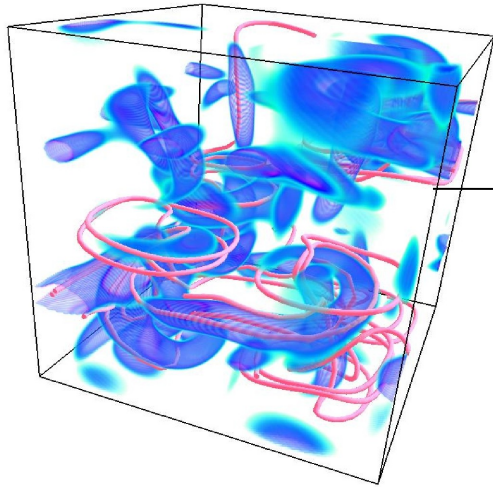


$$E = \sum E(k) = \sum \frac{1}{2} (\hat{u}_x(k)^2 + \hat{u}_y(k)^2 + \hat{u}_z(k)^2) dk$$

$$L_i = \frac{\sum E(k) / k}{\sum E(k)}$$



# Numerical quantities output and experimental observable

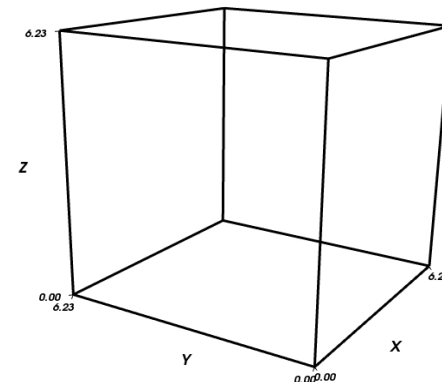


1 D spectra (u, b, w, j, ....)  
 Snapshot 3D fields -> 3D visualization  
**Probes -> time signal**  
**Average in time fields**

Numerical Experiment output :

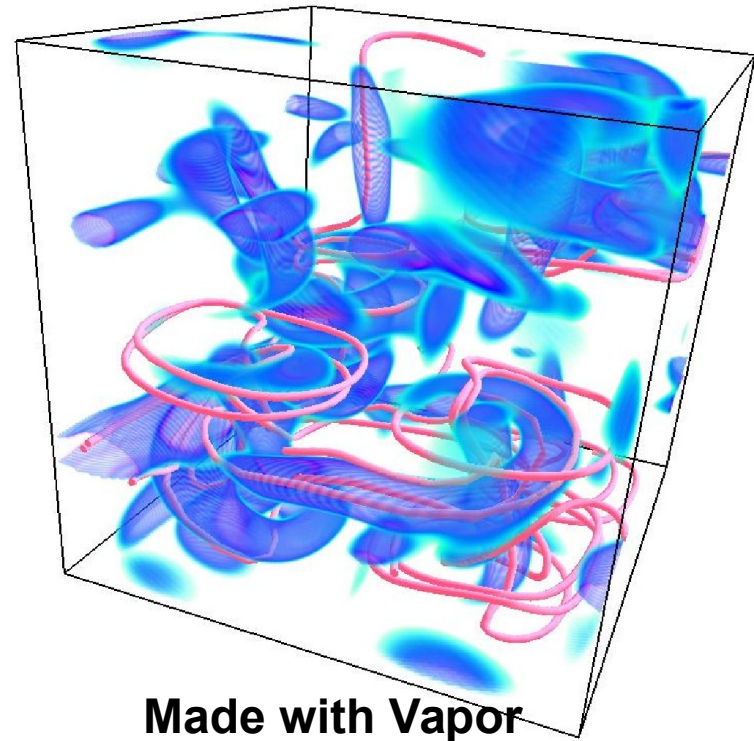
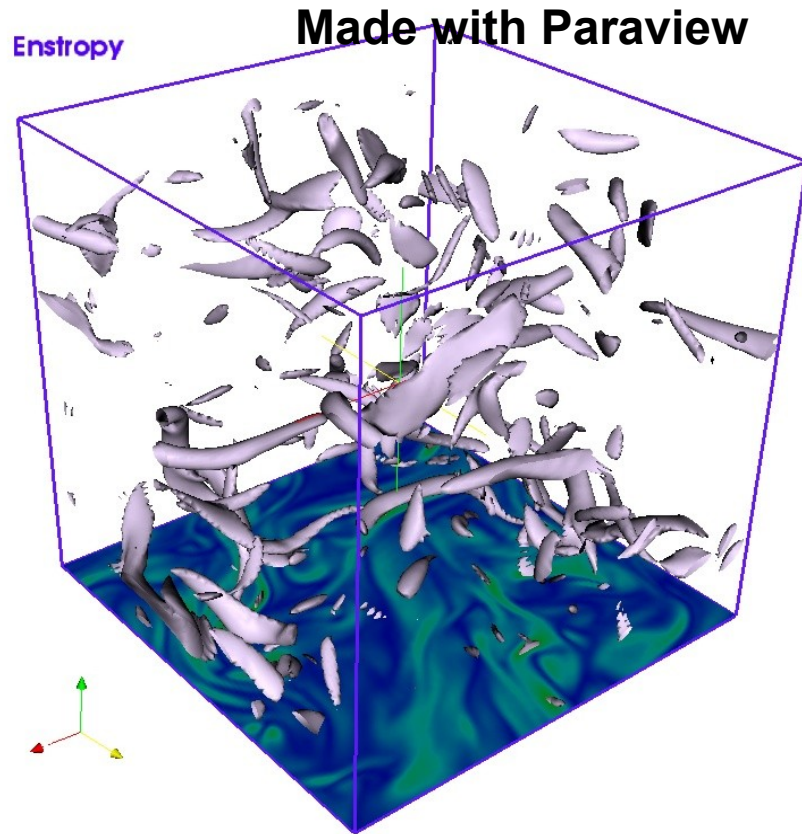
$$\int \vec{u}_{Rv}(\vec{r}, \tau) d\tau \rightarrow \langle \vec{U}(\vec{r}) \rangle \quad \text{Average in time :} \quad \sum \hat{u}_i(\vec{x}, t_n) = \hat{u}_i(\vec{x})$$

Probes : fix location , line -> signal





# visualisation 3D



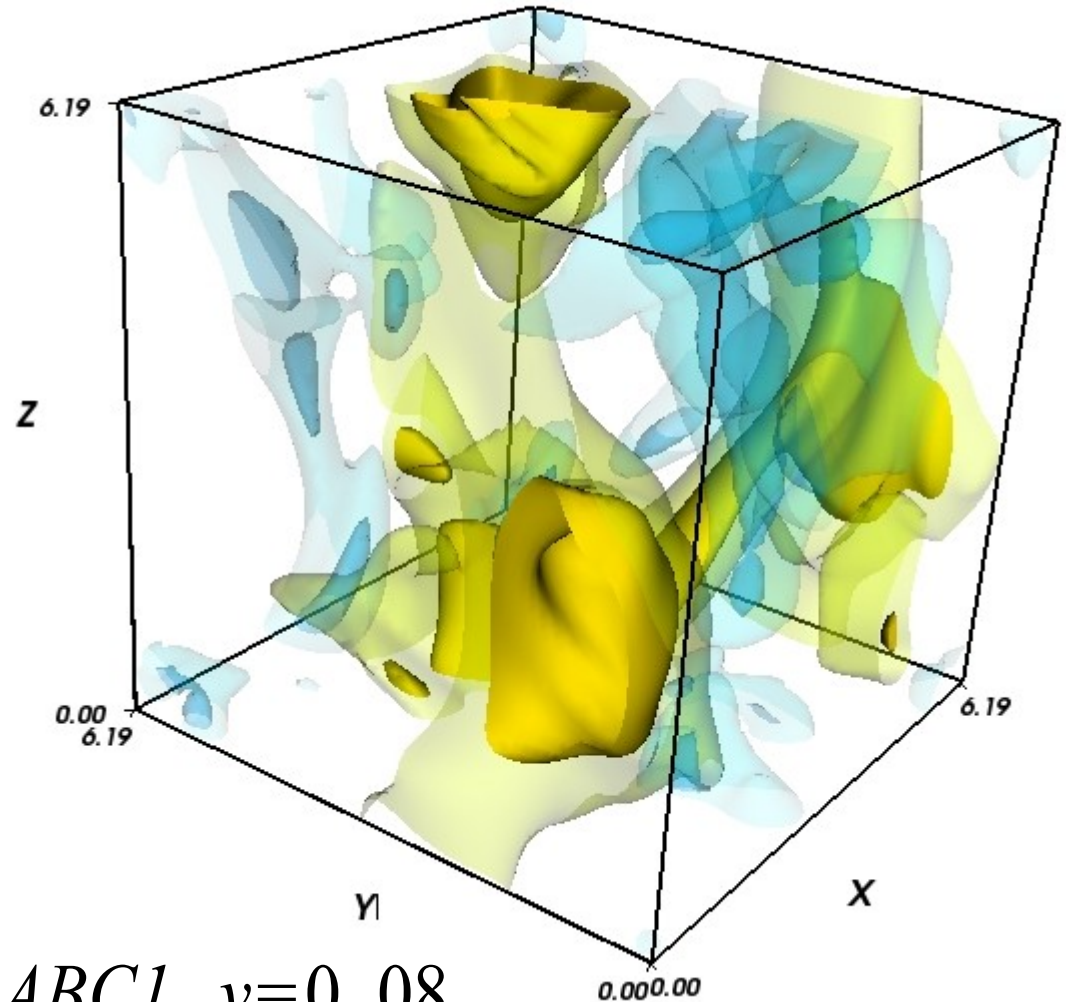
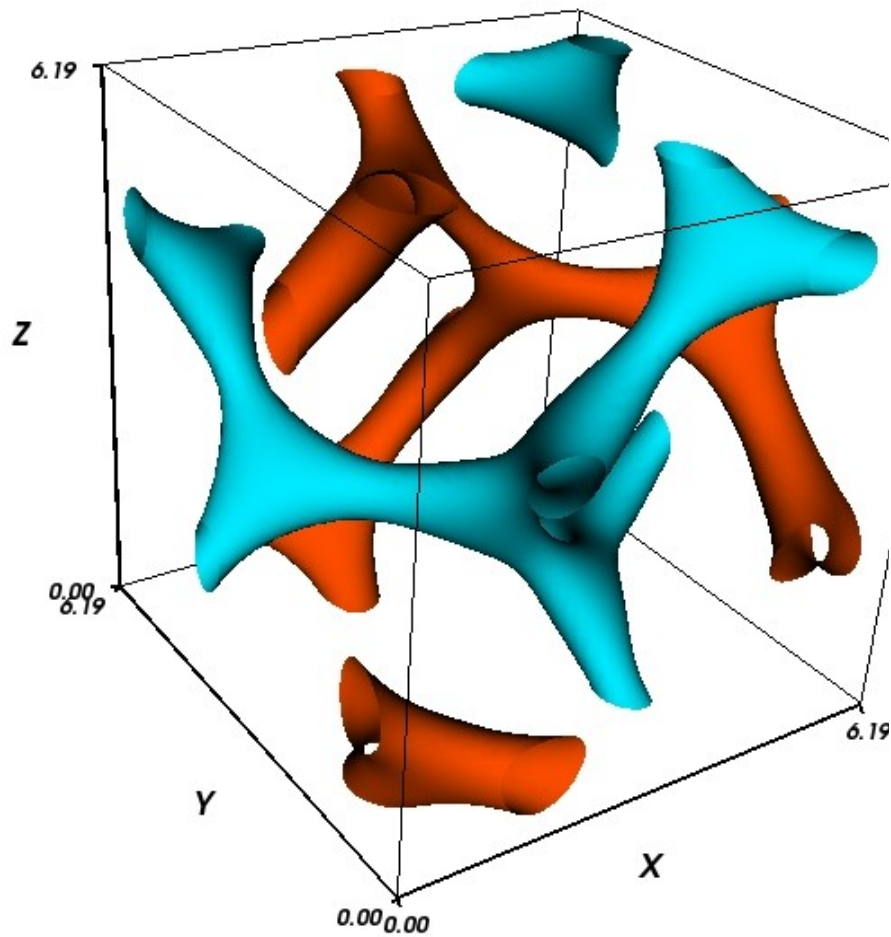
[vapor.ucar.edu](http://vapor.ucar.edu)  
[www.paraview.org](http://www.paraview.org)

Large data set : wavelet decomposition  
volume rendering, field line, periodic box  
spherical geometry, connection with IDL

# Average in time

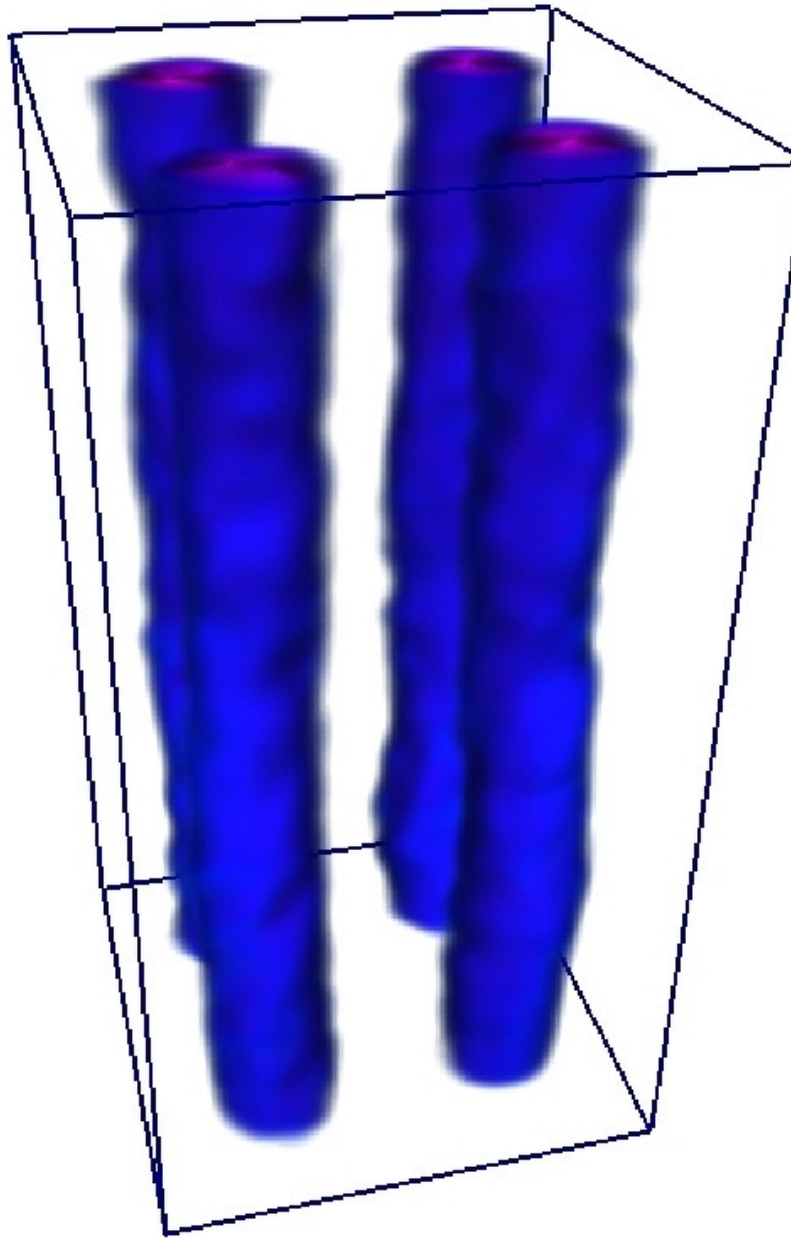
$$\int \vec{u}_{Rv}(\vec{r}, \tau) d\tau \rightarrow \langle \vec{U}(\vec{r}) \rangle$$

$$\text{Average in time : } \sum \hat{u}_i(\vec{x}, t_n) = \hat{u}_i(\vec{x})$$

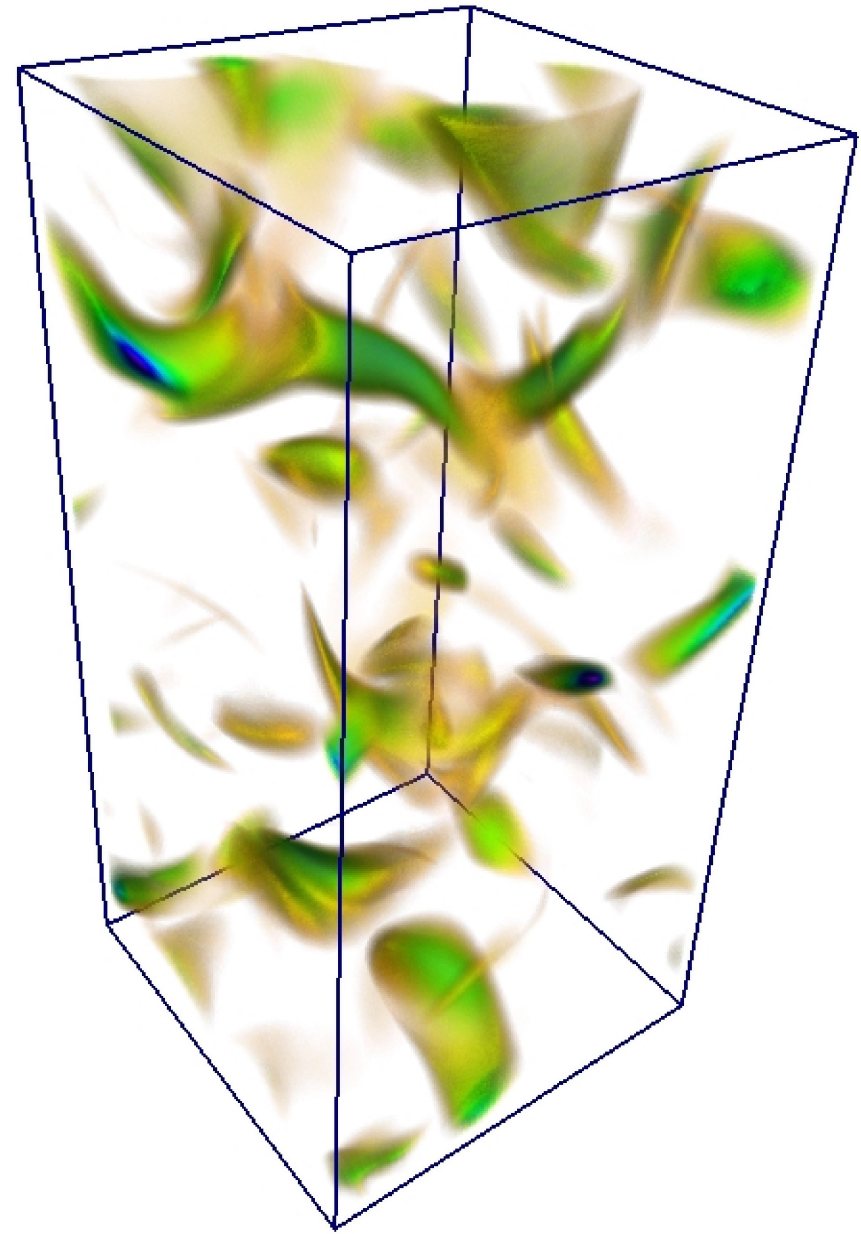


$$F=ABC1 \quad v=0.08$$

# G. O Robert Forcing

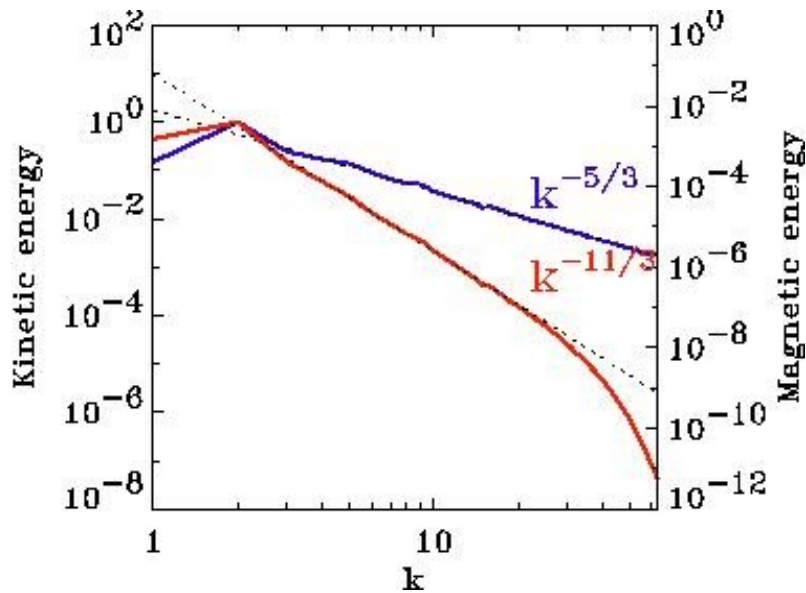


**Average in time**



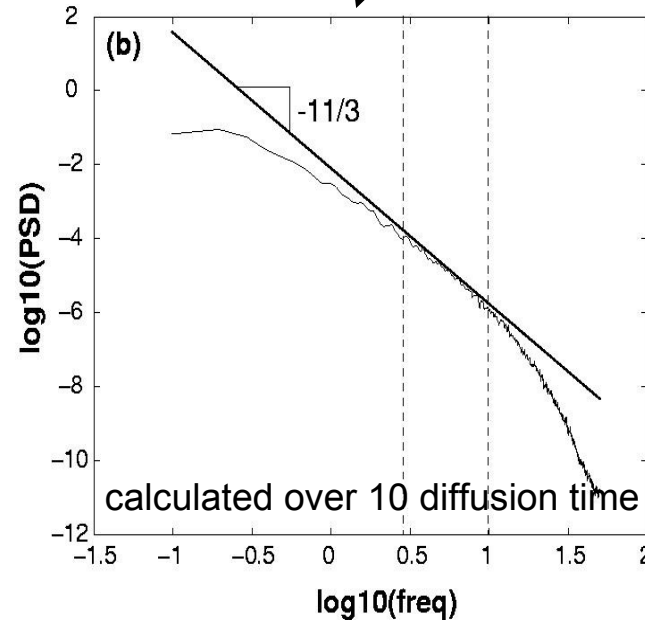
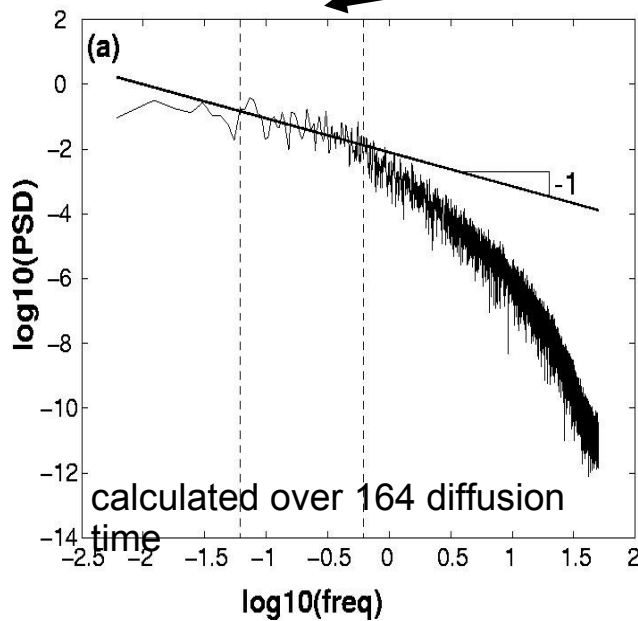
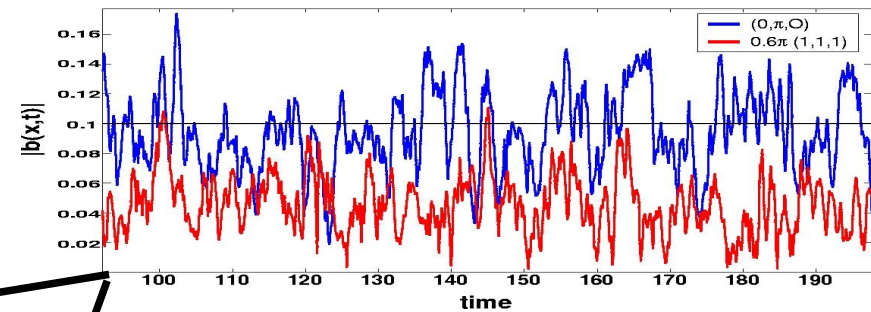
**Snapshot**

# Magnetic induction results Y.Ponty, H. Politano & J-F Pinton , *Phys. Rev. Lett.* 92 (2004)



$$P_m \sim 10^{-3} - 10^{-4}$$

Probes



Power spectra of the temporal fluctuations filtered by an Hanning window box

The 1/f power law has been found in the most of the experimental MHD measurements.

# Eulerian & Lagrangian perspective

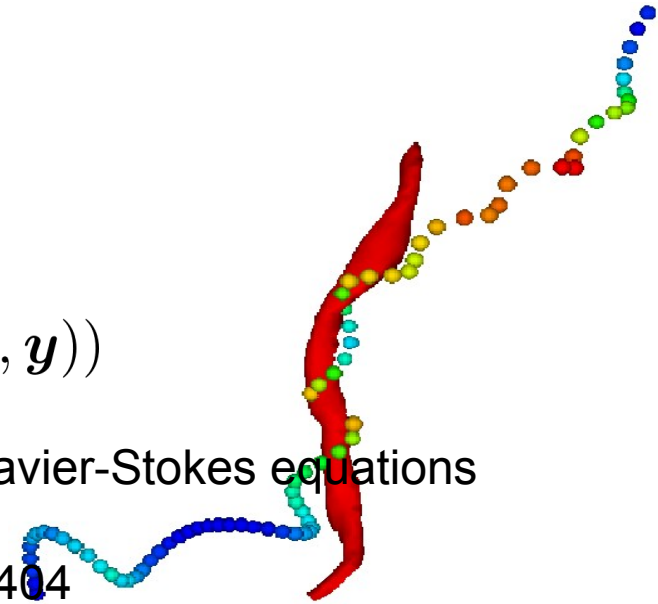
- How to define now the three different regimes?
- What do the fluid trajectories look like?
- Where does the magnetic field grow ?

## Frame of reference

- Euler = Laboratory
- Lagrange = Particle

$$\frac{d\mathbf{X}(t, \mathbf{y})}{dt} = \mathbf{v}(\mathbf{X}(t, \mathbf{y}))$$

$\mathbf{v}(\mathbf{X}(t, \mathbf{y}))$  by Navier-Stokes equations



## Fluid Turbulence

- Toschi F and Bodenschatz E **2009** Ann. Rev. Fluid Mech. 41 375–404  
Biferale Let al 2004 Phys. Rev. Lett. 93 4502  
Yeung P K and Sawford M S **2006** J. Turbulence 7 1–12  
Bec J et al **2006** J. Fluid Mech. 550 349–58  
Yeung P K and Borgas M S **2004** J. Fluid Mech. 503 93–124

## MHD Turbulence

- Homann H et al R **2009** Phys. Plasma 16 082308  
Homann H et al **2007** J. Plasma Phys. 73 821–30  
Homann H et al **2009** New J. Phys. 11 073020

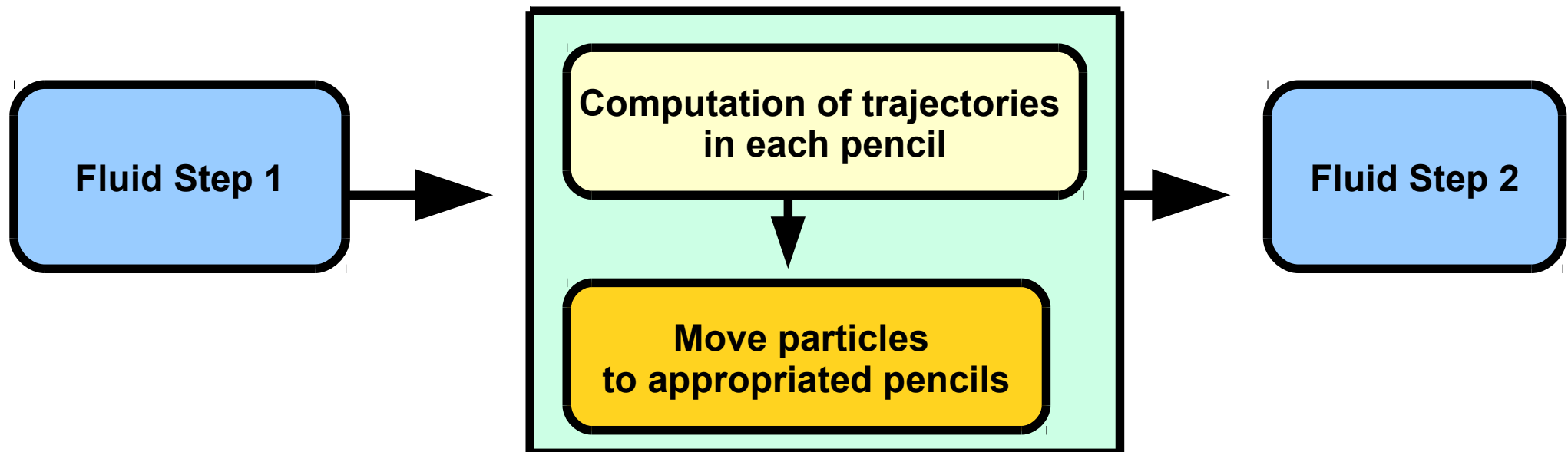
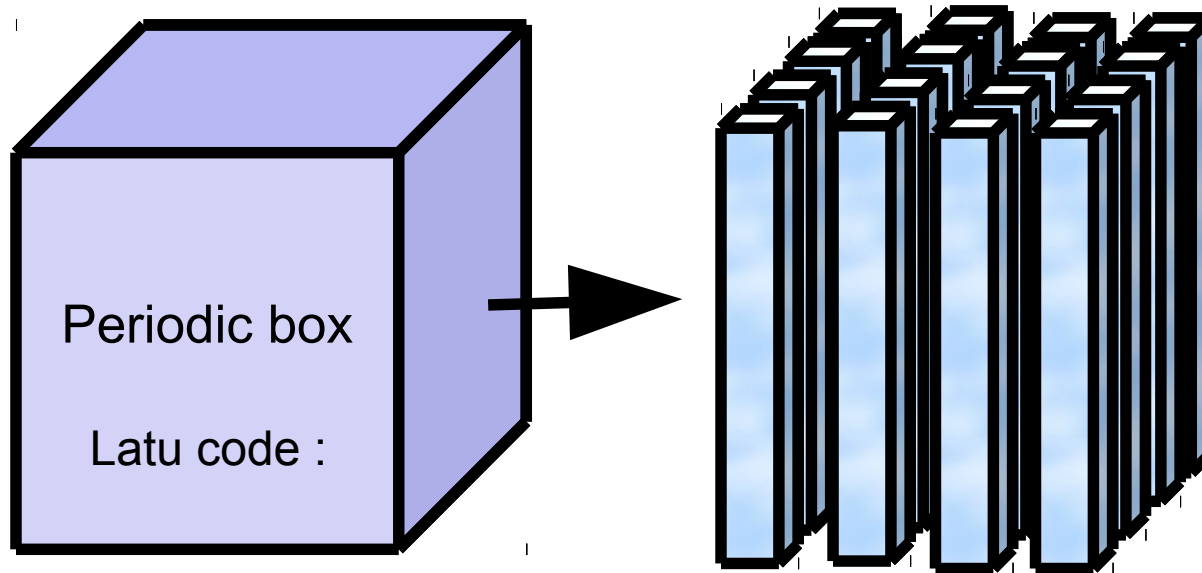
## Dynamo :

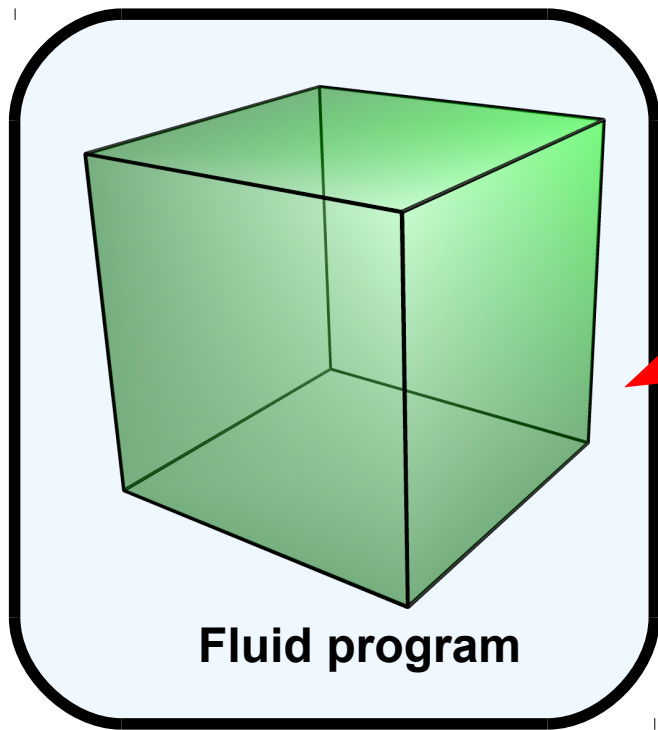
**Holger Homann, Yannick Ponty, Giorgio Krstulovic, Rainer Grauer**  
"Structures and Lagrangian statistics of the Taylor-Green Dynamo"

*New J. Phys.* **16** 075014 (2014)

doi:10.1088/1367-2630/16/7/075014

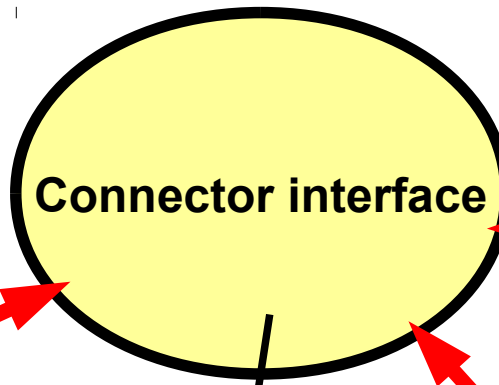
# Parallel strategy for the tracers & particles



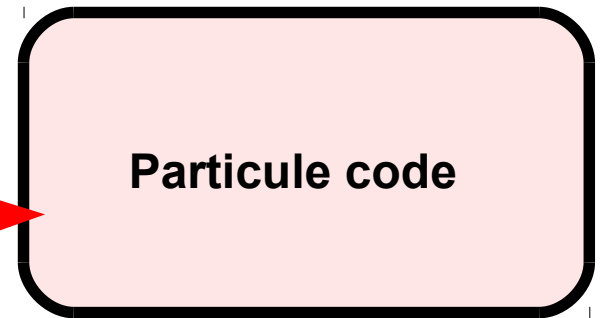


**Fluid program**

**Sub MPI space 1**

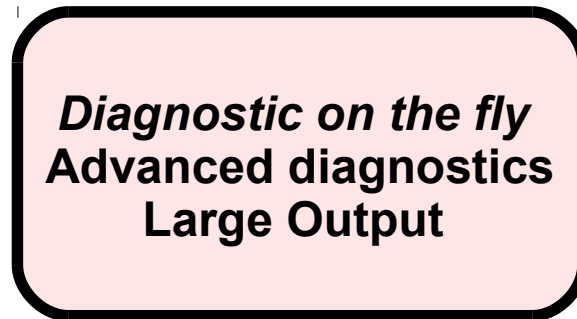


**Connector interface**



**Particle code**

**Sub MPI space 2**



*Diagnostic on the fly*  
**Advanced diagnostics**  
**Large Output**

**Sub MPI space 3**



**Heavy 3D IO**

**Sub MPI space 4**

**Global MPI communicator**

A blue-tinted image of a Rubik's Cube, possibly a 4x4 or 5x5 version, with a glowing light effect emanating from it. The background is dark with a grid pattern.

**Are we able  
to escape from the**

**CUBE**



# Classic Penalization method

$$\partial_t \vec{u} + \vec{u} \nabla \vec{u} = -\nabla P + \nu \Delta \vec{u} + \vec{j} \times \vec{b} - C \xi(\vec{x}) \vec{u}$$
$$\nabla \cdot \vec{u} = 0$$

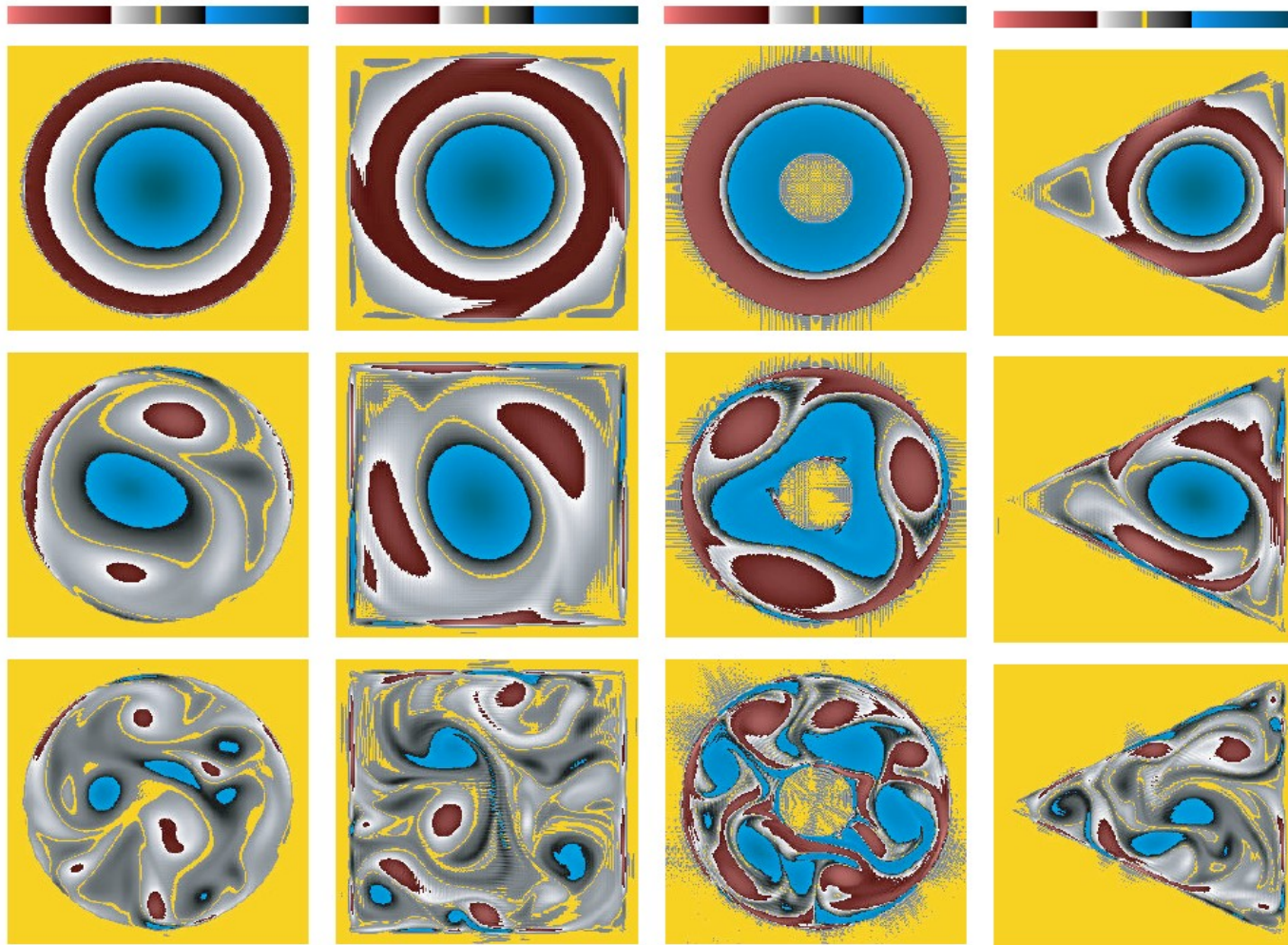
$\xi(\vec{x}) = 1$  *inside boundary*

$\xi(\vec{x}) = 0$  *outside boundary*

Kai Schneider<sup>1</sup> and Marie Farge<sup>2</sup>

Computational Physics and New Perspectives in Turbulence  
Y. Kaneda (Ed.)

Springer, 2007, pp. 241-246



# Pseudo-Penalization method

**M. Minguez, R. Pasquetti and E. Serre**

“A pseudo-penalization method for high Reynolds unsteady flows”

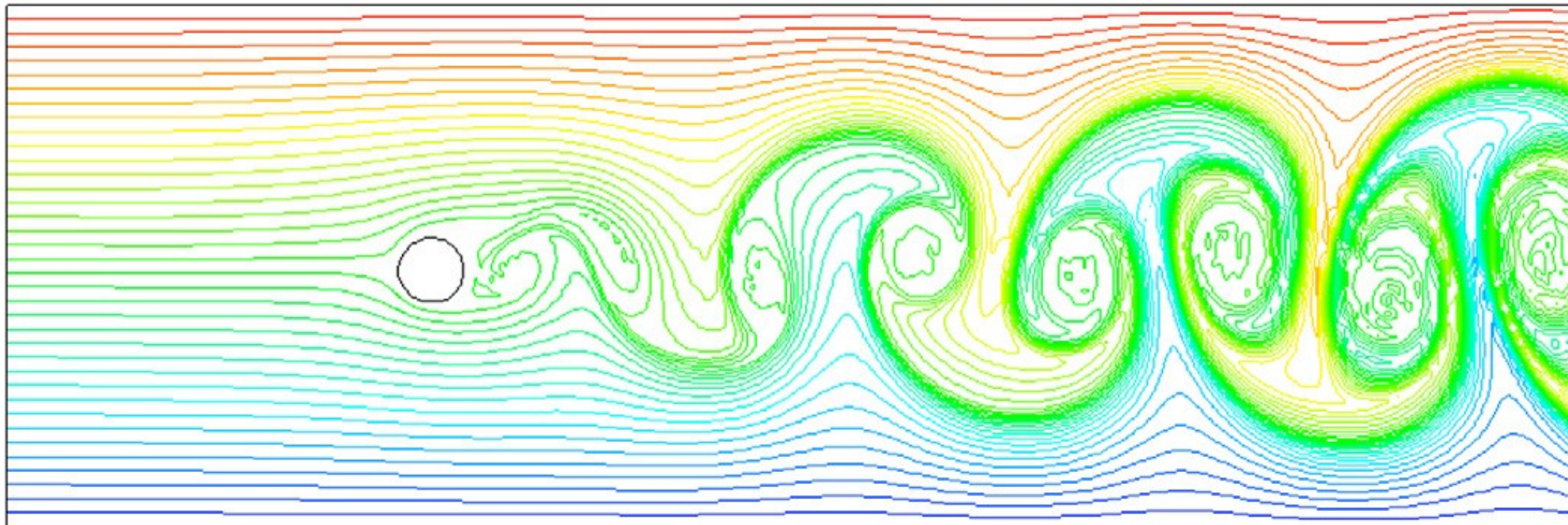
*Applied Numerical Mathematics*

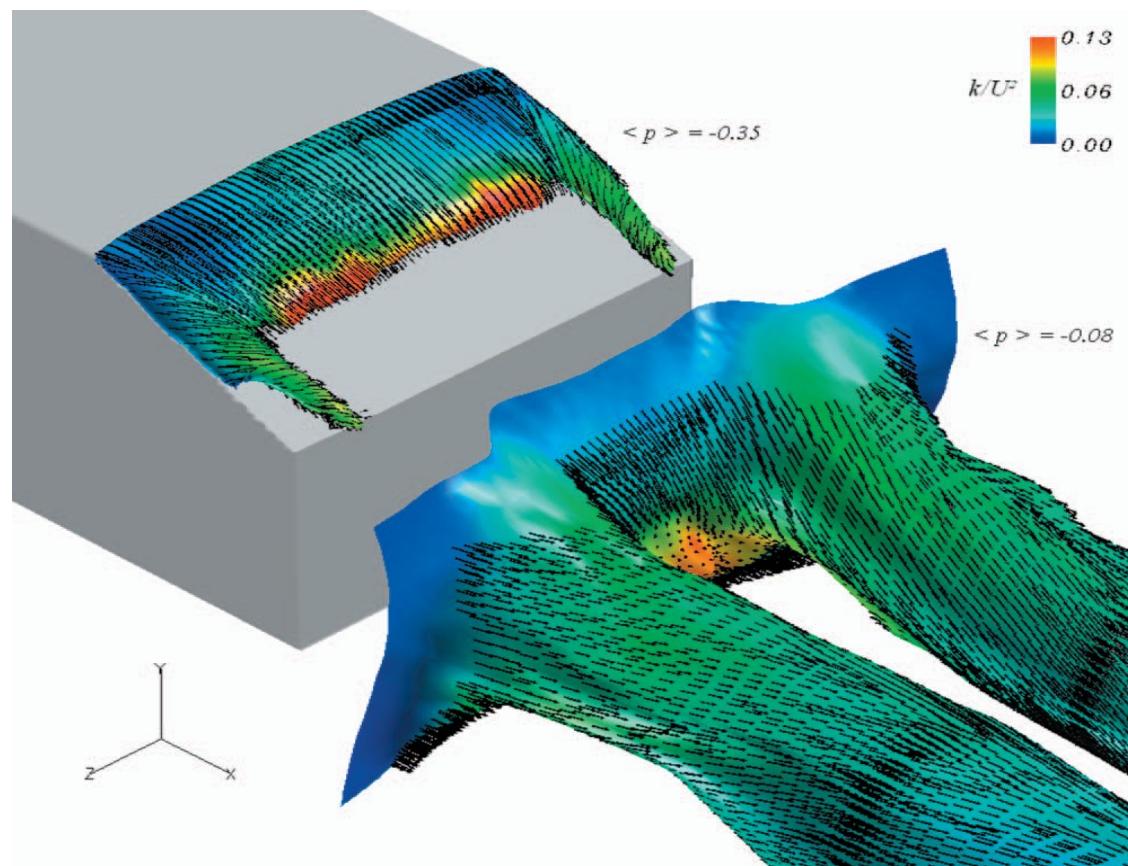
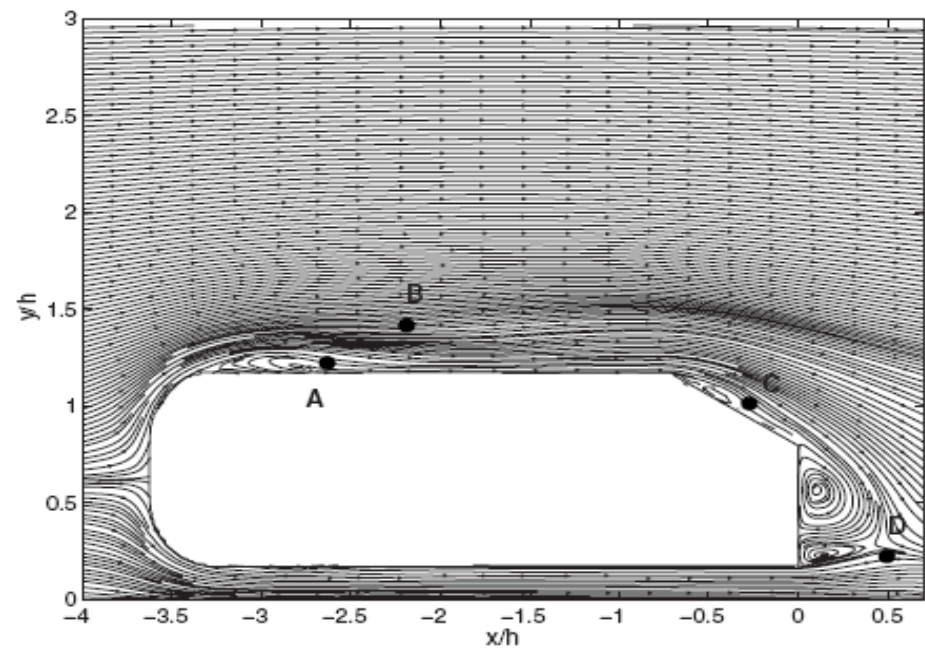
Volume **58**, Issue 7, **July 2008**, Pages 946-954

$$\nu \Delta \mathbf{u}^{n+1} - \frac{\alpha}{\tau} \mathbf{u}^{n+1} - \nabla p^{n+1} = (1 - \chi) f^{n+1} \quad \text{in } \Omega,$$

$$\nabla \cdot \mathbf{u}^{n+1} = 0,$$

$$B(\mathbf{u}^{n+1}) = g^{n+1} \quad \text{on } \Gamma,$$





# Penalisation method : Direct forcing

E. A Fadlum et al JCP 161, 35–60 (2000)

$$\mathbf{u}^{(n+1)} = \mathbf{u}^{(n)} + \Delta t \cdot (\mathcal{L}(\mathbf{u}^{(n)}) + \mathbf{f}_b^{(n)})$$

$$\mathbf{f}_b^{(n)} = -\chi_p(\mathbf{r}, t) \left\{ \mathcal{L}(\mathbf{u}^{(n)}) + \frac{1}{\Delta t} (\mathbf{u}^{(n)} - \mathbf{V}^{(n+1)}) \right\}$$

J. Mohd-Yusof,  
CTR Annual Research Briefs,  
NASA Ames/Stanford University, (1997).

## Volume Fraction method :

$$\bar{\chi}_p(\mathbf{r}) = \frac{1}{2^3 \Delta x \Delta y \Delta z} \int_{-\Delta x}^{\Delta x} \int_{-\Delta y}^{\Delta y} \int_{-\Delta z}^{\Delta z} \chi_p(\mathbf{r} + \mathbf{r}^*) d^3 r^*$$

E. A Fadlum et al JCP 161, 35–60 (2000)

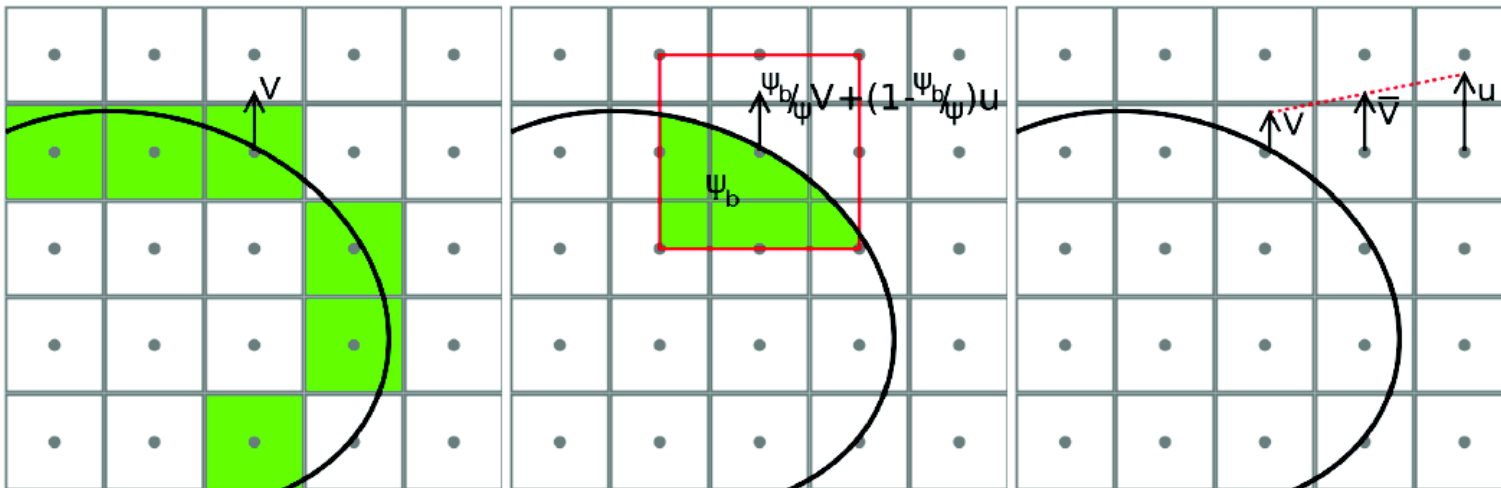


Fig. 3.2.: Stepwise interpolation, volume fraction method and linear interpolation

## The pressure predictor

Brown, D. L., Cortez, R. & Minion, M. L. Accurate Projection Methods for the Incompressible Navier Stokes Equations. J. Comp. Phys. 168 (2), 464–499 (2001).

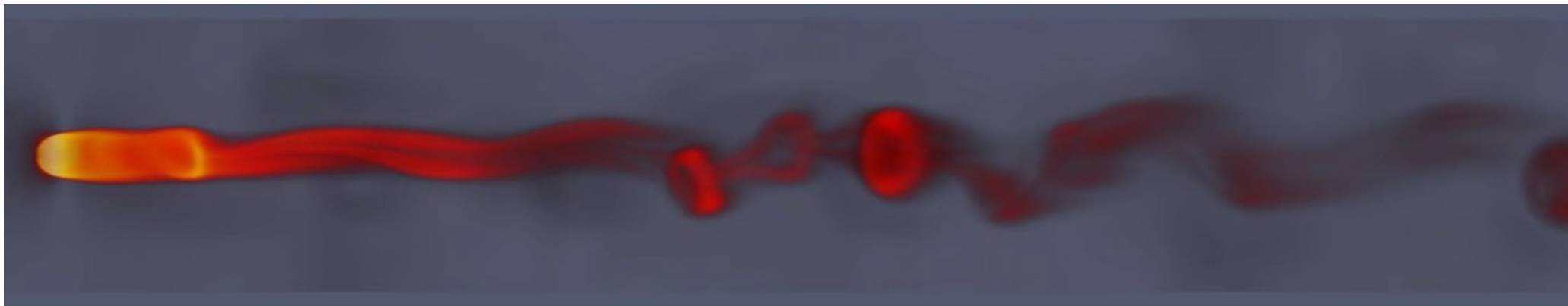
$$\tilde{\mathbf{u}}^{(n+1)} = \mathcal{B}\tilde{\mathbf{u}}^{(n+1)} = \tilde{\mathbf{u}}^{(n+1)} + \mathbf{f}_k^n .$$

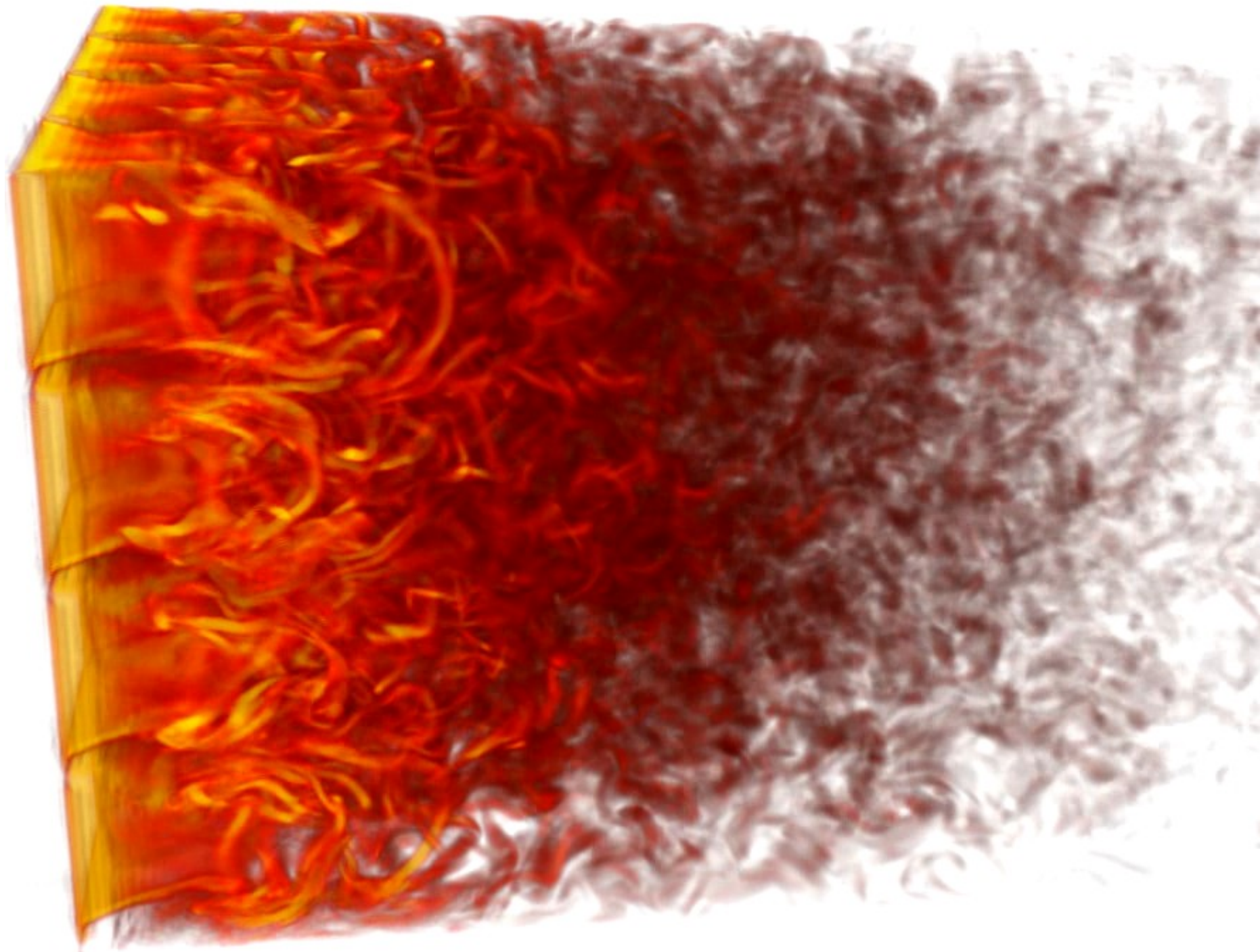
$$p^n = p^{n-1} + \Phi^n , \text{ where } \Delta\Phi^n = \frac{1}{\Delta t} \nabla \cdot \tilde{\mathbf{u}}^{(n+1)} .$$

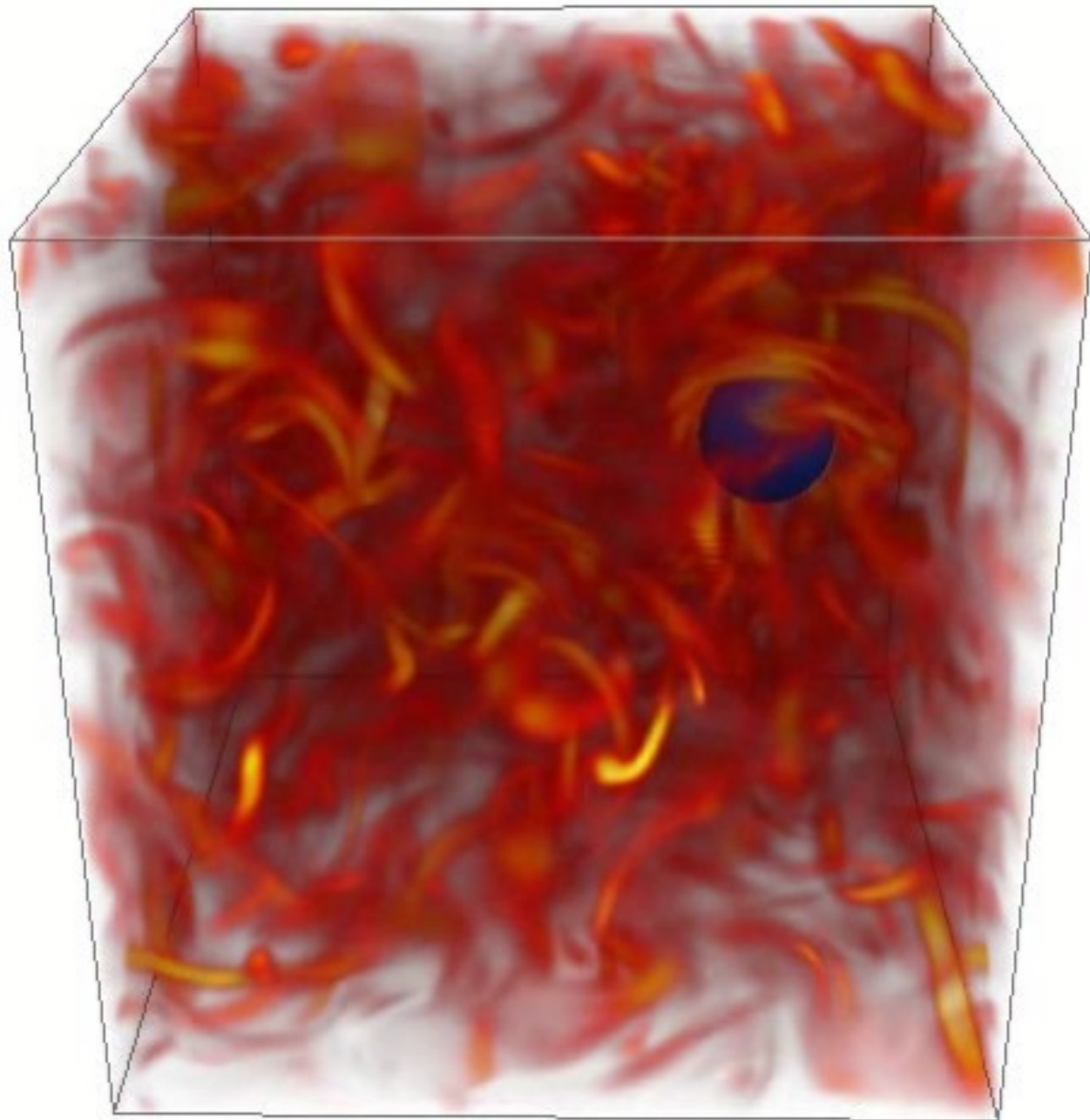
$$\mathbf{u}^{(n+1)} = \mathcal{P}\tilde{\mathbf{u}}^{(n+1)} = \tilde{\mathbf{u}}^{(n+1)} - \nabla p^n$$

H. Homann, J. Bec & R. Grauer JFM 2013.

Effect of turbulent fluctuations on the drag and lift forces on a towed sphere and its boundary layer







---

# Design of the numerical impellers :

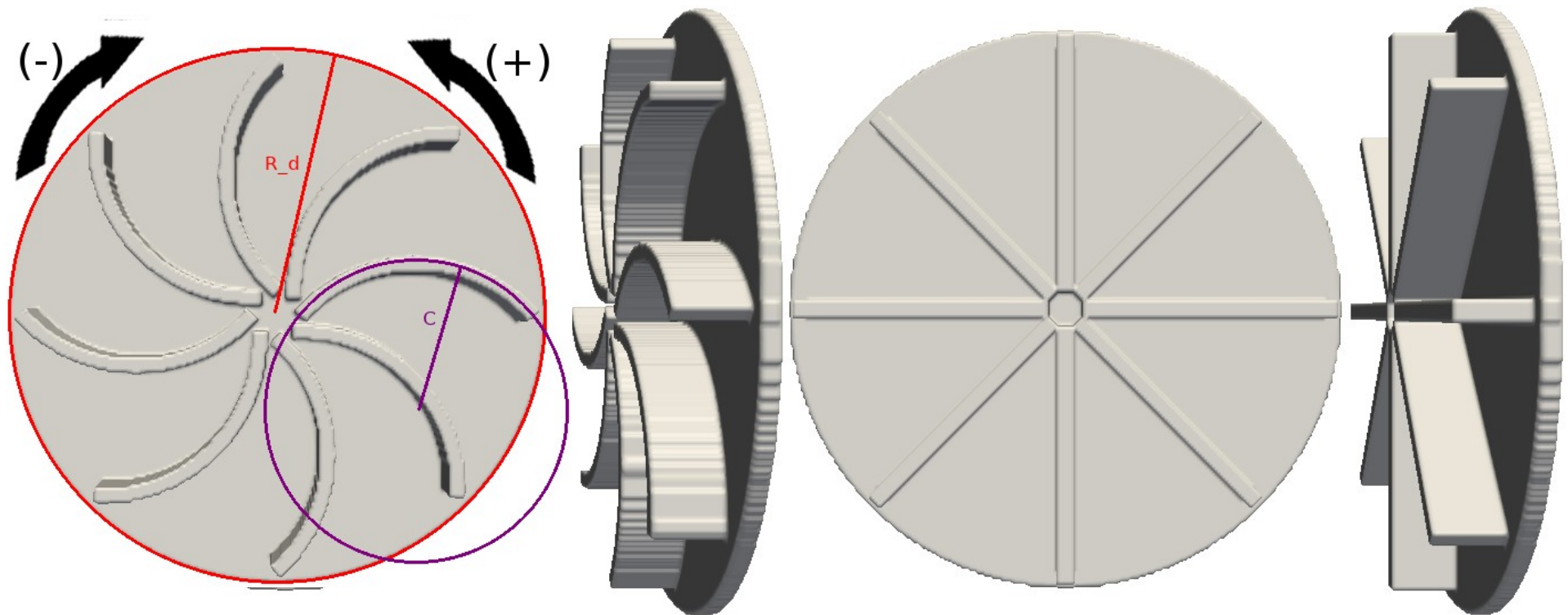
L. Marie et al EPJB 33(4):469-485, June 2003.

TM28 configuration :

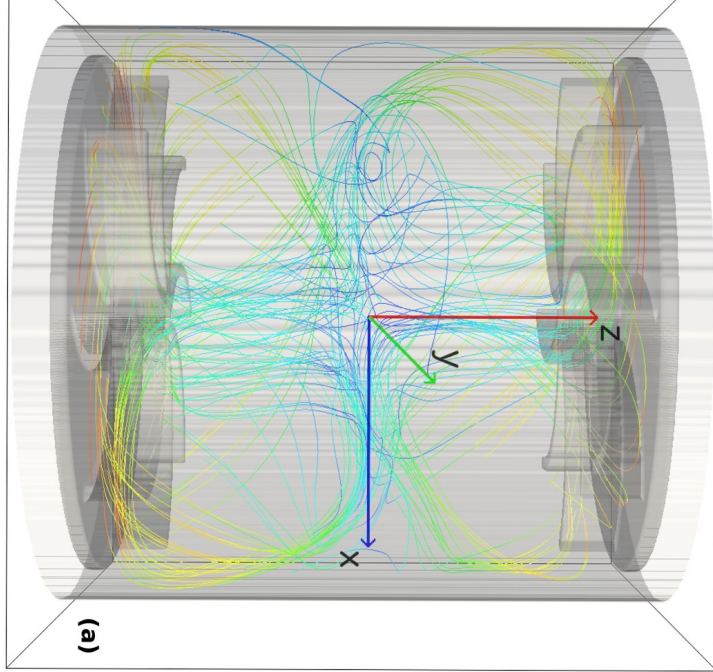
$R_c = 3:0$  ,  $R_d = 0:9R_c$ ,  $C = 0:5R_c$

height of the eight blades is fix at  $0:2R_c$

Expulsion angle  $\alpha = \arcsin (R_d=2C) \sim 1:11976 \text{ rad} \sim 64:15 \text{ deg.}$





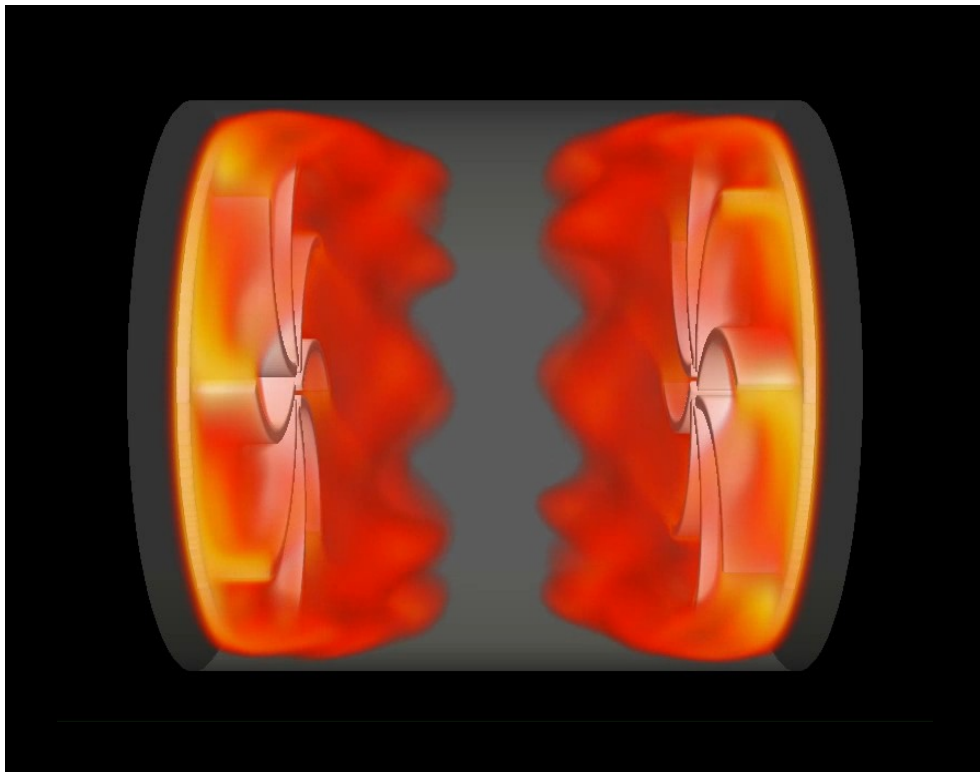


**S. Kreuzahler, D. Schulz, H. Homann, Y. Ponty, R. Grauer**

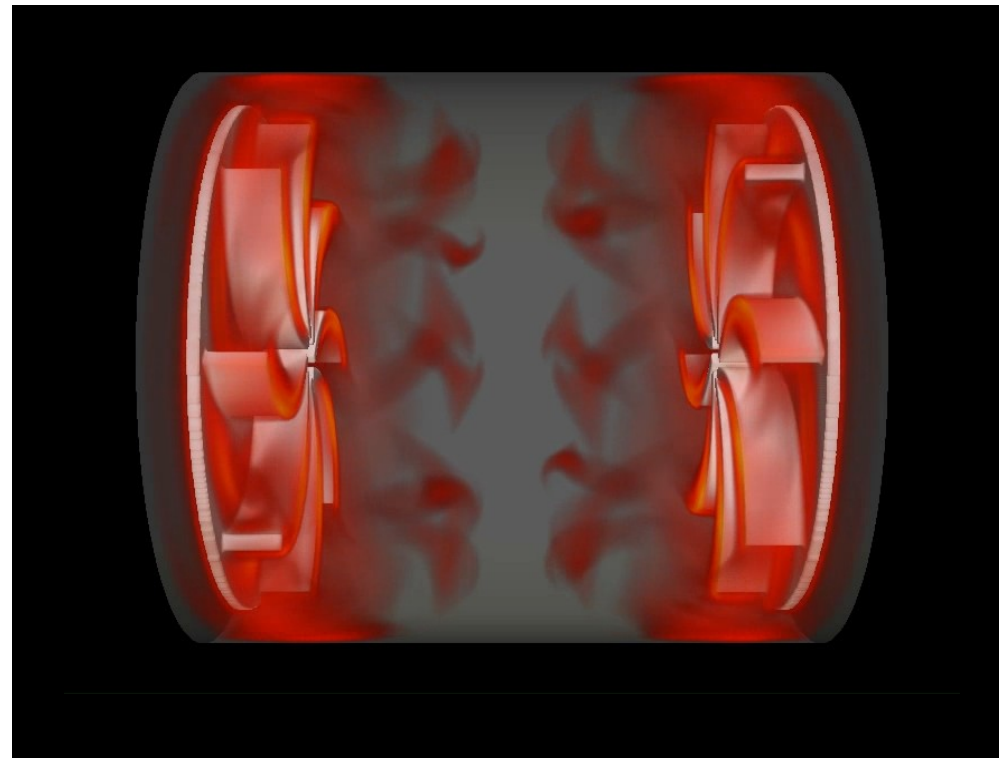
**"Numerical study of impeller-driven von Karman flows via a volume penalization method"**

**New J. Phys. 16 103001 (2014)**

**doi:10.1088/1367-2630/16/10/103001**

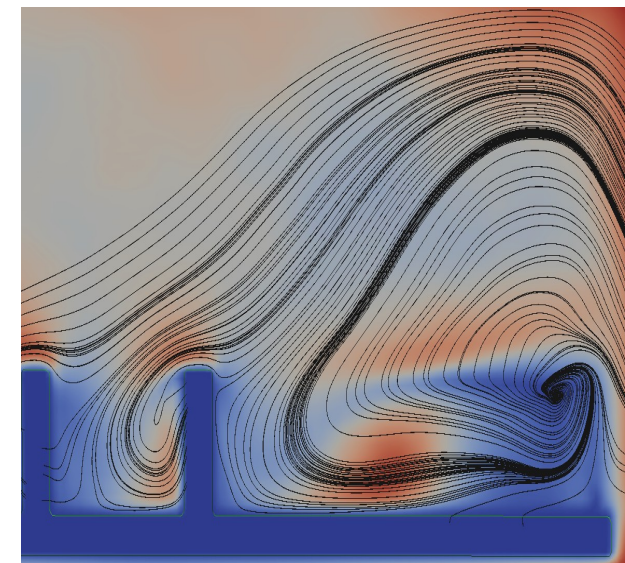
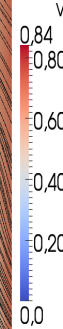
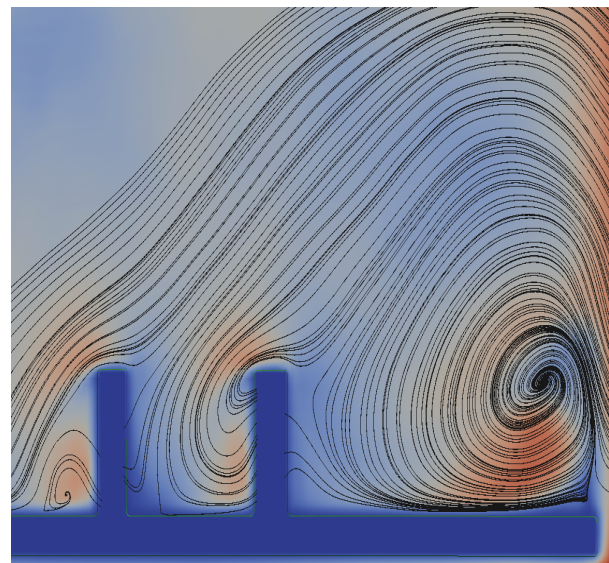
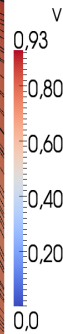
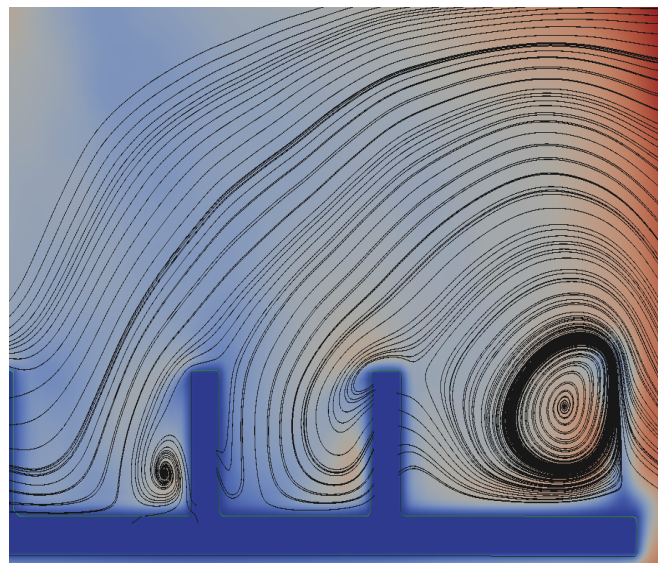
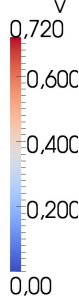
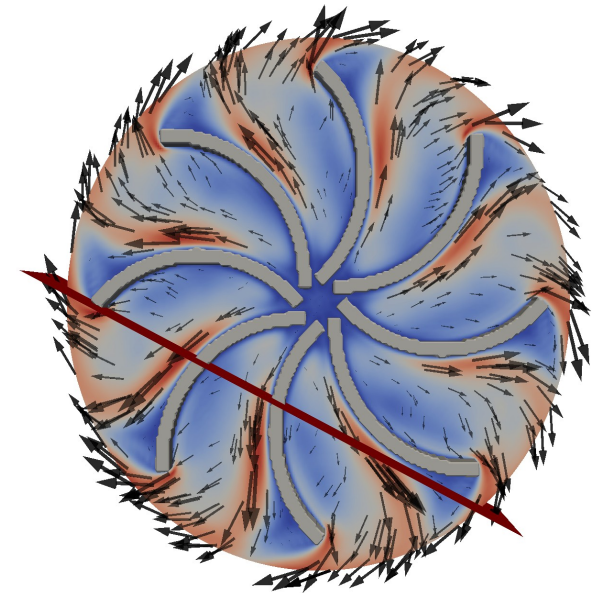
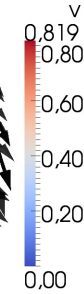
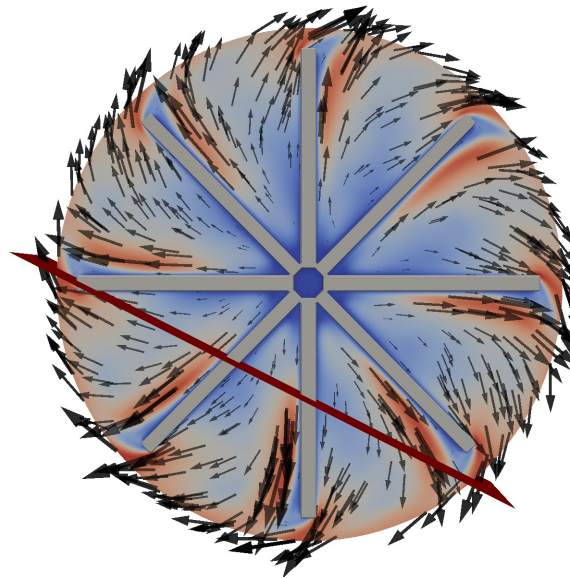
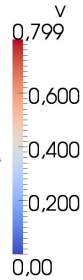
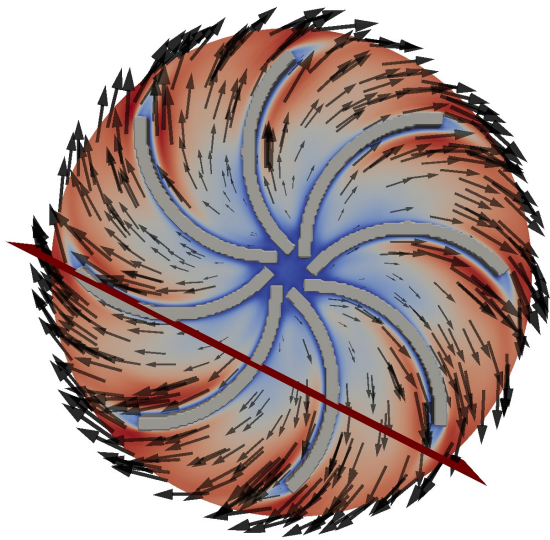


**Kinetic Energy**

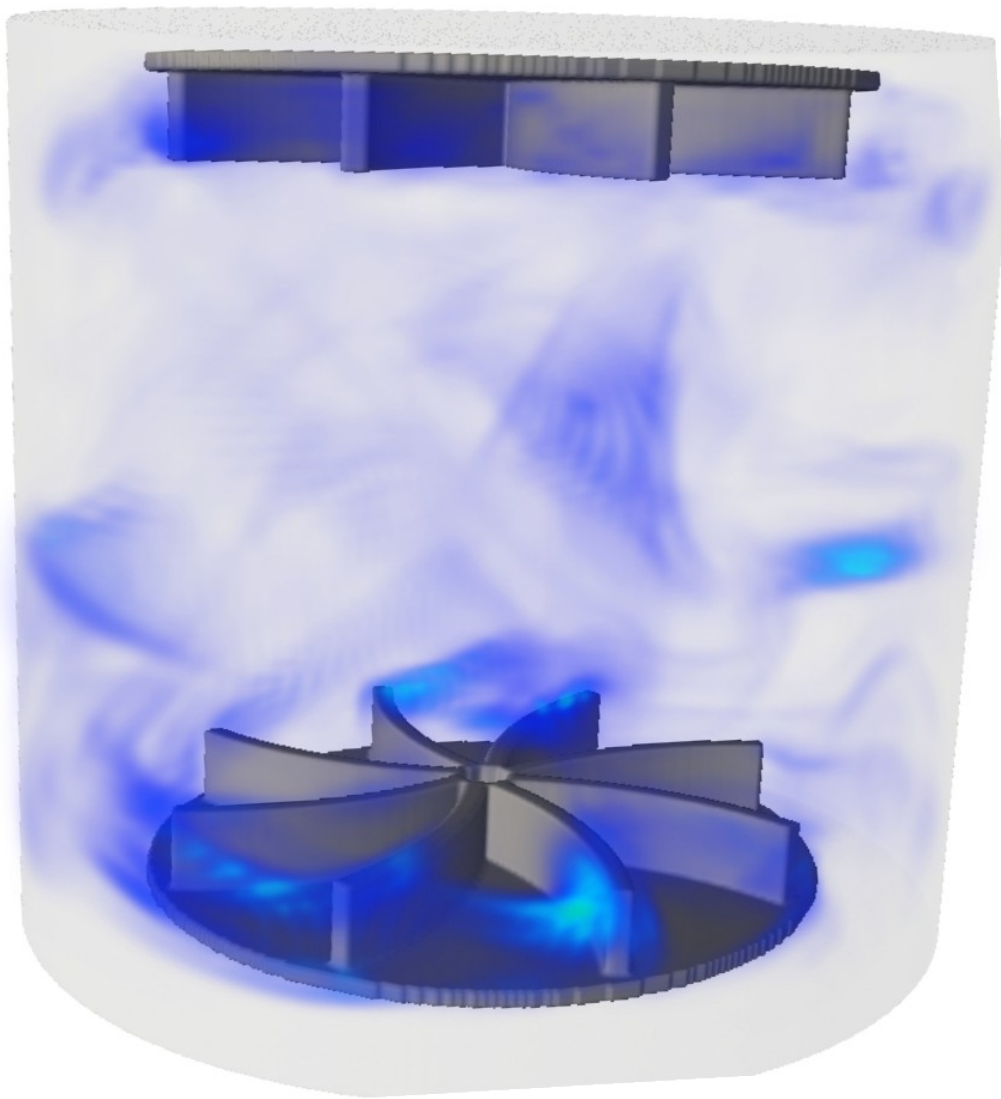


**Enstrophy**

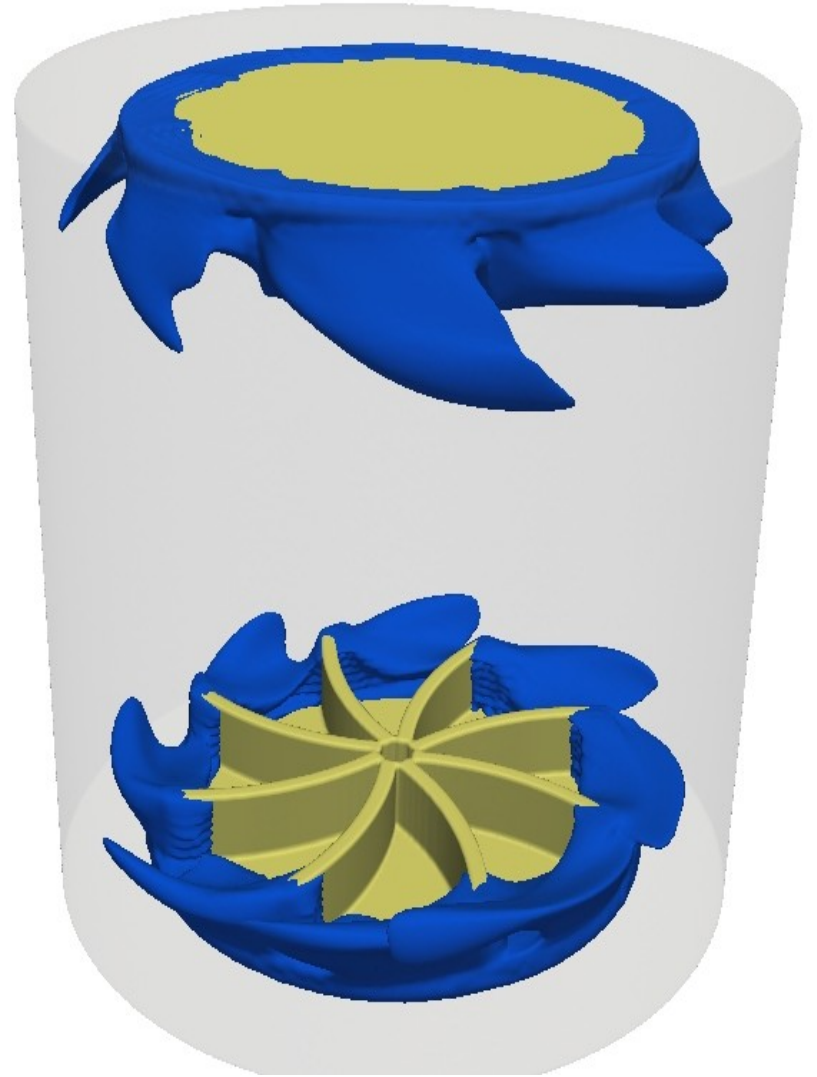
# (+) , straight, (-) configurations



Sebastian Kreuzahler, Daniel Schulz, Holger Homann, Yannick Ponty, Rainer Grauer  
"Numerical study of impeller-driven von Karman flows via a volume penalization method"  
New J. Phys. 16 103001 (2014)

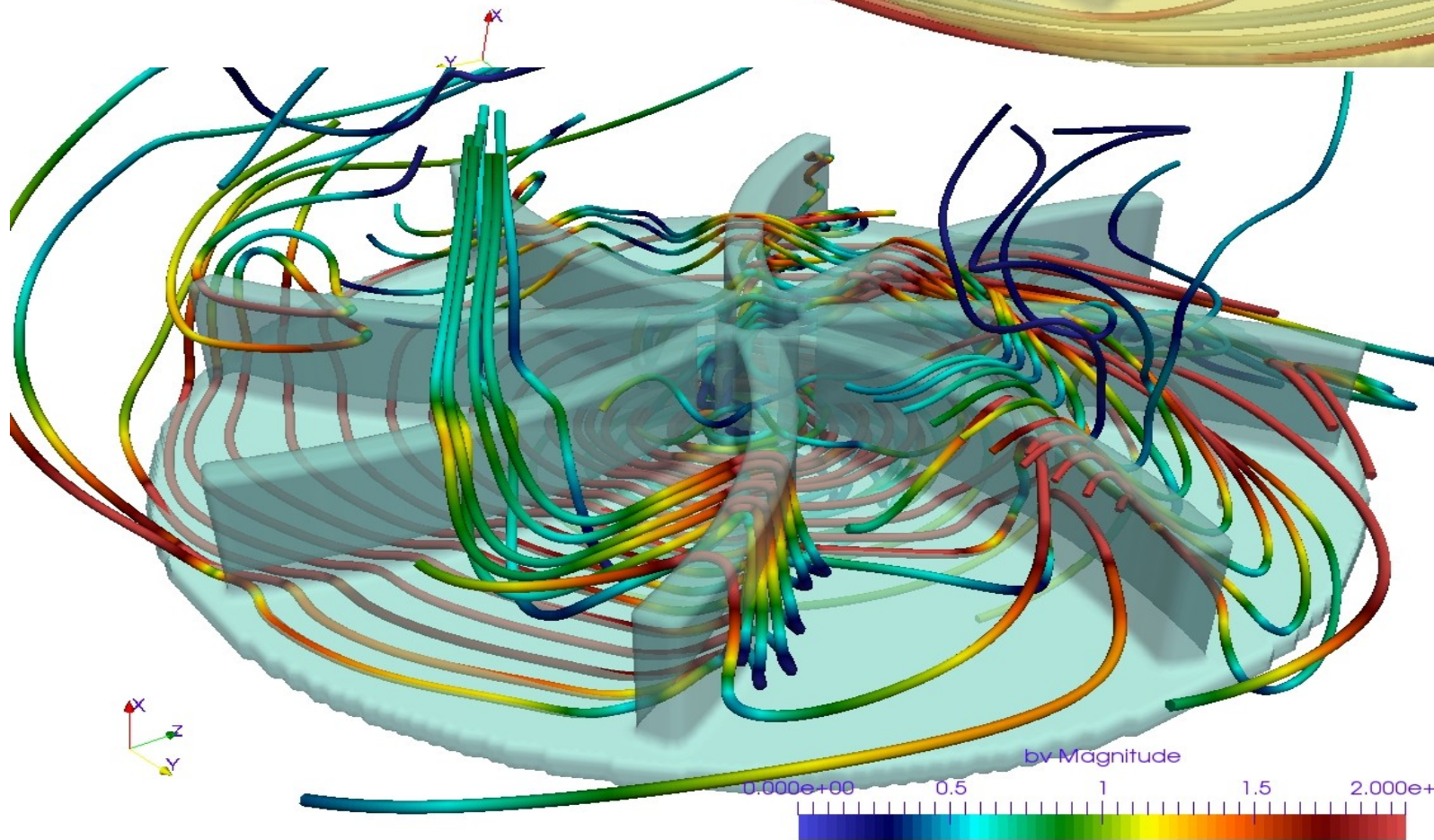
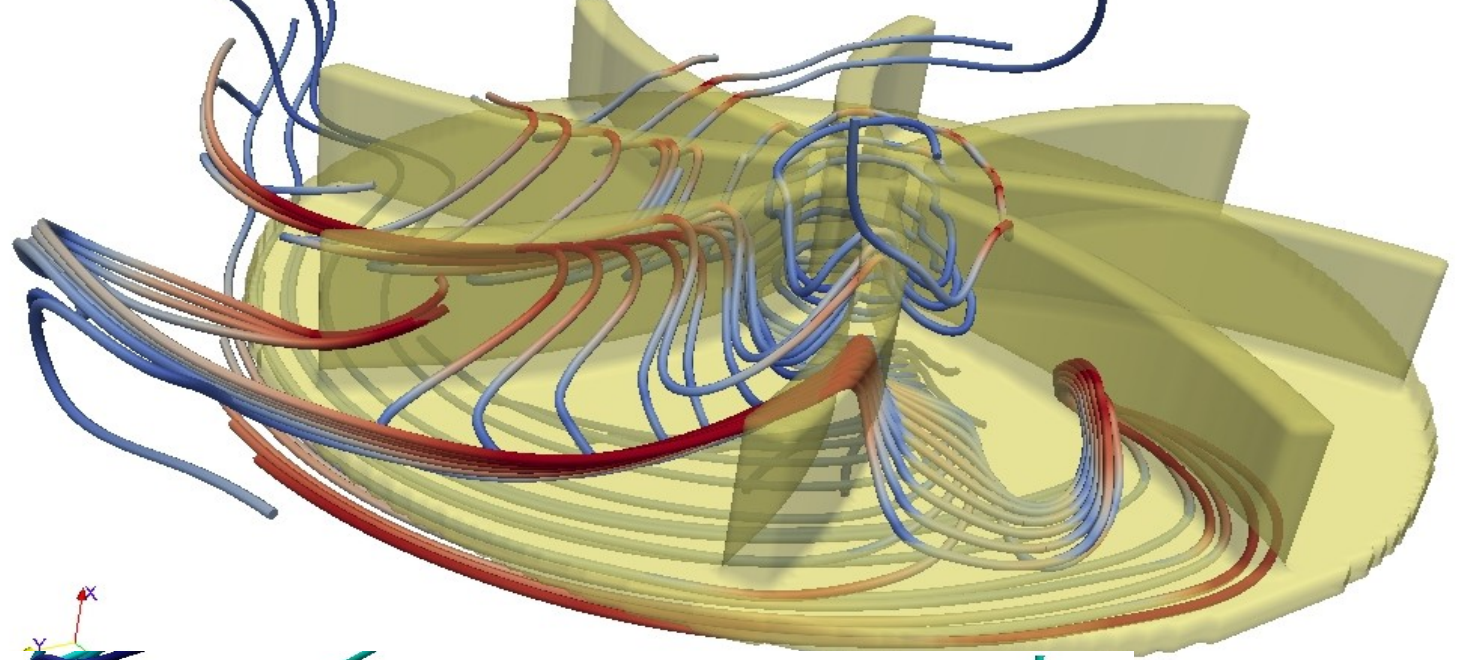


**Enstrophy**



**Kinetic energy**

# Dynamo




## **Pseudo-Penalization method**

**Near the boundaries the schemes is accurate only order h  
-> no spectral accuracy near the boundaries**

**Lost of grid point**

**Easy to implement , versatile  
-> EASY boundaries**

**Prospective numerical tool**

The background is a dark blue, high-tech image of a circuit board. A central component, possibly a microchip or connector, is highlighted with a bright blue glow and lens flare effects. The overall aesthetic is futuristic and digital.

**What do you need  
to built a nice**

**CUBE**

**Editor → emacs, debugger (db)**

**Language → c++ , F90**

**Clusters → ask to your boss or institutes**

**- Version saver → subversion**

**- Wiki, ticket bug → trac**

**Postraitement :**

**3D Visualisation → paraview, vapor**

**1D, 2D plots :**

**Matlab → ask to your boss**

**Python : matplotlib , scipy, ....**

# Subversion (svn) + trac

Firefox browser window showing the Trac interface for the 'cubby' project. The address bar shows the URL: `https://forge.oca.eu/trac/cubby/browser`.

The page displays the project structure under 'root' with a table of directories and their revision history.

Name ▲	Size	Rev	Age	Last Change
▼ cubby		2310	6 days	alainm: It compilespwd! refs <a href="#">#177</a>
▼ branches		2310	6 days	alainm: It compilespwd! refs <a href="#">#177</a>
▸ fftwcont		2184	3 months	alainm: in an alternate engine, do not interleave communication and fftw computing
▸ gpu		2284	2 weeks	alainm: First attempts at using GPU..
▸ io177		2310	6 days	alainm: It compilespwd! refs <a href="#">#177</a>
▼ tags		2145	4 months	ponty: rescale ampl OU process (old branch)
▸ mhd		659	2 years	alainm: The mhd project has been removed. Two notable release have been added into ...
▸ pre_dynafield		221	3 years	alainm: Taged the last version before startiu dynamic fields in the work branch.
▸ yannick_trunk		2145	4 months	ponty: rescale ampl OU process (old branch)
▸ trunk		2306	7 days	alainm: added a generic configuration utility

**Note:** See [TracBrowser](#) for help on using the browser.

View changes...

Powered by **Trac 0.11.4** By Edgewall Software. Visit the Trac open source project at <http://trac.edgewall.org/>



## Changeset 2299 for cubby/trun...

```

274 274     rv(avg_pmagn_d.Wms(space)) = 0;
275 275 }
274 276
275 276     if (avg_pfluid::on()) {
276 276         avg_pfluid::data_type& avg_pfluid_d = avg_pfluid::data();
277 276         rv(avg_pfluid_d.Psims(space)) = 0;
278 276         rv(avg_pfluid_d.Wms(space)) = 0;
279 276     }
280 276
281 276     if (avg_hely::on()) {
282 276         avg_hely::data_type& avg_hely_d = avg_hely::data();
283 276         rv(avg_hely_d.Hm(space)) = 0;
284 276         rv(avg_hely_d.Hms(space)) = 0;
285 276     }
277 277     if (avg_pfluid::on()) {
278 277         avg_pfluid::data_type& avg_pfluid_d = avg_pfluid::data();
279 277         rv(avg_pfluid_d.Psims(space)) = 0;
280 277         rv(avg_pfluid_d.Wms(space)) = 0;
281 277     }
282 277     if (avg_hely::on()) {
283 277         avg_hely::data_type& avg_hely_d = avg_hely::data();
284 277         rv(avg_hely_d.Hm(space)) = 0;
285 277         rv(avg_hely_d.Hms(space)) = 0;
286 277     }
285 287
286 288     }
287 289
288 289     // - - - - Penalization - - - - -
289 291
290 292     if (params.physical.penalization_v) {
291 293         field::scalar P_xi(space, "penalization", field::sp_real);
292 294         physic::make_penalization_field( P_xi,
293 295                                     params.physical.penalization_v,
294 296                                     params.physical.radius,
295 297                                     params.physical.DH);
296 296         io_oengine->store( P_xi, 1.0, 1.0, cst_name( "P_xi.dat" ) );
298 296         io_oengine->store( P_xi, 1.0, 1.0, "P_xi.dat" );
297 299     }
298 300
299 301     if (params.physical.penalization_f) {
300 302         field::scalar P_xi(space, "penalization", field::sp_real);
301 303         physic::make_penalization_field( P_xi,
302 304                                     params.physical.penalization_f,
303 305                                     params.physical.radius_f,
304 306                                     params.physical.DH_f);
304 306         io_oengine->store( P_xi, 1.0, 1.0, cst_name( "P_force.dat" ) );
306 306         io_oengine->store( P_xi, 1.0, 1.0, "P_force.dat" );
305 307     }
306 308
307 309     params.fluid.nu_eff = (params.physical.turbulent_viscosity > 1
308 310                          ? params.fluid.nu + params.fluid.nu_min
309 311                          : params.fluid.nu );
310 312
311 313     out << "After initialisation of v : nu = " << std::scientific << params.fluid.nu
312 314     << " nu_eff=" << std::scientific << params.fluid.nu_eff
313 315     << " nu_min=" << std::scientific << params.fluid.nu_min << std::endl;
314 316

```

# **Python libraries are great and free !**

**Scientific python libraries: scypy**

**Matplotlib : plot 1D, 2D**

**Glue language : one python script could mix :  
“linux command”,  
executable code, ...**

**Interpreted language → no compilation**

**Some part could be compile automatically**

**Or push to be compile ()**

**Parallel script : multiprocessing, mpi, gpu**

# Parser option

```
#!/usr/bin/env python
from pylab import *
from optparse import OptionParser
import string
from scipy import stats

##### Manage the options :

parser = OptionParser()
parser.add_option('-v', '--inputv', action="store", type="string",\
                  dest="inputv", default='energy.log',\
                  help='input velocity energy file (default:energ_v.dat)')
parser.add_option('-l', '--linb', action='store_true',\
                  dest='linb', default=False, \
                  help='linear y axis for magnetic plot (log instead)')
parser.add_option('-t', '--total', action='store_true',\
                  dest='total', default=False, \
                  help='Plot total energy (kinetic + magnetic)')

....
```

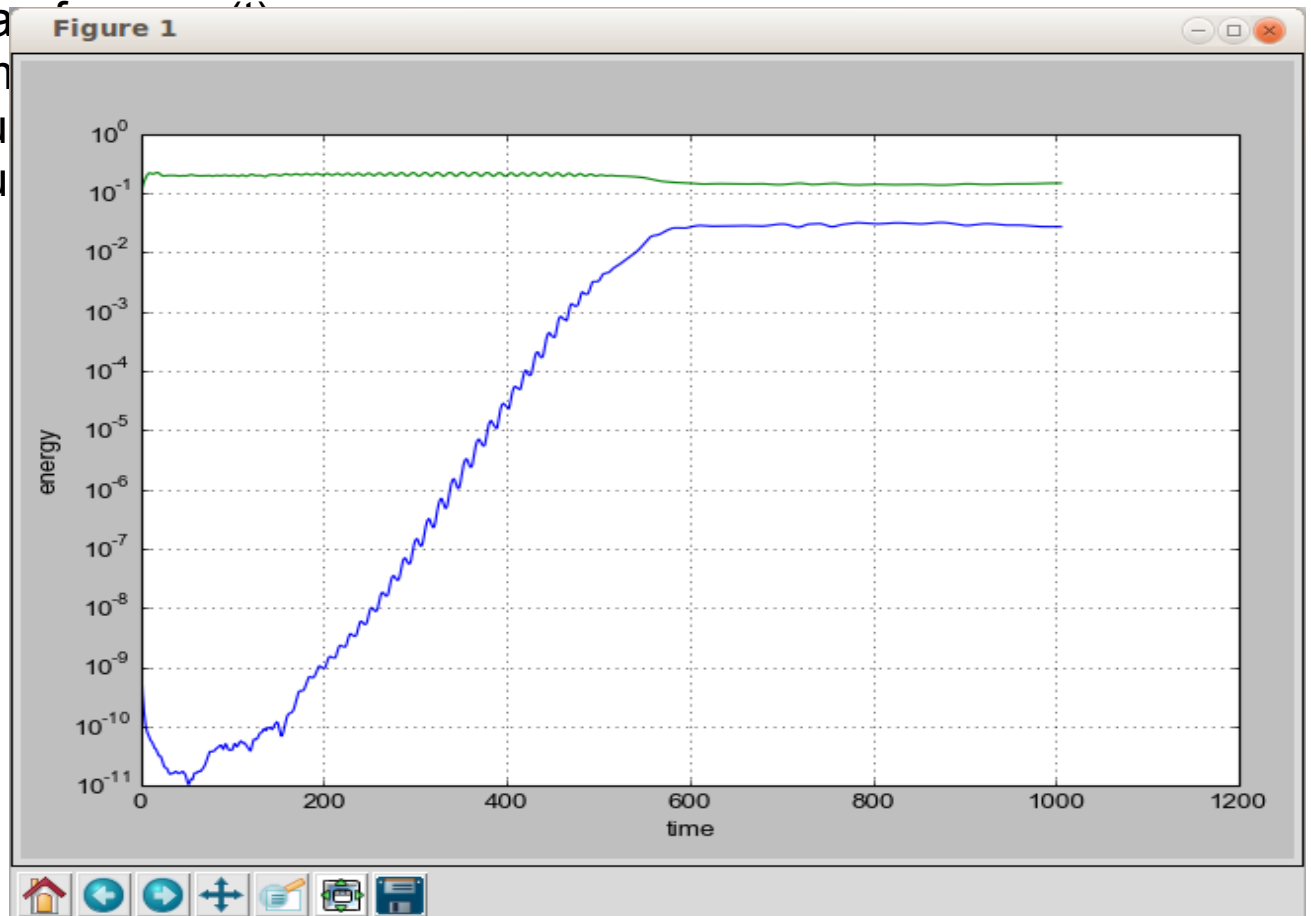
```
ponty@hobbit:~$ plot.py --help
```

```
Usage: plot.py [options]
```

Options:

- h, --help show this help message and exit
- v INPUTV, --inputv=INPUTV  
input velocity energy file (default:energy\_v.dat)
- l, --linb linear y axis for magnetic plot (log instead)
- t, --total Plot total energy (kinetic + magnetic)
- together Plot total energy (kinetic + magnetic) together
- p, --pdf print all the graphic directly inside pdf file
- o, --pdf\_only print all the graphic directly inside pdf file only
- s, --spectra Plot the spectra
- stat Plot the histogram
- five input has five columns
- step plot energy versus

```
plot.py -v energy.log
```



# Parallel task with Python

```
from multiprocessing import Pool
import multiprocessing
```

```
nprocs_max = multiprocessing.cpu_count()
```

```
...
```

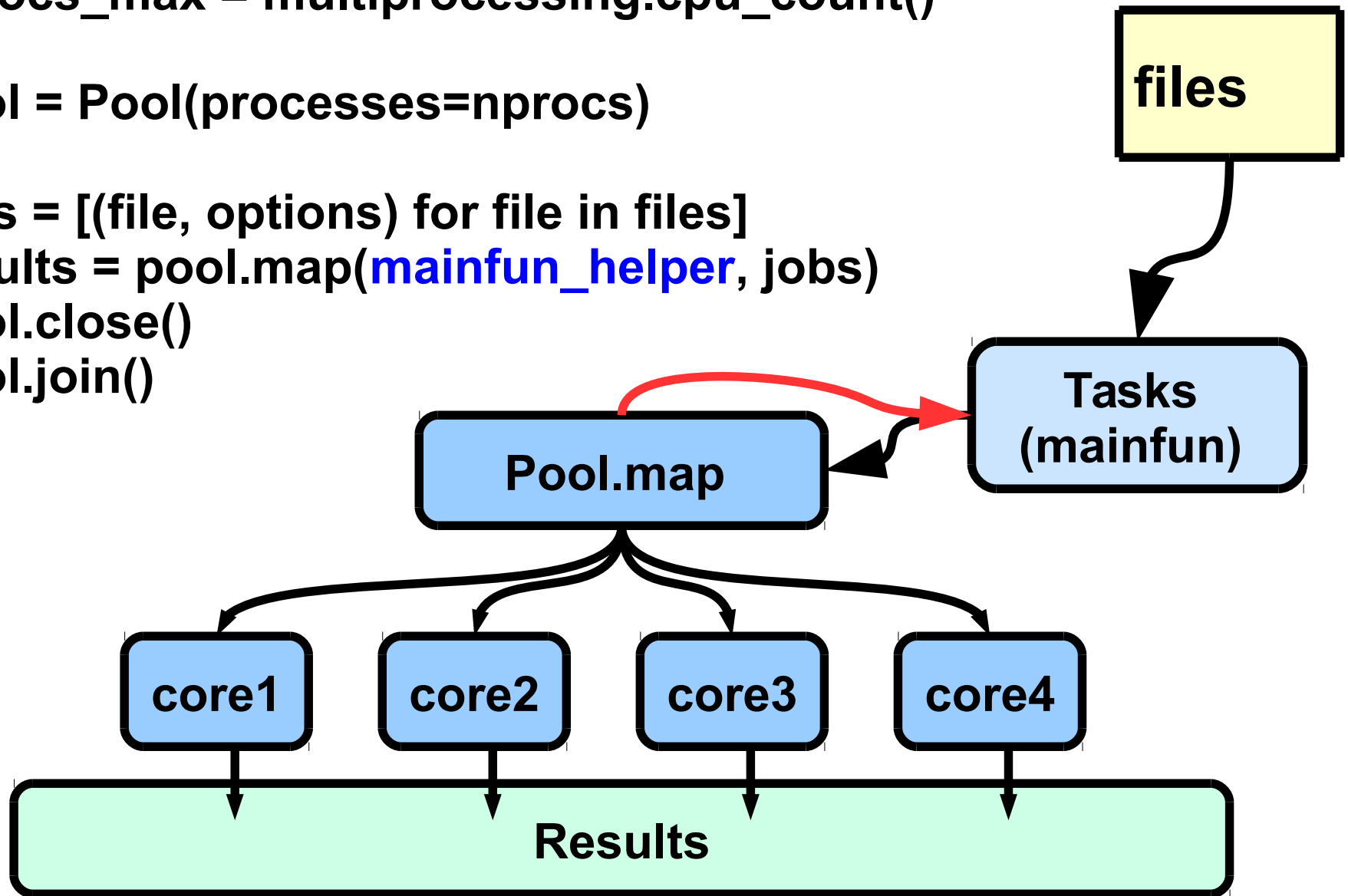
```
pool = Pool(processes=nprocs)
```

```
jobs = [(file, options) for file in files]
```

```
results = pool.map(mainfun_helper, jobs)
```

```
pool.close()
```

```
pool.join()
```





**Impossible to escape**

**Then Join the**

**CUBE**

Thank **Annick** to put me inside  
this **bloody CUBE** !!!



**Scientific CUBE institute at Nice**