

# NUMERICAL MODELS OF NEUTRON STARS: CONNECTING NUCLEAR PHYSICS WITH ASTROPHYSICS

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# PLAN OF THE PRESENTATION

- 1 INTRODUCTION
- 2 NUMERICAL MODELS OF ISOLATED NEUTRON STARS
- 3 GRAVITATIONAL WAVES FROM BINARY NEUTRON STARS

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# Introduction

## THE IDEA OF A NEUTRON STAR

- 1932, Landau (Phys. Z. Sowjetunion, 1, 285)  
Possibility of stars with a central density comparable to that of nuclei
- 1934, Baade and Zwicky (Phys. Rev. 45, 138)  
Prediction of the existence of neutron stars:  
*With all reserve we advance the view that supernovae represent the transition from ordinary stars into neutron stars, which in their final stages consist of extremely closely packed neutrons.*
- 1939, Tolman, Oppenheimer, and Volkov  
General relativistic neutron star models:  
 $M \approx 0.7M_{\odot}$  and  $r \sim 10$  km  
 $\Rightarrow$  density  $\sim 0.1 \text{ fm}^{-3}$

WALTER BAADE



Fritz Zwicky



## DISCOVERY OF PULSARS

ANTONY HEWISH



In 1967, at Cambridge University Antony Hewish lead an observational survey of extra-galactic radio-sources.

In August, his student Jocelyn Bell detects important signal fluctuations, which are observed to be periodic with a period of 1.337 s.

JOCELYN BELL



⇒ **pulsar** or *Pulsating Source of Radio* (PSR)

## PULSARS ARE NEUTRON STARS

Originally, the sources were associated with pulsations of **neutron stars** or **white dwarfs** after thinking of little green men: the first pulsar has been called LGM before PSR B1919+21...



Thomas Gold, in 1968 identifies pulsars with rotating magnetized neutron stars and predicts a slight increase of their period due to energy loss.

⇒ with the discovery of the Crab pulsar (PSR B0531+21) with a period of 33 ms, white dwarfs are ruled out...

At maximal rotating speed, centrifugal and gravitational forces cancel exactly:

$$R\Omega^2 = \frac{GM}{R^2}, \text{ and } P_{\min} \simeq \sqrt{\frac{3\pi}{G\rho}}$$

$P_{\min} \sim 1$  s for white dwarfs and  $P_{\min} \sim 1$  ms for neutron stars.



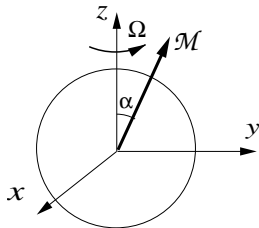
## ROTATING DIPOLE MODEL

To emit electro-magnetic waves, magnetic axis must not be aligned with the rotation one  $\Rightarrow$  oblique dipole (simplest analytic) model.

Accelerated dipole  $\Rightarrow$  radiation of its rotational energy  $E = \frac{1}{2} I \Omega^2$ :

$$\dot{E} = -\frac{2\pi}{3c^2\mu_0} \Omega^4 R^6 B_p^2 \sin^2 \alpha$$

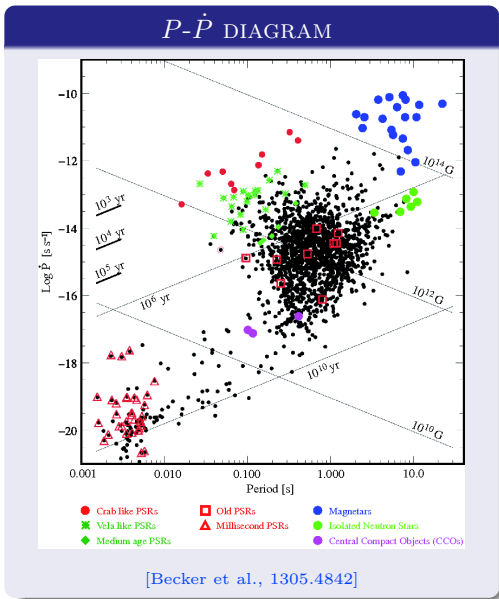
$\Rightarrow$  Deduce polar magnetic field  $B_p$  from observations of  $\dot{P}$ , assuming the star is a sphere.



For the Crab pulsar  $B_p \sin \alpha \simeq 5.3 \times 10^8$  T.

## ON THE OBSERVATIONAL SIDE

- Almost 3000 neutron stars have been observed as pulsars, among others Crab, Vela, Geminga, Hulse-Taylor double pulsar, ...
- Several NS-NS binary systems known
- Some NSs observed via surface emission

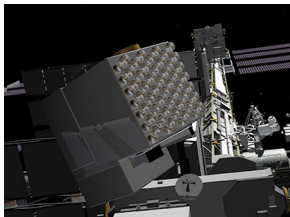


## CONSTRAINTS FROM OBSERVATIONS

Observations	Quantities detected	Dense matter properties
Orbital parameters in binary systems	Neutron star masses	Equation of state (EoS), high densities
GW from binary systems	Tidal deformability	Compactness, EoS
$X$ -ray observations	Surface temperature	Heat transport/neutrino emission, superfluidity
	Radii	EoS, also low and intermediate densities (crust)
Pulsar timing	Rotation frequencies	EoS via mass-shedding limit
Pulsar timing	Glitches	Evidence for superfluid component
GWs from isolated stars	Oscillations	Eigenmodes (EoS, crust properties)
QPO	Radii	EoS
	Asterosismology	Eigenmodes

Observables:  $f$ ,  $M$ ,  $B_{\text{pole}}$ ,  $T_{\text{surf}}$ ,  $R$ , ...

# SOME NEUTRON STAR OBSERVATION PROJECTS



Neutron star Interior  
Composition Explorer (NICER)

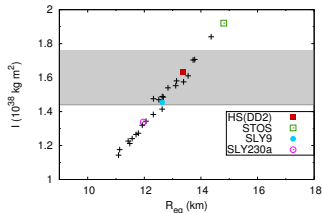
NICER is a soft X-ray telescope onboard the ISS, to observe X-ray binaries with millisecond pulsars

⇒ determination of radius with  $\sim 5\%$  accuracy.

⇒ results expected very soon. . .

SKA shall bring many accurate observations able to constrain neutron star:

- Masses (increase of mass determination by a factor  $\sim 10$ )
- Moments of inertia ⇒ radius
- Rotation frequencies / glitches . . .



French SKA whitebook (2017)

Need for very accurate models!

# Numerical models of isolated neutron stars

## SIMPLEST MODELS

Compute equilibrium between gravitation and pressure...

### NEED FOR GENERAL RELATIVITY:

- Most of quantities are wrong by  $\sim 20 - 40\%$
- Notions such as **maximal mass** do not exist with the use of Newtonian theory.

First models (TOV, 1939) hydrostatic equilibrium in spherical symmetry, using General Relativity, the perfect fluid model and the ideal Fermi gas EoS for degenerate neutrons.

### NEED FOR NUCLEAR PHYSICS:

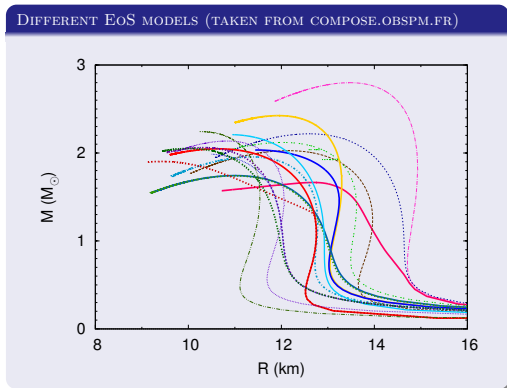
- With the TOV approach, maximal mass of  $0.7M_{\odot}$ , incompatible with all observations.

$\Rightarrow$  Pauli exclusion principle applied to neutrons cannot support neutron star gravity.

# MASS-RADIUS RELATION

- $M$  and  $R$ 
  - GR, staticity + spherical symmetry
  - Equation of state (EoS)

⇒ solving TOV-system
- Matter in old NSs can be considered as cold and in weak equilibrium  
⇒ EoS:  $p(\epsilon)$  (Oertel *et al.* 2017).

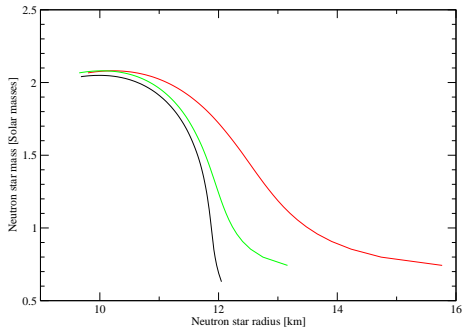


⇒ Determining mass **and** radius of (one) object considered as the Holy Grail...

Beyond spherical symmetry and perfect fluid (alone), other models need to be considered.

# NEED FOR ROTATION?

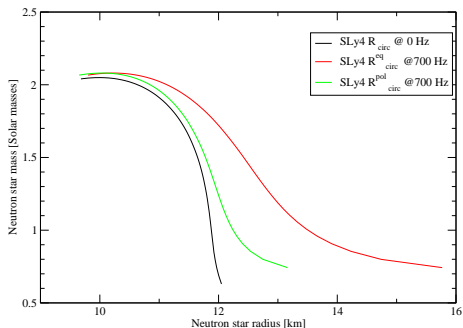
## THREE DIFFERENT EoSs ...





# NEED FOR ROTATION

ONE EOS : SLY4 DOUCHIN & HAENSEL (2001)



Rotation is important when dealing with radii.

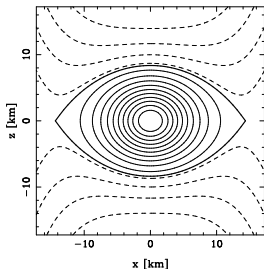
⇒ Need for numerical models or perturbative approach.

# RAPIDLY ROTATING MODELS

USING LORENE LIBRARY (LORENE.OBSPM.FR)

- GR with stationarity and axisymmetry (+ circularity)
- Perfect fluid with rigid or differential rotation
- EoS from nuclear physics
- Rotation frequency limited by **mass-shedding limit**: matter leaving the star at the equator, due to centrifugal force.
- Impossible to describe in slow-rotation limit.
- Developed since the late 1970's, several publicly available codes.

Enthalpy isocontours Keplerian frequency



Lin & Novak (2006)

⇒ needed to determine moment of inertia  $I$  and maximal rotational frequency for a given EoS.

# MAGNETIC FIELD

## MOTIVATIONS

### THEORETICAL

- conservation of magnetic flux: 1 G for an *O*-type star of  $\sim 10R_{\odot} \Rightarrow \sim 10^{12}$  G.
- More if magneto-rotational instability in core-collapse supernovae : magnetars  $\Rightarrow$  influence on structure?

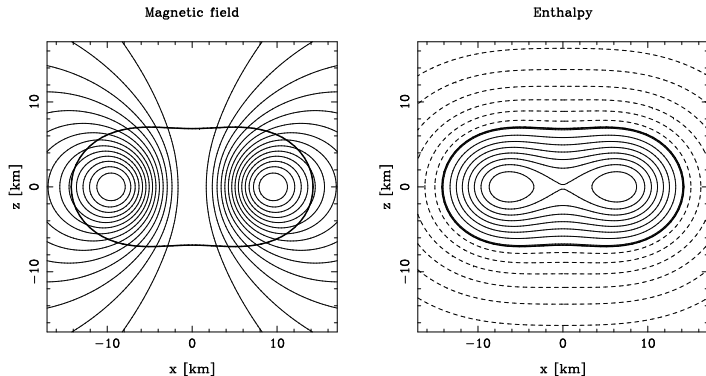
### OBSERVATIONAL

- Magnetic slowdown measured through the spin-down  $\dot{P}$  gives values of  $B_{\text{pole}}$  up to  $10^{16}$  G,
- **magnetars** could represent as much as 10% of all pulsars (Muno *et al.* 2008);
- they can produce very strong X- and  $\gamma$ -ray bursts, from the glitch-like rearrangement of the crust, in which magnetic field is pinned (*e.g.* Dec. 2004 with SGR 1806-20).

# MAGNETIC FIELD

## NUMERICAL MODELS

- Perfect conductor + independent currents
- Maxwell equations and equilibrium with Lorentz force.
- Poloidal magnetic field, moment aligned with rotation axis



$B_p \sim 5 \times 10^{16}$  G, Bocquet *et al.* (1995)

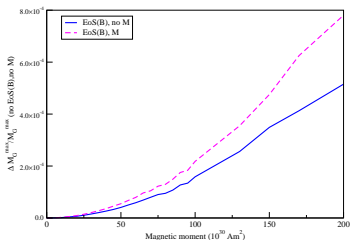
⇒ determination of a “universal” profile for  $\|\vec{B}\|$  (Chatterjee *et al.* 2018).

# MAGNETIC FIELD

## EFFECT ON THE EoS

Many studies on effect of strong ( $\gtrsim 10^{16}$  G) magnetic fields on properties of nuclear matter: EoS  $p(\varepsilon, B)$ .

⇒ Starting from a microscopic Lagrangian density of fermions coupled to an electromagnetic field, new matter model and contribution to Einstein-Maxwell equations (Chatterjee *et al.* 2015)



Chatterjee *et al.* (2015)

- First global model including magnetization (microscopic interaction between matter and magnetic field)
- Model shows no additional term in equilibrium equation (cancellation)
- Effect small with quark matter EoS: other choices?

# SUPERFLUIDITY

## MOTIVATIONS

### THEORETICAL

At nuclear density the critical temperature:  $T_{\text{crit}} \sim 1$  MeV  $\Rightarrow$  superfluid component some minutes after their birth.

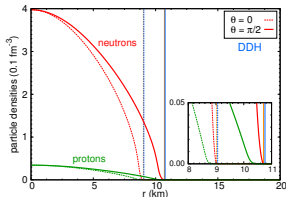
### OBSERVATIONAL: GLITCHES

Some pulsars exhibit sudden changes in the rotation period: instead of regularly slowing down, it shows rapid speed-up.

$\Rightarrow$  Within the two-fluid framework:

- outer crust (+fluid) is slowed down, not the inner fluid;
- until the stress (or interaction) between both becomes larger than some threshold.

$\Rightarrow$  models in the two-fluid approach in [Prix et al. \(2005\)](#), [Sourie et al. \(2016\)](#)

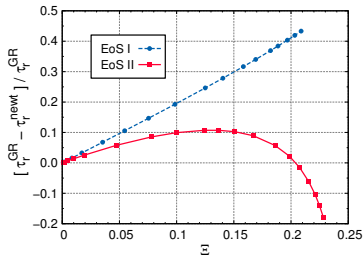
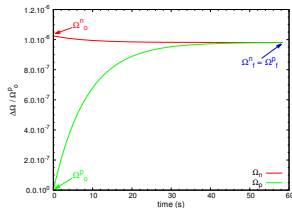
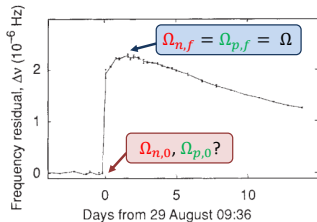


Sourie et al. (2016)

# SUPERFLUIDITY

## GLITCH MODELS

Modelling of the glitch rise time, *e.g.* for the Vela pulsar



- Observational constraints  $\tau \lesssim 30$  s.
- Constraint on superfluid properties in neutron stars (drag to lift ratio).
- Effects of General Relativity are very strong.

Compactness  $\Xi = \frac{GM}{Rc^2}$  with  $\Xi_{\text{NS}} \simeq 0.2$ , see also Sourie *et al.* (2017)...

## THERMAL EFFECTS

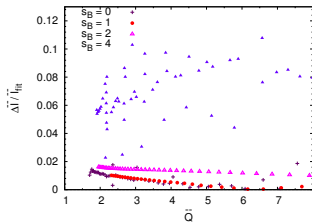
“Standard” models consider matter at zero temperature :  
neglect temperature effects.

⇒ Important for proto-neutron stars (birth) or, possibly, in the last phases of binary neutron star evolution (tidal heating).

First attempt by [Goussard \*et al.\* \(1997\)](#) to build models with temperature-dependent EoS.

Similar approach to study universality of  $I$ - $\Lambda$ - $Q$  relations ( $\Lambda$  determined by GW observations).

⇒ breaking of universality for high, but realistic entropy effects.



Marques *et al.* (2017)



# Gravitational waves from binary neutron stars

# GRAVITATIONAL WAVES FROM BINARY NEUTRON STAR MERGERS

GW170817: first detection of a NS-NS merger with LIGO/Virgo detectors: information from different phases:

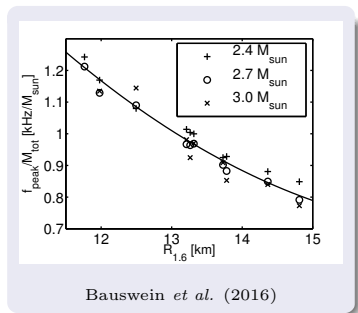
- Inspiral  $\Rightarrow$  masses of objects
- Late inspiral  $\Rightarrow$  tidal deformability  $\tilde{\Lambda}$  depends on matter properties

**GW170817**

$70 < \tilde{\Lambda} < 720$  (90%  
confidence level)

(low spin prior) *Abbott et al.*

(2018)



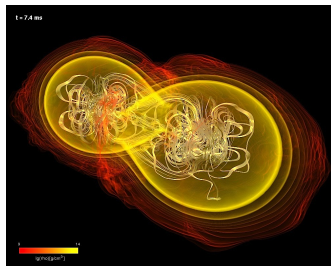
- Post merger oscillations  $\Rightarrow$  peak frequency strongly correlated with NS radius

# SIMULATIONS OF BINARY NEUTRON STARS

First simulations of binary neutron star merger by [Shibata & Uryū \(2000\)](#). However, contrary to black holes, these simulations are not completely mature:

- Mostly using polytropic EoS (or piecewise polytropic)
- Magnetic field implementation ongoing (MHD approximation, surface,...)
- High numerical viscosity
- No neutrino treatment (almost)
- No crust...

⇒ Initial data from the LORENE numerical library (<http://lorene.obspm.fr>)

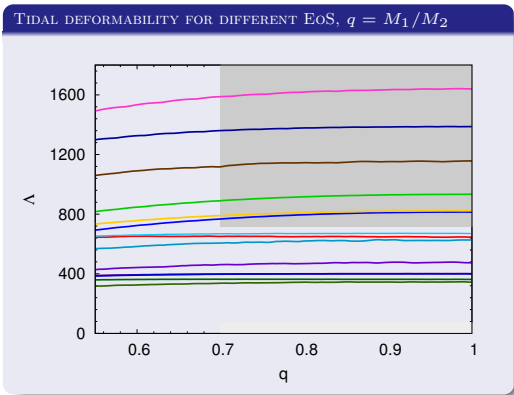


Courtesy of ITP, Frankfurt

# TIDAL DEFORMABILITY

## CONSTRAINTS ON THE EoS

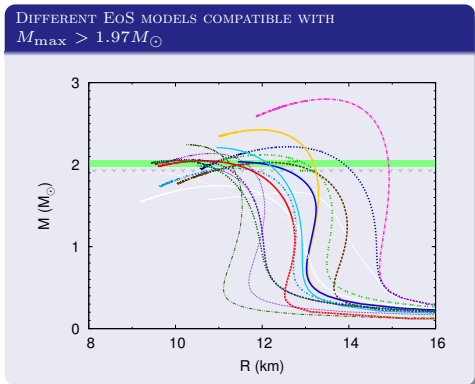
- Tidal deformability  $\tilde{\Lambda}$  depends on matter properties
- $\tilde{\Lambda}(M_{chirp}, q, \text{EoS})$
- $\sim 5\%$  uncertainty from crust treatment
- $\lesssim 10\%$  uncertainty from thermal effects



Assuming  $I$ - $\Lambda$ - $Q$  relations are universal,  $\Rightarrow$  stronger constraints on the moments of inertia  $I$  / mass  $M$ .

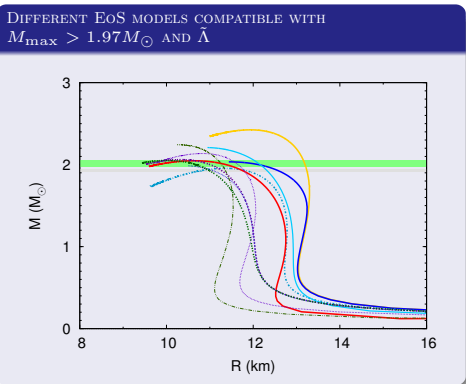
# MASS-RADIUS RELATIONS WITH BINARY NS CONSTRAINTS

- Some EoS (giving less compact NSs) excluded by limit on  $\tilde{\Lambda}$
- Additional (model dependent) constraints from relation with EM observations
- $M_{tot}$  + no prompt BH collapse [Bauswein et al. \(2017\)](#)
- $M_{tot}$  + estimate of energy loss to ejecta [Margalit & Metzger \(2017\)](#)
- Ejecta masses + composition [Shibata et al. \(2017\)](#)
- $\tilde{\Lambda} \gtrsim 450$  from ejecta masses [Radice et al. \(2017\)](#)



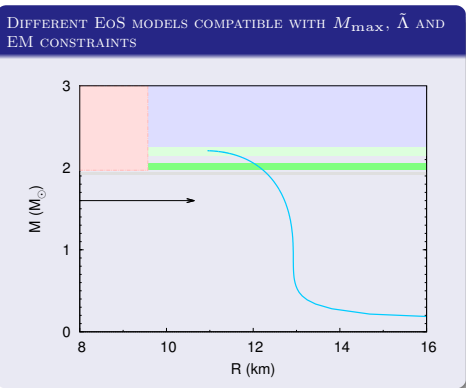
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## CONCLUSIONS

### OBSERVATION / ANALYSIS

- Neutron stars are now well-observed objects.
- Observations acquire much better accuracy (NICER, SKA,...)
- Better accuracy in the determination of the EoS?

### NUMERICAL MODELS

- Rotation, magnetic field easy to take into account.
- Superfluid models can give insight on glitch phenomena (more observations?).
- Elastic crust should be modelled, too (tough!).

⇒ Gravitational waves bring a lot of new information into the game. Now, a lot of effort must be devoted to better modelling of binary neutron stars: no group in France...