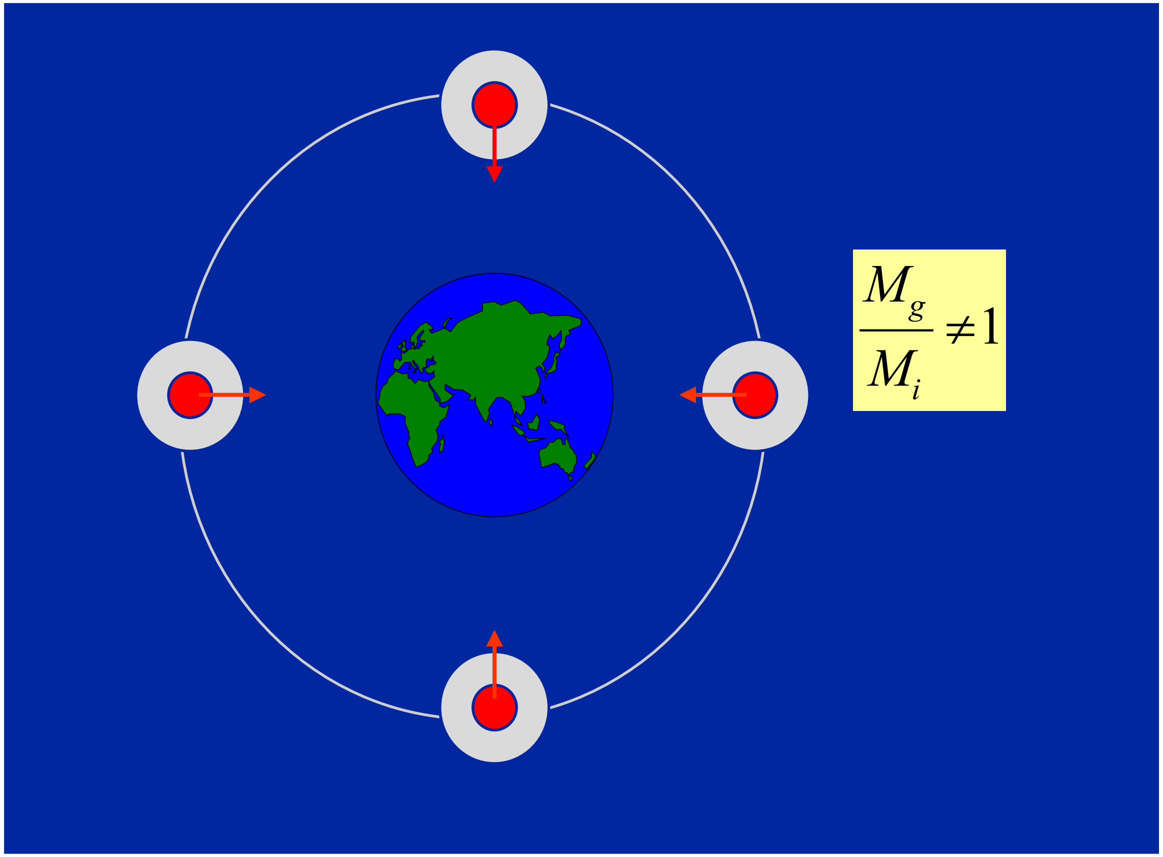
Aspects de l'analyse d'erreur pour MICROSCOPE

G. Métris - Ph. Berio

Observatoire de la Côte d'Azur

Collaboration : ONERA- CNES-ESA-ZARM

MICROSCOPE = measurement of the differential acceleration



$$\frac{M_g}{M_i} \neq 1$$

Equation for the differential acceleration

$$\begin{aligned}
2\vec{\gamma}^{(d)} = {} & \gamma^{(1)} - \gamma^{(2)} && \\
= {} & \vec{g}(\tilde{O}_2) - \vec{g}(\tilde{O}_1) && \textit{gravity gradient} \\
& +\delta_2\,\vec{g}(\tilde{O}_2) - \delta_1\,\vec{g}(\tilde{O}_1) && \textit{EP violation} \\
& +2\vec{\Omega}\wedge\overset{\circ}{\overrightarrow{O_2O_1}} + \overset{\circ\circ}{\overrightarrow{O_2O_1}} && \textit{relative motion of the masses} \\
& +\vec{\Omega}\wedge(\vec{\Omega}\wedge\overrightarrow{O_2O_1}) + \dot{\vec{\Omega}}\wedge\overrightarrow{O_2O_1} && \textit{inertial acceleration} \\
& +\frac{\vec{fp}_2}{m_2} - \frac{\vec{fp}_1}{m_1} && \textit{perturbations}
\end{aligned}$$

Linearization of the gravity

$$\begin{aligned}
2\vec{\gamma}^{(d)} = {} & -[\mathbf{I}]\,\overrightarrow{O_1O_2} && \textit{inertial acceleration} \\
& +[\mathbf{T}]\,\overrightarrow{\tilde{O}_1\tilde{O}_2} && \textit{gravity gradient} \\
& +\delta\,\vec{g}(\tilde{G}) && \textit{EP violation} \\
& +2\vec{\Omega}\wedge\overset{\circ}{\overrightarrow{O_2O_1}} + \overset{\circ\circ}{\overrightarrow{O_2O_1}} && \textit{relative motion of the masses} \\
& +\vec{\gamma p}_2 - \vec{\gamma p}_1 && \textit{perturbations}
\end{aligned}$$

$$\boxed{\delta = \delta_{1,2} = \frac{m_G^{(1)}}{m_I^{(1)}} - \frac{m_G^{(2)}}{m_I^{(2)}} \simeq \eta_{1,2}}$$

← Controlled to 0 in MICROSCOPE

← Not considered here

On the x sensitive axis

$$\boxed{2\gamma_x^{(d)} = \delta g_x^{(s)} + (T_{xx}-I_{xx})\Delta_x + (T_{xy}-I_{xy})\Delta_y + (T_{xz}-I_{xz})\Delta_z}$$

Earth's gravity : monopole in « nominal » frame

- *Nominal frame :* • *y axis normal to the (osculating) orbital plane*
  - • *in plan axes (x and z) in uniform rotation w.r.t. nodal frame*
- *ω_0 = angular position of satellite along the orbit measured from the node (~ orbital frequency)*
- *S = angle between z and the ascending node (rotational frequency)*

Gravity acceleration :

$$g_x^{(s)} = -\frac{\mu}{r^2}\sin(\omega_o - S) \longleftarrow \text{Sensitive axis => EP signal}$$

$$g_y^{(s)} = 0 \longleftarrow \text{Axis normal to the osculating orbital plane}$$

$$g_z^{(s)} = -\frac{\mu}{r^2}\cos(\omega_o - S)$$

Gravity gradient :

$$\left.\begin{aligned} T_{xx}^{(s)} &= \frac{1}{2}\frac{\mu}{r^3}(1 - 3\cos 2(\omega_o - S)) \\ T_{xy}^{(s)} &= 0 \\ T_{xz}^{(s)} &= \frac{3}{2}\frac{\mu}{r^3}\sin 2(\omega_o - S) \end{aligned}\right\} \longleftarrow \text{Quantifies the impact of the off-centring}$$

$$T_{yy}^{(s)} = -\frac{\mu}{r^3}$$

$$T_{yz}^{(s)} = 0$$

$$T_{zz}^{(s)} = \frac{1}{2}\frac{\mu}{r^3}(1 + 3\cos 2(\omega_o - S))$$

Quasi-inertial configuration

$S = S_0 \Longrightarrow aep = 2\pi f_o(t - t_0) + (u_0 - S_0) \Longrightarrow fep = f_o$

$g_x = -\frac{\mu}{a^2}$ [redacted] $+ 2e\sin(2aep - \omega + S_0)]$ (1)

* *Main EP signal is at frequency fep =* f_o

$T_{xx} = \frac{1}{2}\frac{\mu}{a^3}\Big[$ [redacted]

$+$ [redacted] $- \frac{21}{2}\cos(3aep - \omega + S_0)\Big)\Big]$

$T_{xy} = 0$

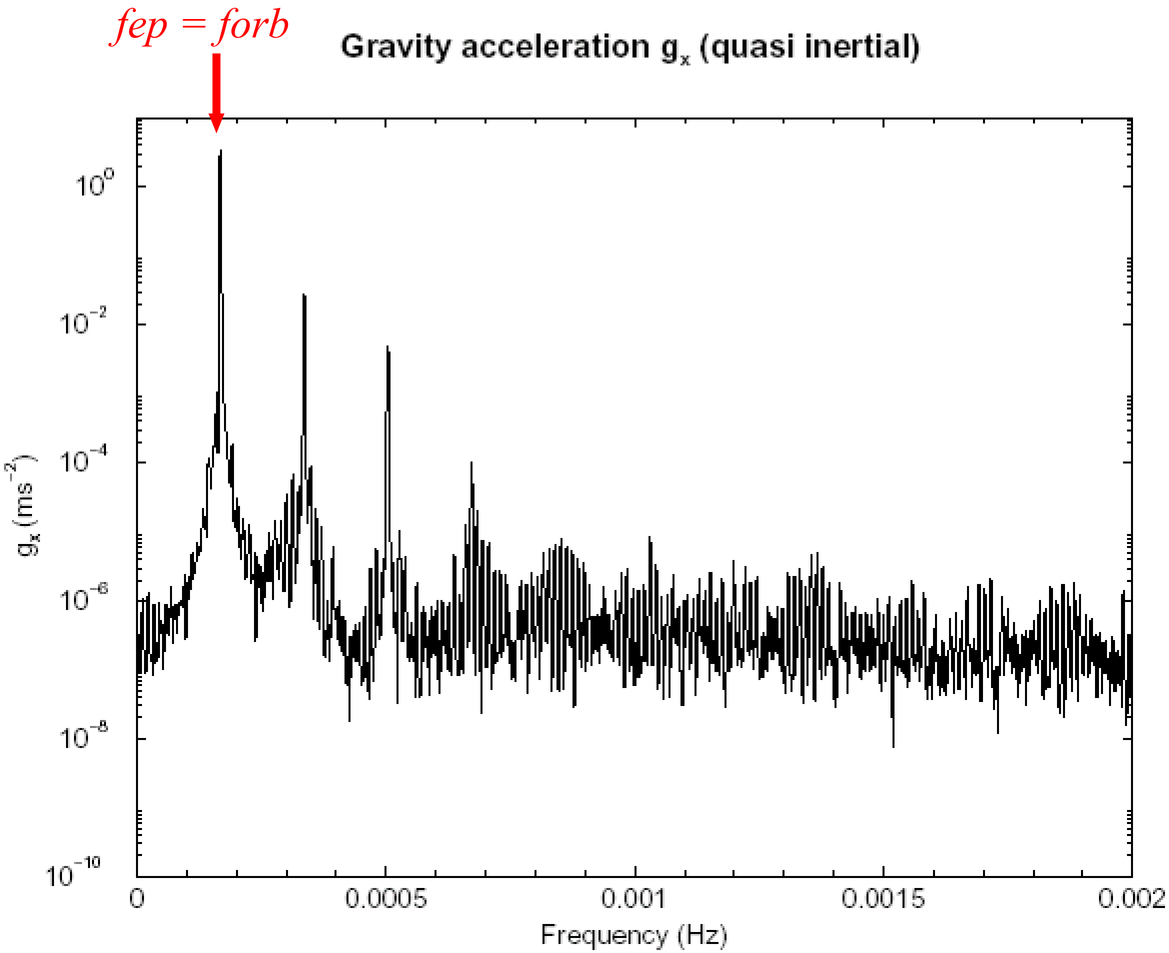
$T_{xz} = \frac{1}{2}\frac{\mu}{a^3}\Big[$ [redacted]

$+$ [redacted] $\frac{21}{2}\sin(3aep - \omega + S_0)\Big)\Big]$

* *Main gradient signal at frequency 0 and 2 fep (uncorrelated)*

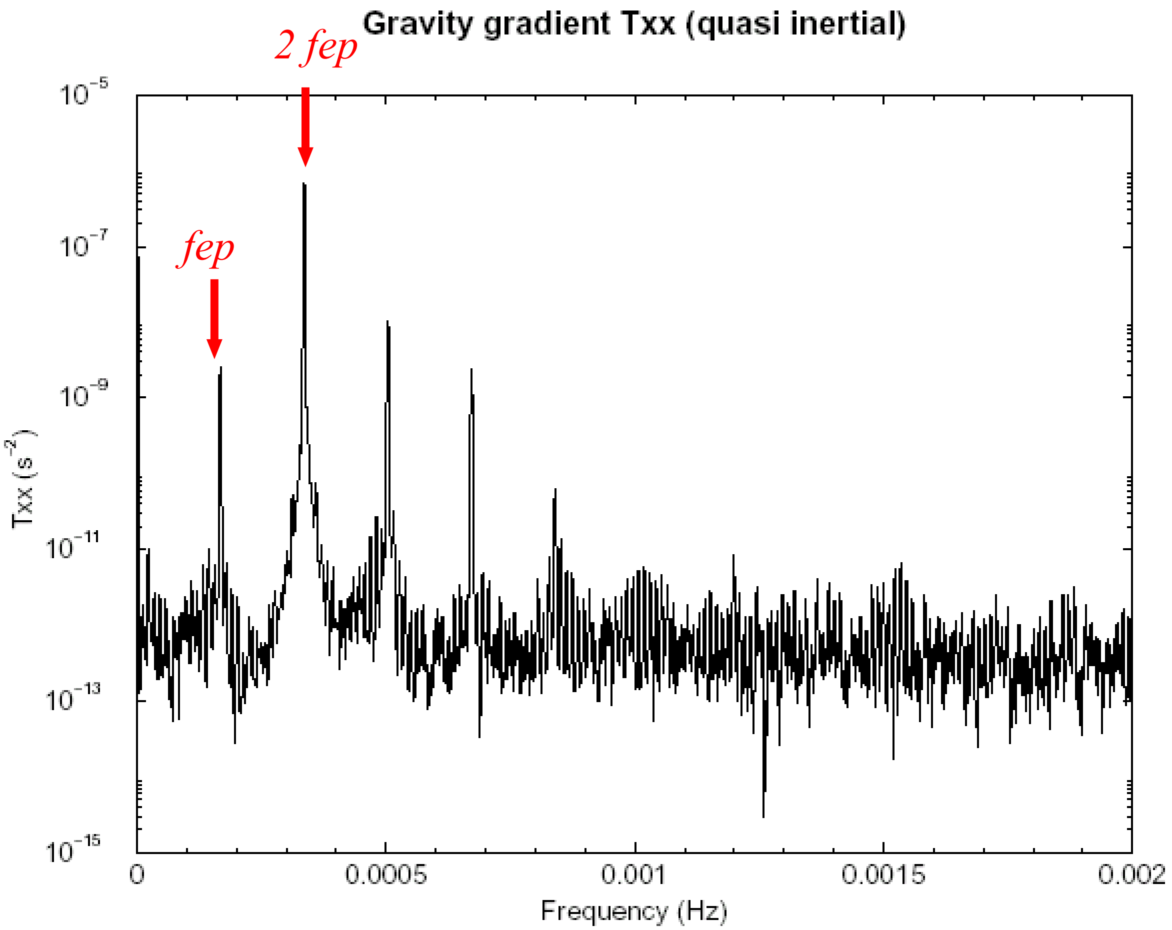
Secondary signal at EP frequency due to in plane off-centring (Δx and Δz)
⇒ must be minimized / corrected

$e = 0.005$



fep = forb
Gravity acceleration g_x (quasi inertial)
g_x (ms^{-2})
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0
0.0005
0.001
0.0015
0.002
Frequency (Hz)

e = 0.005


Gravity gradient Txx (quasi inertial)
2 fep
fep
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
Txx (s^{-2})
0
0.0005
0.001
0.0015
0.002
Frequency (Hz)

Constrains on the eccentricity and on the estimation of the off-centring

Error on the EP parameter (inertial case) : $Er(\delta) = \frac{3}{4}\frac{e\Delta_x}{a}\sin(S_0 - \omega) + \frac{3}{4}\frac{e\Delta_z}{a}\cos(S_0 - \omega)$

For h = 700 km : $10^{15}||Er(\delta)|| \leq 100\, e\Delta(\mu\text{m}) = 10 \times \frac{e}{5\,10^{-3}} \times \frac{\Delta}{20\mu\text{m}}$

Error after correction : $10^{15}||Er(\delta)|| = 100\, [\underbrace{e \times Er(\Delta)} + \underbrace{Er(e) \times \Delta}]$

e as small as possible
+ estimation of Δ*x* and Δ*z*

Relaxed in rotating mode

From manufacturing off-centring as small as possible
+ estimation of *e* (more generally of the position)

Positioning considered below

How to estimate Δx and Δz?

We use the main gravity gradient signal at 2 fep frequency :

$T_{xx}^{(2fep)}$ [redacted] $\simeq$ [redacted] $\cos(2aep)\ \mathrm{ms}^{-2}$

$T_{xz}^{(2fep}$ [redacted] $\simeq$ [redacted] $\sin(2aep)\ \mathrm{ms}^{-2}$

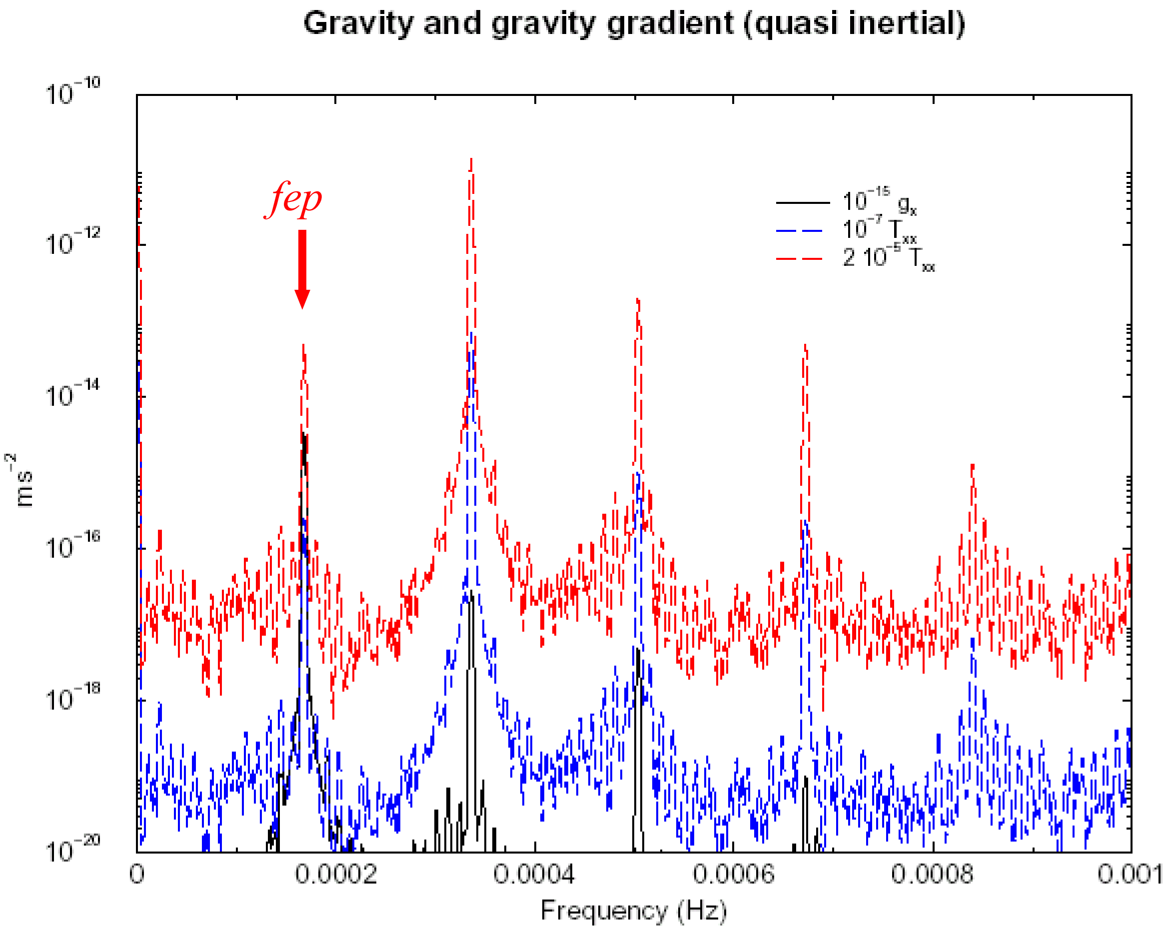
2 uncorrelated signals

To compare to the 8 10^{-15} ms^{-2} needed for the EP signal

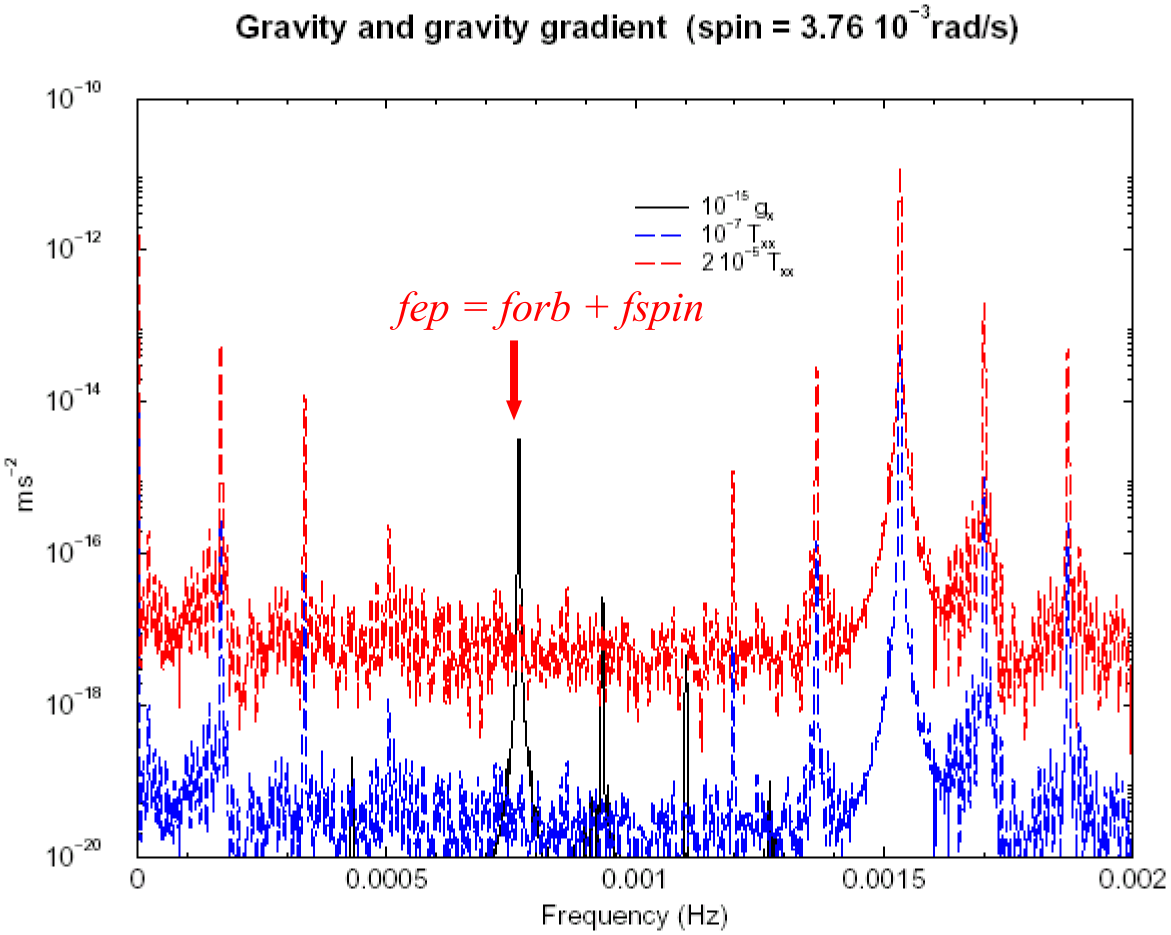
Our requirement is to estimate Δx and Δz to 0.1 µm : OK

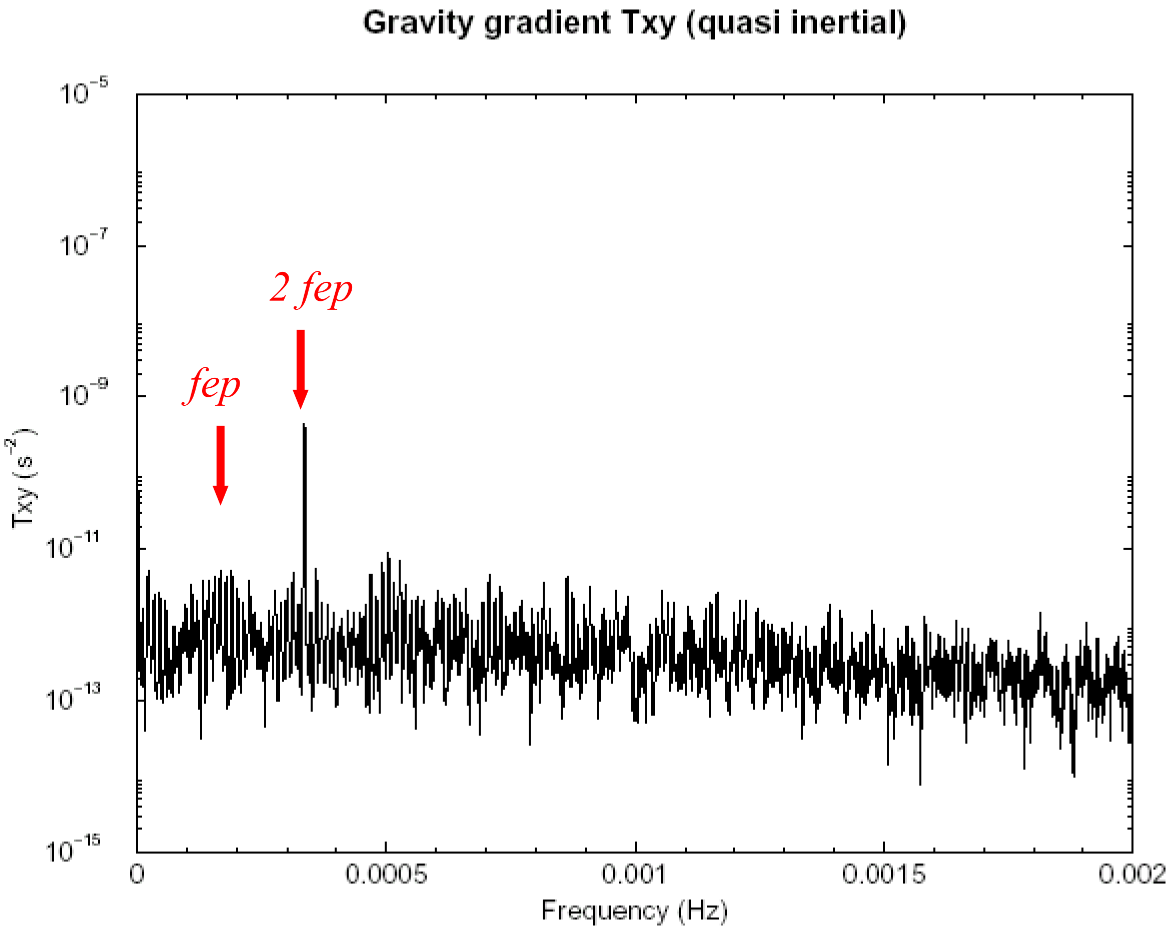
We cannot estimate Δy by this method but Δy is not required

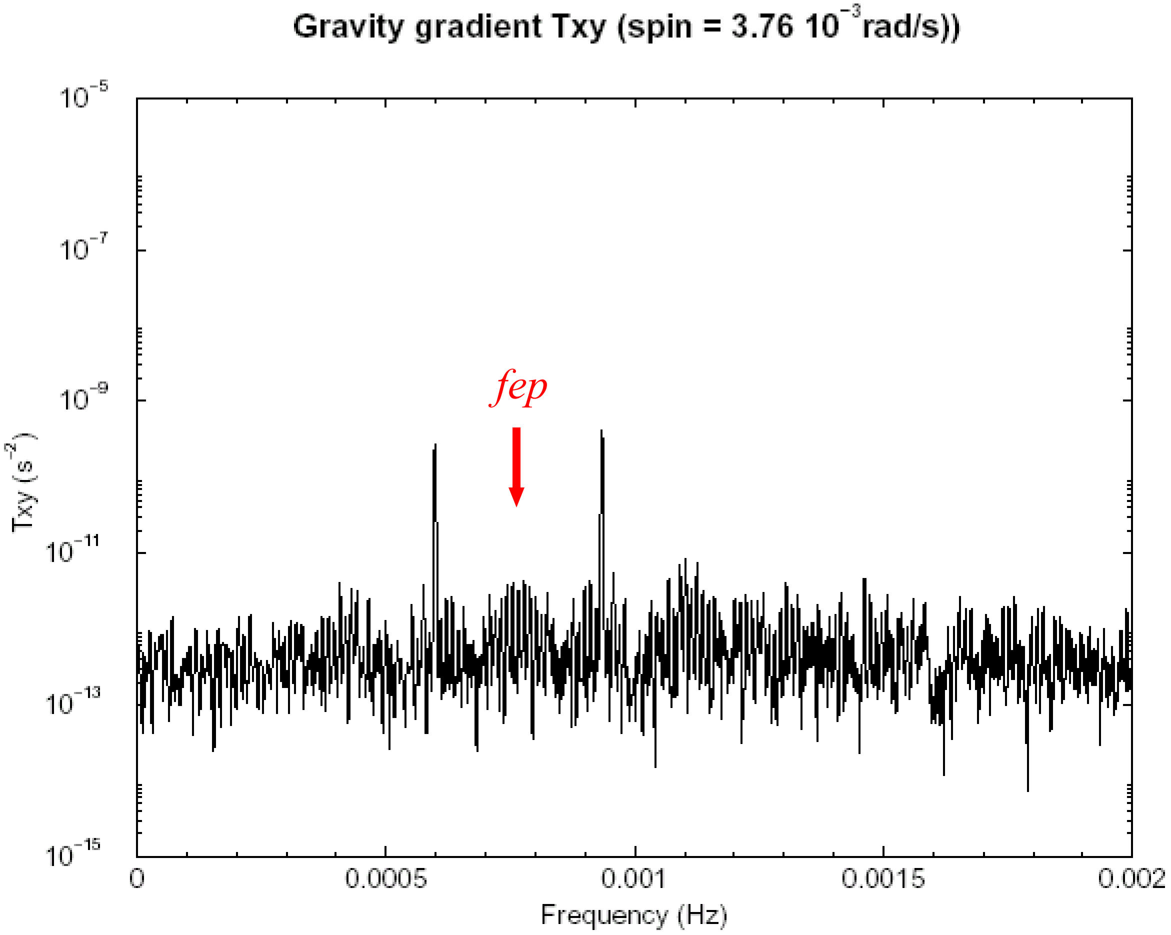
e = 0.005


Gravity and gravity gradient (quasi inertial)
fep
10^{-15} g_x
10^{-7} T_{xx}
$2\ 10^{-5}$ T_{xx}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
ms^{-2}
0
0.0002
0.0004
0.0006
0.0008
0.001
Frequency (Hz)

e = 0.005


Gravity and gravity gradient (spin = 3.76 10^{-3}rad/s)
10^{-15} g_x
10^{-7} T_{xx}
2 10^{-5} T_{xx}
fep = forb + fspin
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
ms^{-2}
0
0.0005
0.001
0.0015
0.002
Frequency (Hz)


Gravity gradient Txy (quasi inertial)
Txy (s^-2)
10^-5
10^-7
10^-9
10^-11
10^-13
10^-15
0
0.0005
0.001
0.0015
0.002
Frequency (Hz)
fep
2 fep



Gravity gradient Txy (spin = 3.76 10^{-3} rad/s))

Satellite positioning

@ Errors in the estimated position at some particular frequencies yield errors on the gravity gradient at EP frequency

$$\begin{aligned}
\Delta X_r &= \Delta X_{r,1}\cos(aep+\tau_{r,1}) + \Delta X_{r,3}\cos(3aep+\tau_{r,3}) && = \text{error on radial component}\\
\Delta X_t &= \Delta X_{t,1}\cos(aep+\tau_{t,1}) + \Delta X_{t,3}\cos(3aep+\tau_{t,3}) && = \text{error on along track component}\\
\Delta X_n &= \Delta X_{n,0} + \Delta X_{n,2}\cos(2aep+\tau_{n,2}) && = \text{error on normal component}
\end{aligned}$$

$$\begin{aligned}
10^{15}Err_2(\delta) = 0.015\Big\{\Big[&5\Delta X_{r,1}\sin(\tau_{r,1}) + 2\Delta X_{t,1}\cos(\tau_{t,1})\\
&- 3\Delta X_{r,3}\sin(\tau_{r,3}) - 2\Delta X_{t,3}\cos(\tau_{t,3})\Big]\Delta x\\
+ \Big[&- 3\Delta X_{r,1}\cos(\tau_{r,1}) + 2\Delta X_{t,1}\sin(\tau_{t,1})\\
&+ 3\Delta X_{r,3}\cos(\tau_{r,3}) - 2\Delta X_{t,3}\sin(\tau_{t,3})\Big]\Delta z\Big\}
\end{aligned}$$

- off centering in µm
- position error in km

Requires position errors smaller than 100 meters

Dating and synchronization

- *The gravity gradient is not constant*

- *If we use the difference of accelerations not measured at the same time or if we correct the signal using a gravity gradient computed at a wrong date…*

- *… we can get a false signal*

→ *Necessity to specify dating and synchronisation.*

- *The requirements depends on the frequencies and on the configuration; typically :*
  - ►*bias of absolute dating < 0.5 s*
  - ►*bias of synchronization < 0.04 s*
  - ► *error of dating at fep and 3 fep frequencies < 0.004 s*
  - ► *error of synchronization at fep and 3 fep frequencies <* 10^{-4} *s*

Spacecraft self gravity

Nominally, no moving mass in the spacecraft ➔ no temporal variation of the gravity, but possible vibrations and thermal distorsions

- ► *Monopole part of proof masses ➔ variation of the gravity gradient due to small deformations of the spacecraft:*

$$dT_{X,X} = \sum_{m_i} 3\,\frac{Gm_i}{{R_i}^7}\left((3\,X_i{R_i}^2 - 5\,{X_i}^3)dX_i + (Y_i{R_i}^2 - 5\,Y_i{X_i}^2)dY_i + (Z_i{R_i}^2 - 5\,{X_i}^2 Z_i)dZ_i\right)$$

e.g. : 1 kg at 30 cm with deformations of 10 µm ➔ ~ 10^{-18} ms^{-2}

10 g at 5 cm with deformations of 10 µm ➔ ~ 2 10^{-17} ms^{-2}

- ► *Imperfect spherical symmetry of proof masses ➔ differences of moments of inertial :*

$$d\Delta g_{x_i,2+3} = \sum_{m_i}\left(\frac{\partial\Delta g_{x_i,2+3}}{\partial X_i}dX_i + \frac{\partial\Delta g_{x_i,2+3}}{\partial Y_i}dY + \frac{\partial\Delta g_{x_i,2+3}}{\partial Z_i}dZ_i\right)$$

Proof mass with moments of 5 10^{-4} kg m^2 with differences of 500 ppm

1 kg at 30 cm with deformations of 10 µm ➔ ~ 10^{-19} ms^{-2}

10 g at 5 cm with deformations of 10 µm ➔ ~ 5 10^{-18} ms^{-2}

All these effects are computed by CNES using mass distribution in the spacecraft and thermal model ➔ OK

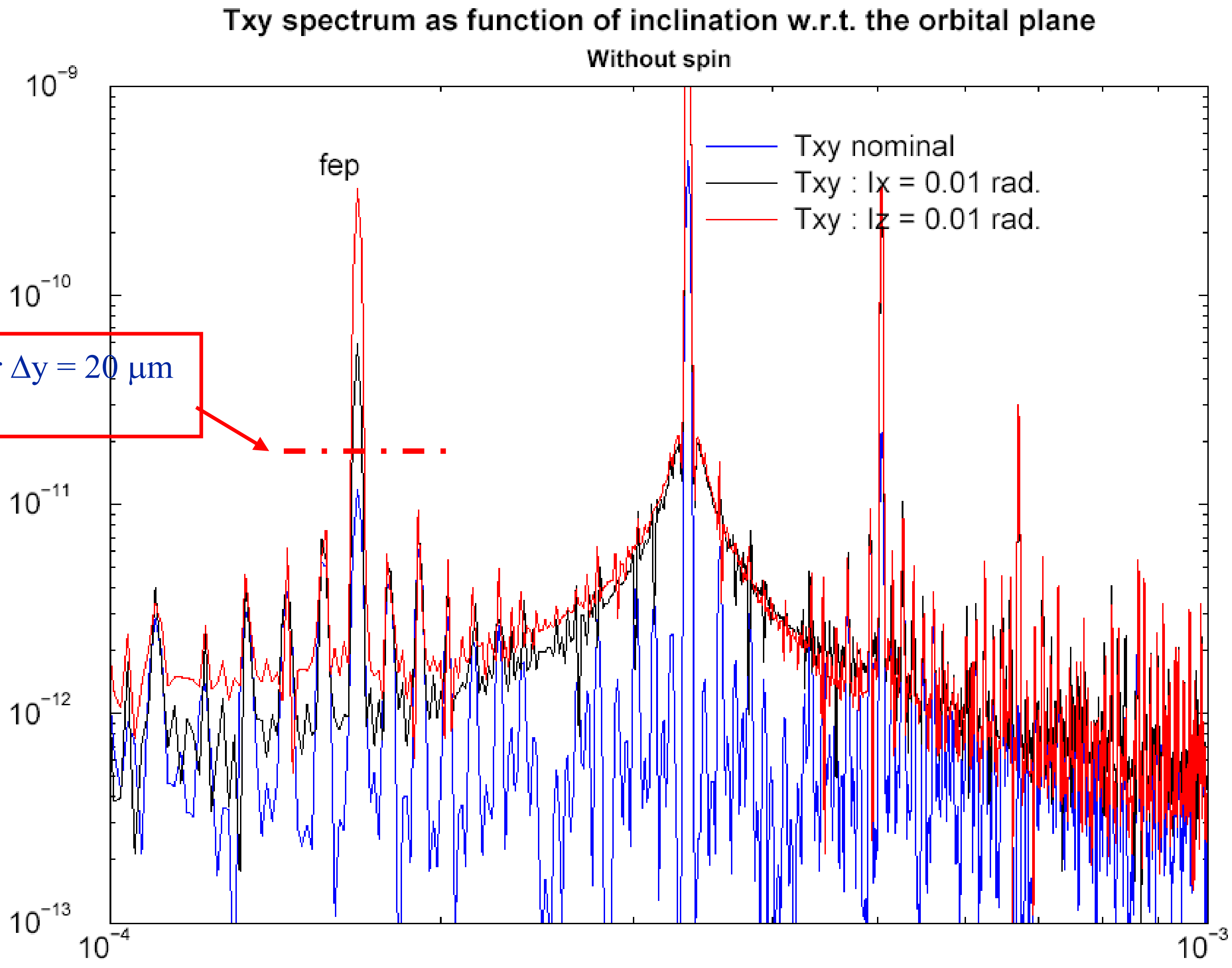
Attitude knowledge and control

- *Attitude knowledge necessary to correctly remove the gravity gradient signal (*$T_{xx}\ \Delta x$ *and* $T_{xz}\ \Delta z$*)*

- *Attitude control necessary to keep sensitive axis // to the orbital plane (allows to minimize* T_{xy}*)*

- *Here also, the requirements depends on the frequencies, but we need typically :*
  - ► 10^{-3} *rad in bias*
  - ► 10^{-5} *rad at fep and 3 fep frequencies*

Why we want to control the attitude



Txy spectrum as function of inclination w.r.t. the orbital plane

Without spin

Requirement for Δy = 20 μm and not known

Stability of angular velocity and acceleration

$$2\gamma_x^{(d)} = \delta g_x^{(s)} + (T_{xx} - I_{xx})\Delta_x + (T_{xy} - I_{xy})\Delta_y + (T_{xz} - I_{xz})\Delta_z$$

$$\begin{aligned} dI_{xx} &= -2\Omega_y\, d\Omega_y - 2\Omega_z\, d\Omega_z \\ dI_{xy} &= d\Omega_x\, \Omega_y + \Omega_x\, d\Omega_y - d\dot{\Omega}_z \\ dI_{xz} &= d\Omega_x\, \Omega_z + \Omega_x\, d\Omega_z + d\dot{\Omega}_y \end{aligned}$$

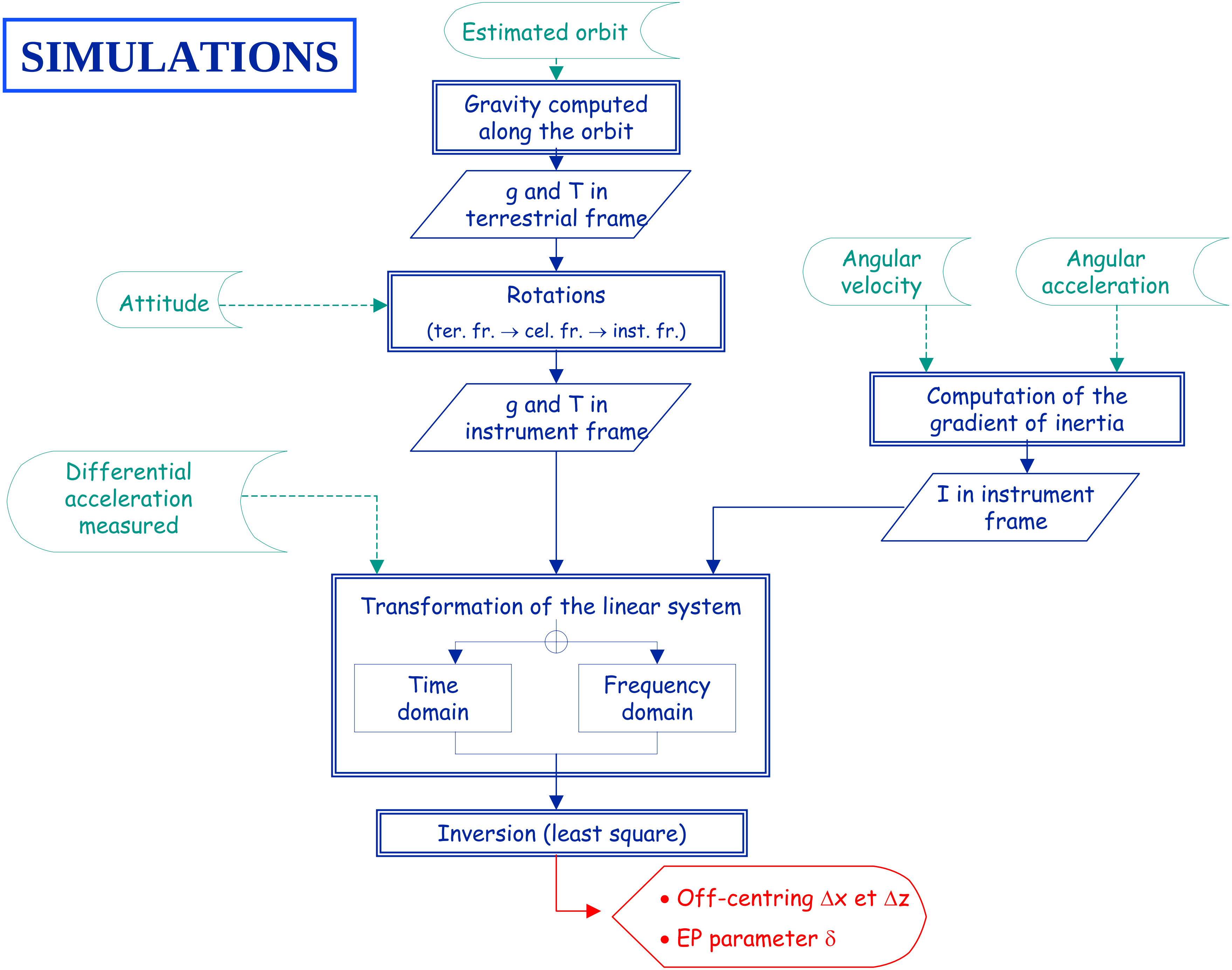
$$dI_{x\alpha}(fep) < \frac{2\,10^{-16}\mathrm{ms}^{-2}}{20\,10^{-6}\mu\mathrm{m}} < 2\,10^{-11}\mathrm{rad\,s}^{-2}$$

$\Omega_x(fep) < 10^{-6}$ rad s^{-1} $\quad \Omega_y < 5 10^{-3}$ rad s^{-1}(spinné) $\quad \Omega_z(fep) < 10^{-6}$ rad s^{-2}

$\Downarrow$

$$\begin{aligned} d\Omega_x(fep) \text{ and } d\Omega_y(fep) &< 10^{-9}\mathrm{rad/s} \\ d\dot{\Omega}_y \text{ and } d\dot{\Omega}_z &< 10^{-11}\mathrm{rad/s}^2 \end{aligned} \longrightarrow \begin{cases} < 0.2\ \mu\textit{rad for attitude stability in rotating mode} \\ < 10\ \mu\textit{rad for attitude stability in inertial mode} \end{cases}$$

SIMULATIONS



Estimated orbit

Gravity computed along the orbit

g and T in terrestrial frame

Attitude

Rotations

(ter. fr. → cel. fr. → inst. fr.)

Angular velocity

Angular acceleration

g and T in instrument frame

Computation of the gradient of inertia

Differential acceleration measured

I in instrument frame

Transformation of the linear system

Time domain

Frequency domain

Inversion (least square)

- Off-centring Δx et Δz
- EP parameter δ

How to handle the differential signal ?

Equation to solve :

Design matrix A

observations Y

$$\begin{pmatrix} g_x(t_1) & T_{xx}(t_1) & T_{xz}(t_1) \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ g_x(t_N) & T_{xx}(t_N) & T_{xz}(t_N) \end{pmatrix} \begin{pmatrix} \delta \\ \Delta x \\ \Delta z \end{pmatrix} = \begin{pmatrix} \Delta\gamma_x(t_1) \\ \cdot \\ \cdot \\ \cdot \\ \Delta\gamma_x(t_N) \end{pmatrix}$$

parameters X

$$\Rightarrow \quad X = (A^T P A)^{-1} A^T P Y$$

P : weight matrix = inverse of the covariance matrix

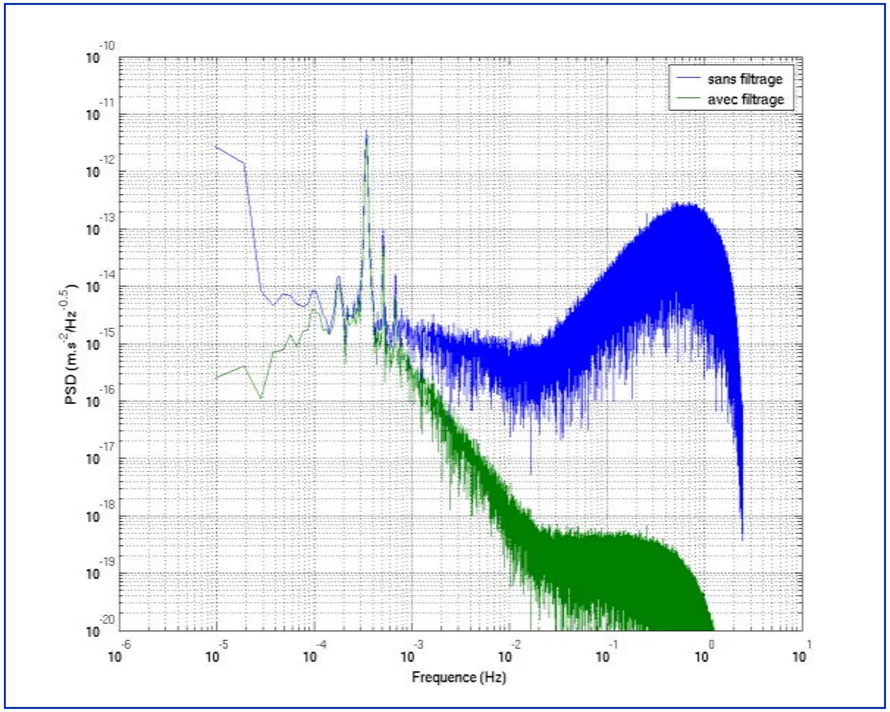
Difficulties :

- P non diagonale for non white noise
- Covariance matrix difficult to know accurately
- Even if known, problem of inversion (typical dimension = 1 000 000)

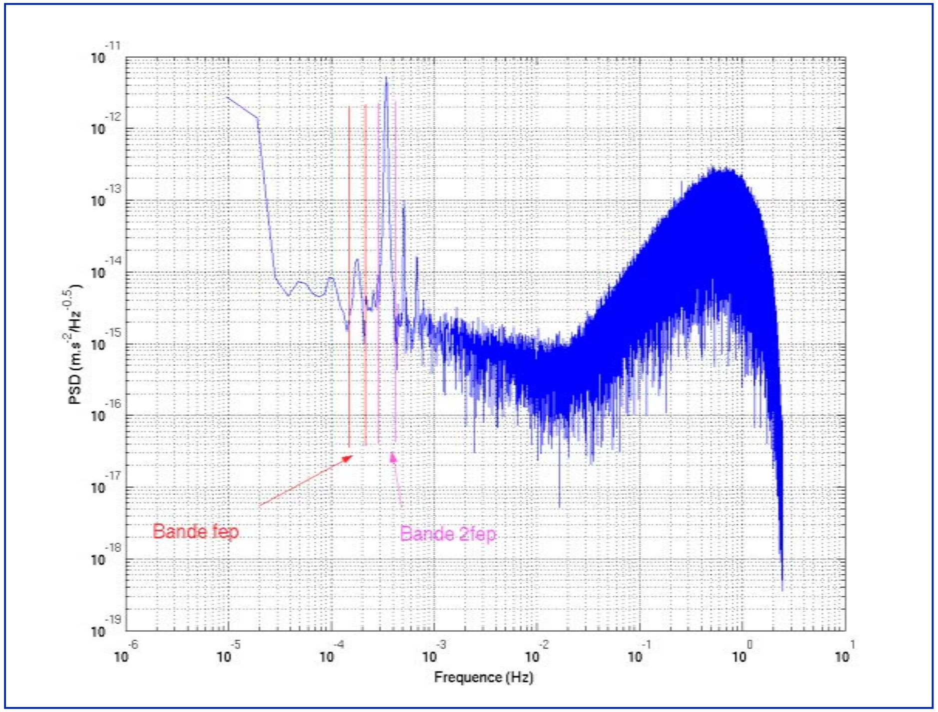
Our solutions :

➡ Transform the linear system

TIME DOMAIN

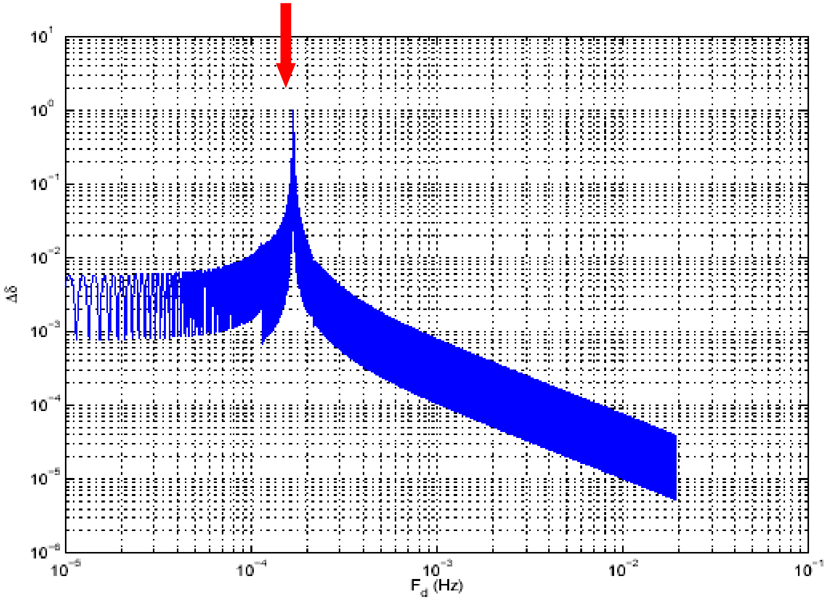


FREQUENCY DOMAIN



How do we reject a perturbing signal ?

Maximum error on δ due to a perturbing signal of amplitude 1
as a function of the frequency of the perturber

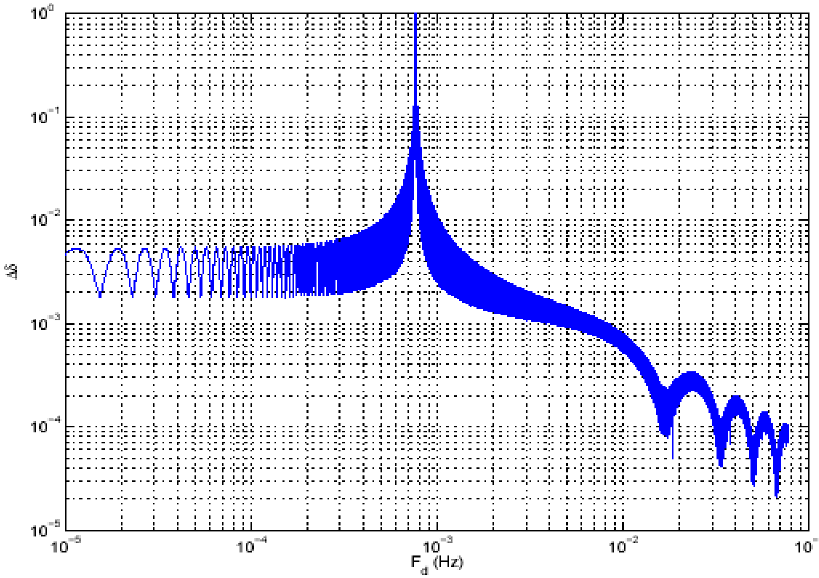


EP frequency

Inertial mode (88 orbits ~ 6 days)

Influence of a data gap

Maximum error on d due to a perturbing signal of amplitude 1 as a function of the frequency of the perturber :

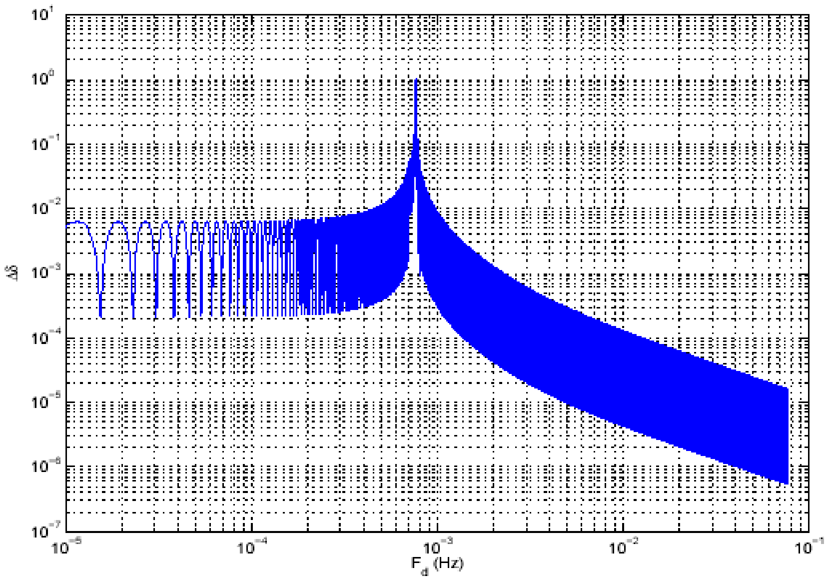


Gap of 1 minute (22 orbits ~ 1.5 days)

Conclusion

- *If nothing is done, the differential signal measured at fep is dominated by the gravitational gradient ($e=5\ 10^{-3}$, $\Delta=20\ \mu m$).*
- *Estimation of the off-centring along x and z with an accuracy of 0.1 µm is sufficient to correct the signal.*
- *This estimation can be performed by using the larger signal of the gravitational gradient at frequency 2 fep.*
- *This requires the knowledge of the satellite position (100 m) and attitude (10^{-5} rd) and absolute time of measurement.*
- *Satellite self gravity is not a real problem.*
- *Analysis of simulated data allows to check these results.*
- *We have also studied the effect of an unknown perturbation at different frequencies and the impact of possible gaps in the data.*

Rotating mode :



Influence of the window :