Relativity tests with Gaia

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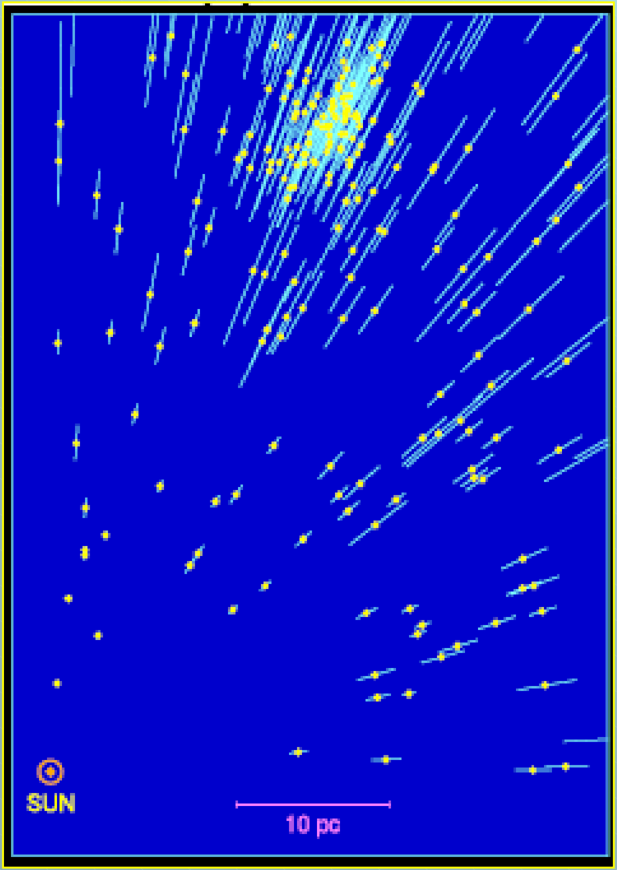
Summary

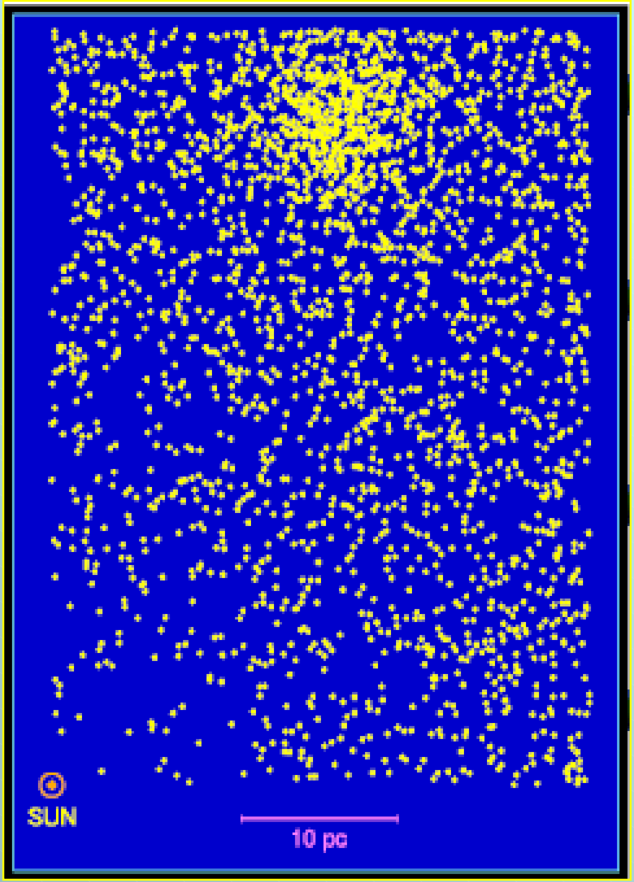
- **The space astrometric mission Gaia**
  - relativistic astrometry and fundamental physics
- **Gaia as an experiment for general relativity**
  - light deflection
  - quadrupole effect
  - perihelion precession

The astrometric context

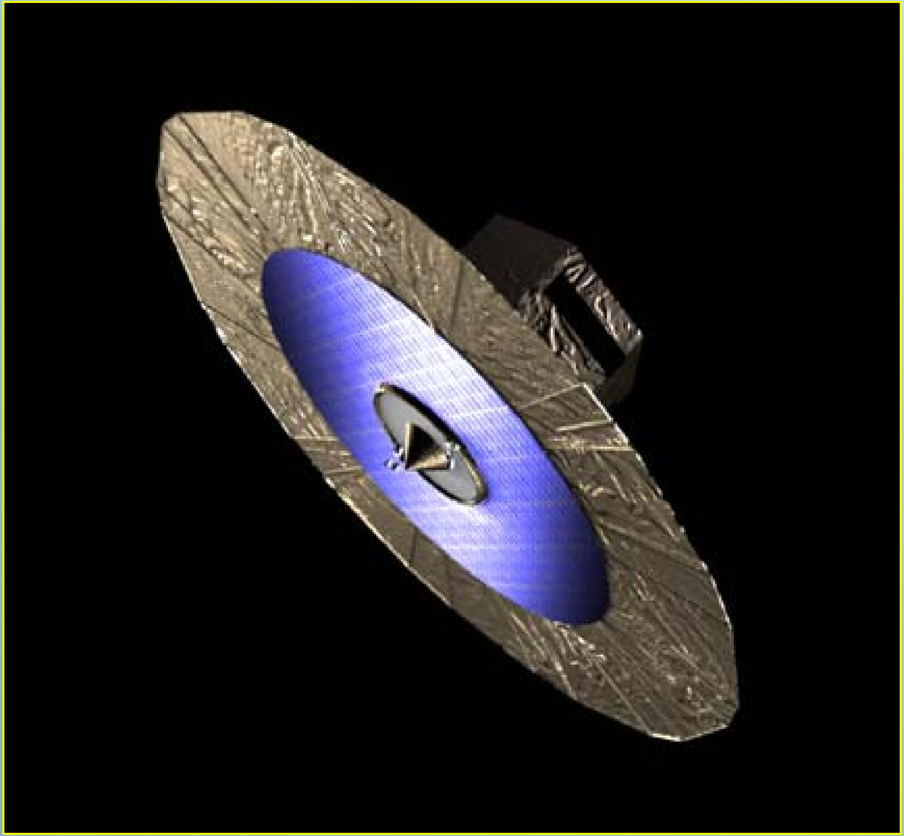
- Hipparcos, launched by ESA in 1989

- Gaia, the successor, selected by ESA as Cornerstone within the Cosmic Vision program, will be launched in 2012

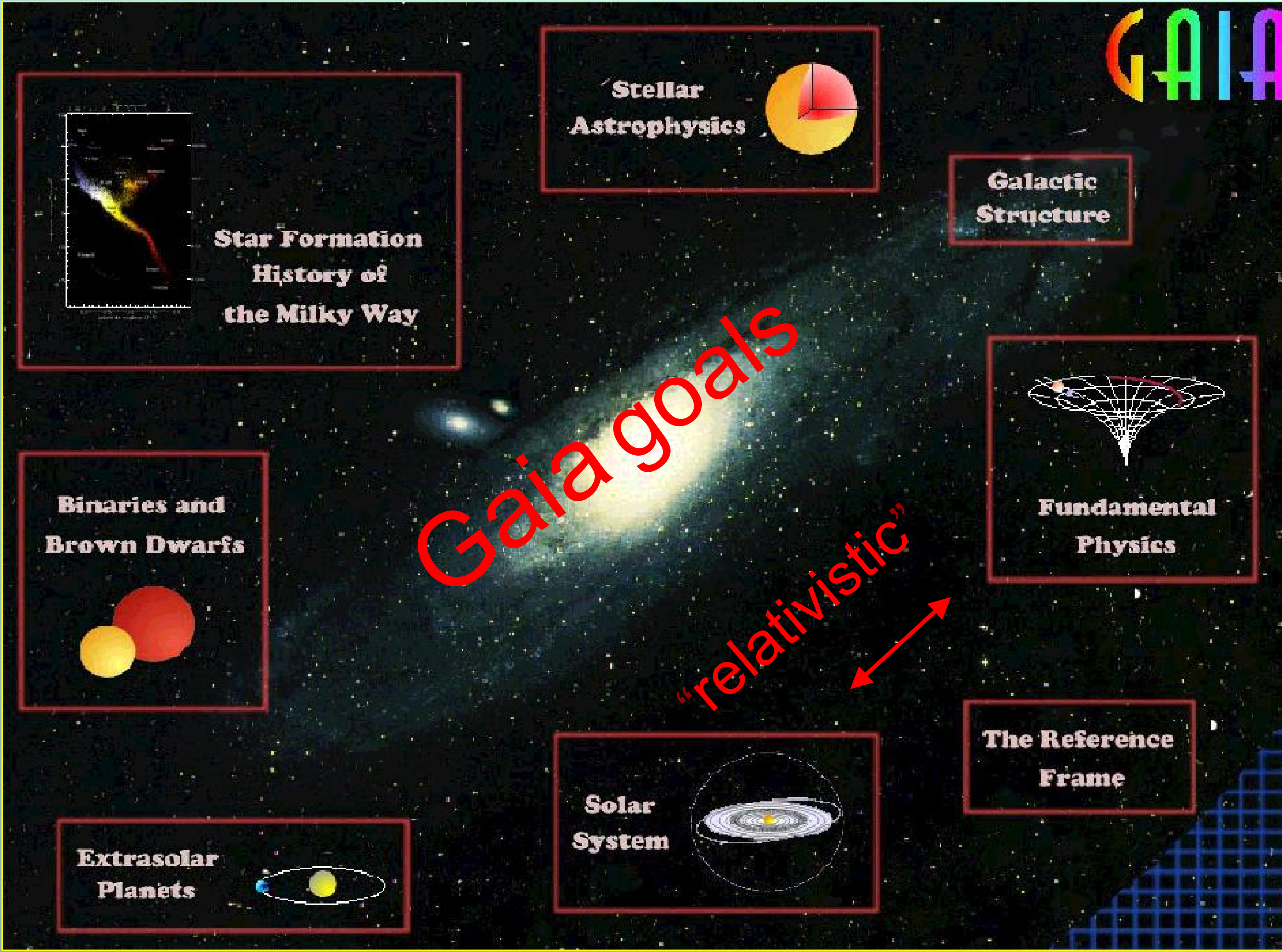




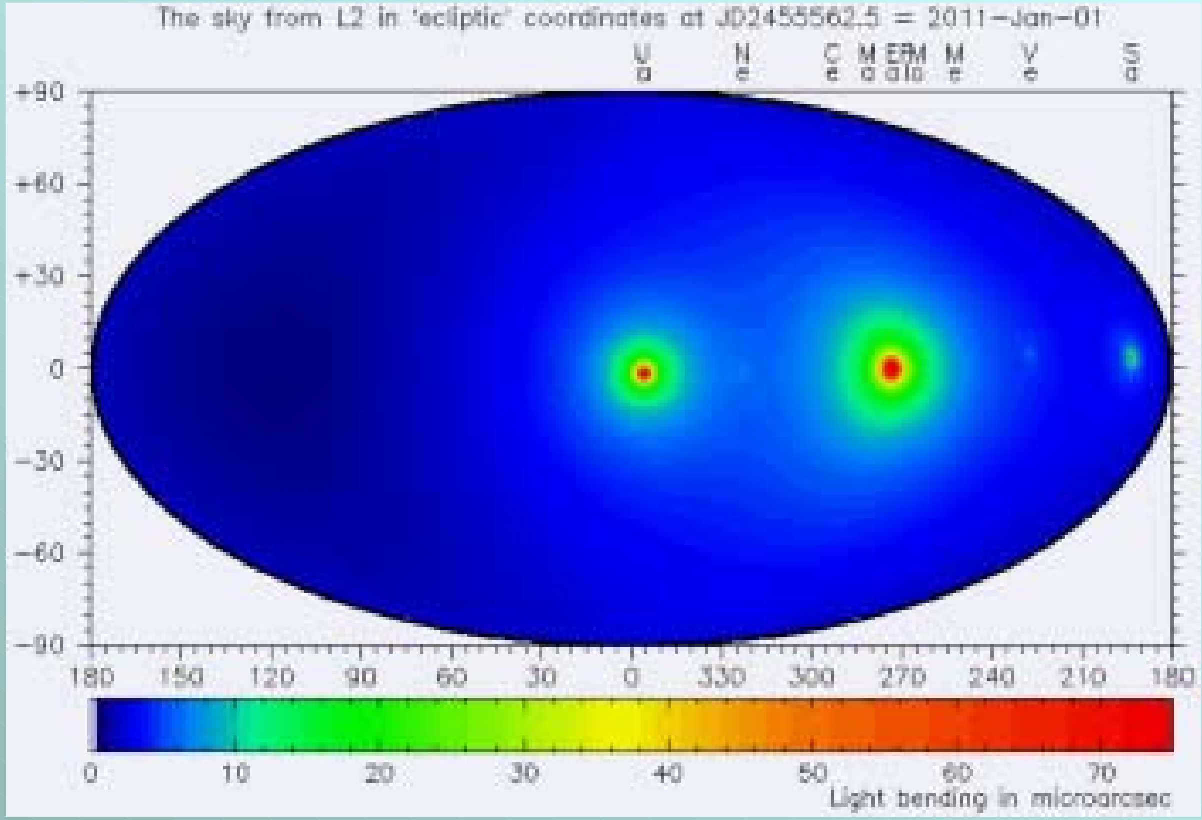
The Gaia mission



- Lissajous orbit around L2 (Sun-Earth/Moon system)
- 5 years of continuous observations
- Same scanning law of Hipparcos
- 10^9 observed stars down to V=20; final accuracy of 10 μas at V=15
- multi-epoch, multi-colour photometry and radial velocity to 1-10 km/s down to V=16-17


GAIA
Gaia goals
Stellar Astrophysics
Galactic Structure
Star Formation History of the Milky Way
Binaries and Brown Dwarfs
Fundamental Physics
“relativistic”
The Reference Frame
Solar System
Extrasolar Planets

Relativistic astrometry & fundamental physics



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astro.estec.nl/Gaia

- High accuracy!!! At the µas level of accuracy, new relativistic effects in light deflection are detectable
- a careful choice of the relativistic model will allow to use the satellite measurements for a highly accurate test of GR and fundamental physics.

Gaia relativity tests in the Solar System

Deflections of the light

PPN parameter γ

Effects due to a planet

Precession of the perihelion

PPN parameter β

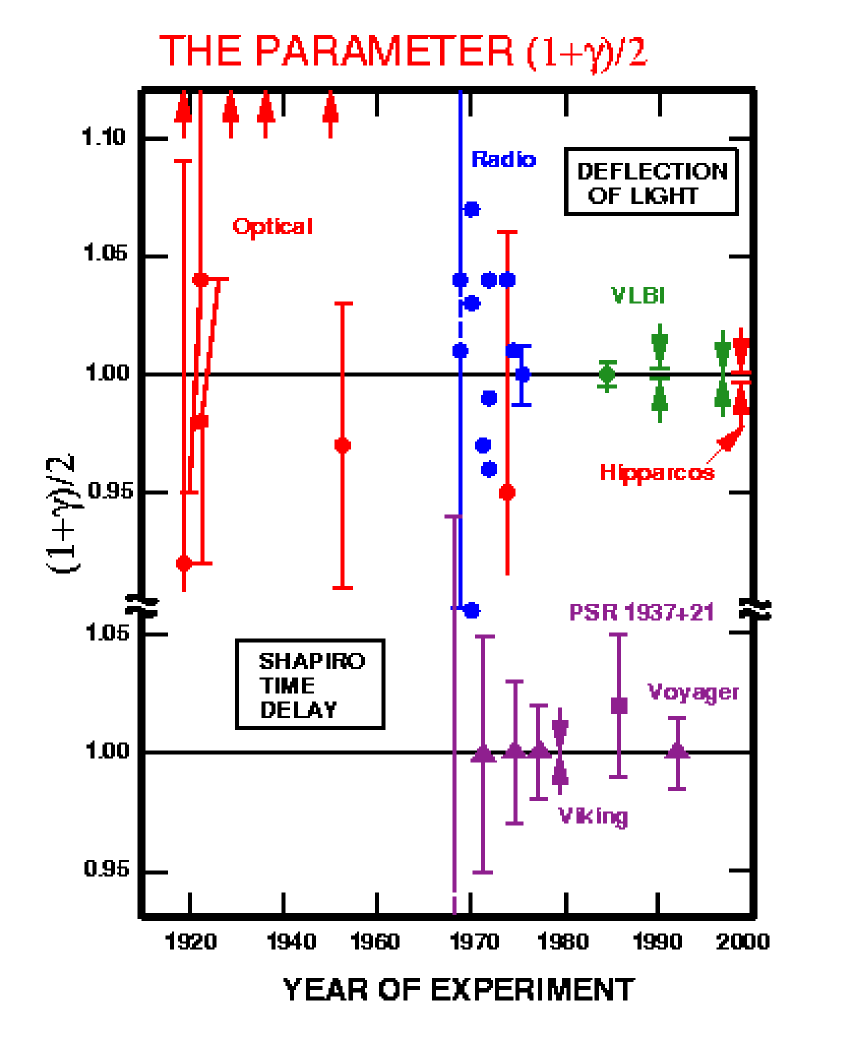
What γ and β measure

- The PPN parameter γ measures the excess of curvature or amount of space-curvature produced by unit rest mass

- The PPN parameter β measures the amount of non-linearity in the superposition law of gravitational fields

In GR γ and β are both equal to 1; other theories, called *scalar-tensorial*, predict small deviations from GR value

Current Limits



$\sigma_{\gamma-1}$

Time delay	2×10^{-3}	Viking
Deflection	3×10^{-4}	VLBI

❖Cassini-Earth-Sun conjunction has updated the estimation of γ (B. Bertotti, L.Iess & P.Tortora, Nature, ***425***, 2003)

$$\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$$

$\sigma_{\beta-1}$

Perihelion shift	3×10^{-3}	$(J_2 = 10^{-7})$
Nordtvedt (LLR)	6×10^{-4}	$(4\beta-\gamma-3)$

from Will, 1998

GPB determination of γ

- Geodesic precession : $$8.4\left[\frac{2\gamma+1}{3}\right]\left(\frac{R}{a}\right)^{5/2} \text{ "/yr}$$
- Frame dragging : $$0.055\left[\frac{\gamma+1}{2}\right]\left(\frac{R}{a}\right)^{3} \text{ "/yr}$$
- Measurements at 0.1 mas/yr ➔ γ should come at $3\text{x}10^{-5}$

The physical implication of γ

- If the universe is evolving, today we can expect a very small discrepancy of γ by unit in the range

$$|\gamma - 1| \approx 10^{-5} - 10^{-7}$$

A remnant of a long range scalar field would violate General Relativity and also the assumptions in the equivalence principle (i.e. lack of universality of the constants of microphysics).

The exact amount of the violations depends on the particular scalar-tensor theory adopted

Light deflection-1: the PPN γ experiment with Gaia

The adopted metric is the PPN expression for the Schwarzschild metric in isotropic coordinate (in geometrized units)

$$ds^2 = -\left[(1 - \frac{2M_{Sun}}{r} + 2\beta\left(\frac{M_{Sun}}{r}\right)^2\right]c^2dt^2 + \left[1 + \gamma\frac{2M_{Sun}}{r}\right]\left[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

- Geodesics for light rays : $k^\nu k^\mu{}_{;n} = 0$

$$\cos\psi = \frac{h_{\alpha\beta}k_1^\alpha k_2^\beta}{\sqrt{h_{\iota\pi}k_1^\iota k_1^\pi}\sqrt{h_{\rho\sigma}k_2^\rho k_2^\sigma}}.$$

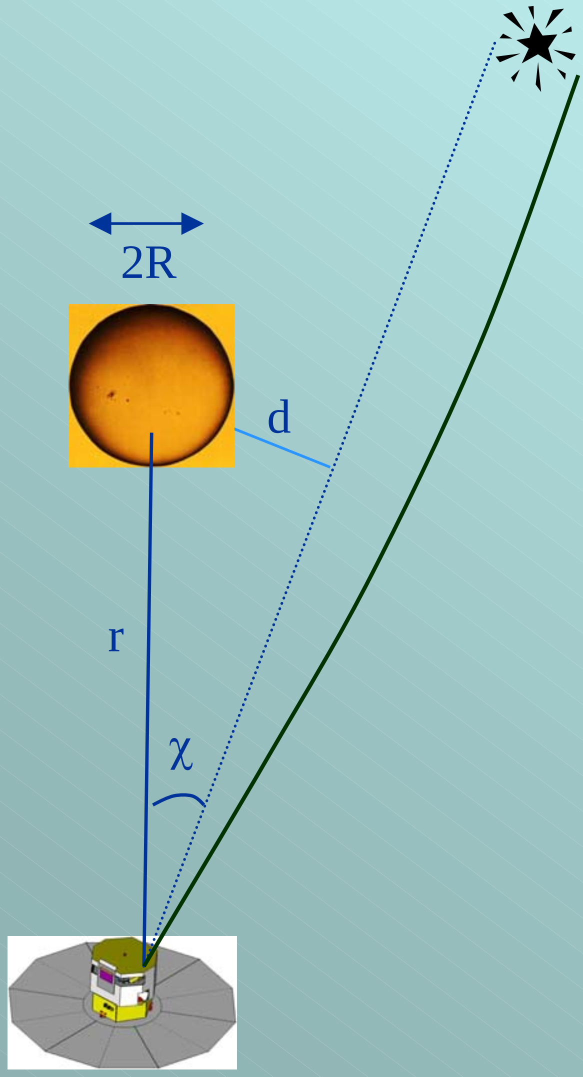
1b: PPN Ramod

$$\cos\psi = f(\lambda_i, \beta_i, p_i, \mu_{\lambda_i}, \mu_{\beta_i}, \gamma)$$

$$\mathbf{r}(t) = \mathbf{r}(t_0) + c\sigma(t - t_o) + \delta\mathbf{r}(t)$$

1a: Eddington-like experiments without eclipse

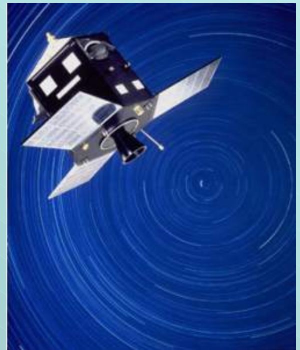
Light Deflection - 1a

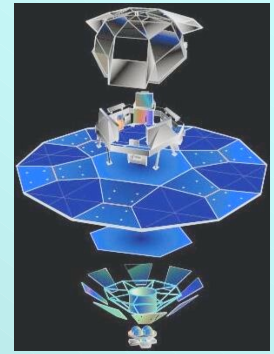


$$\delta\alpha = \frac{1+\gamma}{2} \cdot \frac{4GM}{c^2 R} \cdot \frac{R}{r} \cdot \frac{1}{2\tan\frac{\chi}{2}}$$

if $\chi << 1 = R/r \Rightarrow \delta\alpha_g = \frac{4GM}{c^2 R}$ = Grazing deflection

	Grazing (mas)	*Gaia min χ (mas)*	*Gaia $\chi = 45$ deg*
Sun	1750	13	10 mas
Earth	0.5	0.003	2.5 μas
Jupiter	16	16	2 μas
Saturn	6	6	0.3 μas





Hipparcos

- 10^5 stars V < 10
- 2.5 x 10^6 abscissas
- ♦~ 3 to 8 mas
- χ > 47 degrees

⇓

$$\gamma = 1 \pm 3 \times 10^{-3}$$

(Froeschlé, Mignard & Arénou, 1997, ESA SP-402)

Gaia

σ_H / σ_G

- 8 x 10^6 stars V < 13
- 2.5 x 10^8 abscissas ⇨ 10
- ♦~ 10 μas ⇨ 400
- χ > 40 degrees ⇨ 2
- + fainter stars

⇓

$$\sigma_\gamma \approx 1 \times 10^{-6} \text{ to } 3 \times 10^{-7}$$

(F.Mignard, in “Gaia: a European Space project”, pag 105-121, EAS,EDP Sciences 2002)

Light deflection-1b: the PPN Ramod simulation

Experimental campaign based on realistic end-to-end simulation of the PPN Relativistic Astrometric MODel in the case of astrometric mission like GAIA

- First step: computing the catalogue values of the true quantities (i.e. γ=1) + root-mean-square error

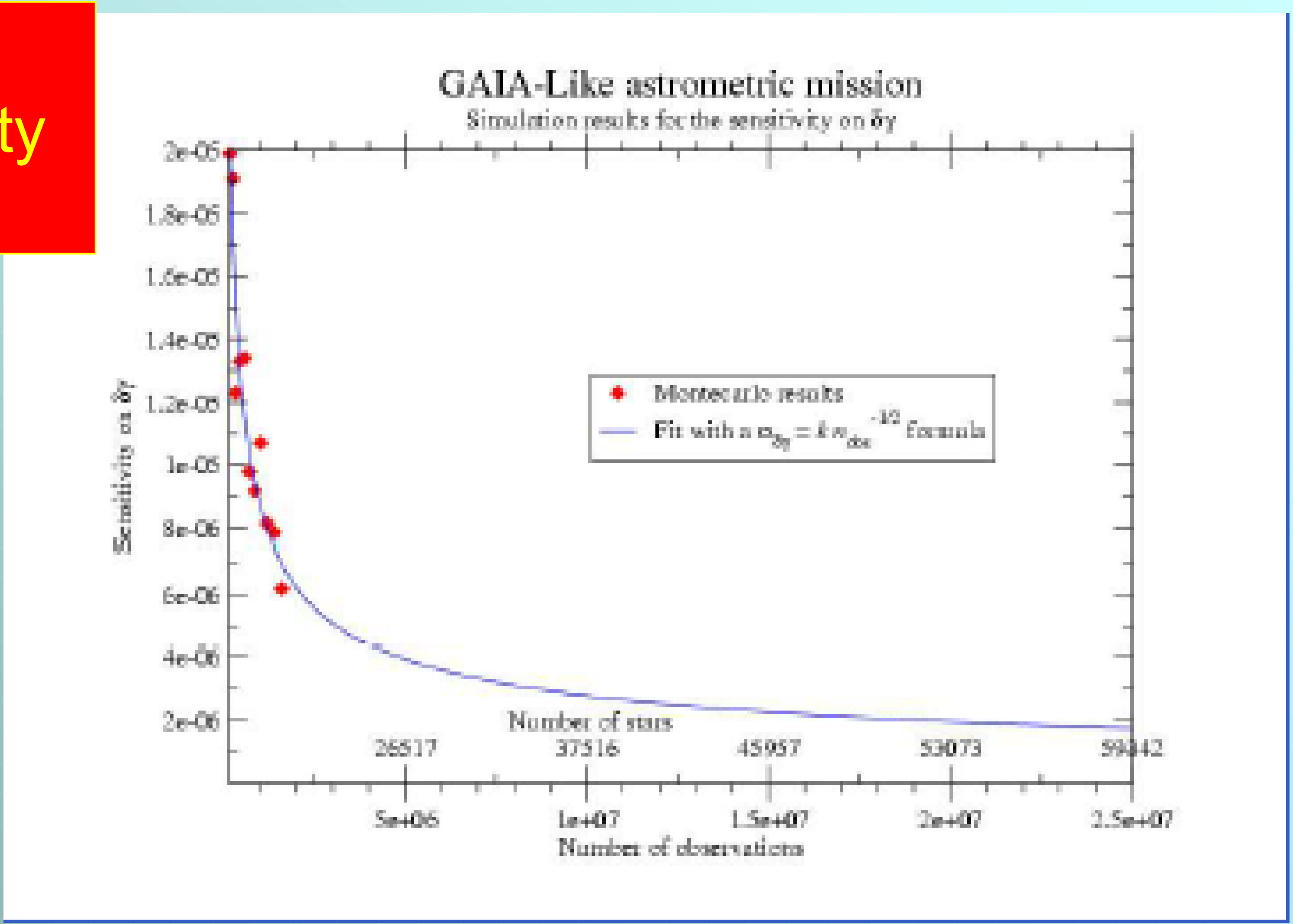
$$-\sin\psi\,\delta\psi = \sum_{n=1}^{n.unk} \frac{\partial f}{\partial x_n} \mathrm{d}x_n.$$

- Second step: generating the satellite observations by perturbing the true arcs with the observational errors expected (i.e. $\sigma_\gamma \cong 2 \times 10^{-3}$)

$$x_{new} = x_{cat} + \mathrm{d}x.$$

Reference: Vecchiato, A., Lattanzi, M. G., Bucciarelli, B., Crosta, M., de Felice, F., Gai, M., (2003), "Testing general relativity by micro-arcsecond global astrometry", *Astron. Astrophys.*, **399**, 337

Gaia sensitivity to γ



For V<15, a realistic case for Gaia is $n^* \sim 10^6$

1 yr-long Gaia-like mission could measure a value 3×10^{-7} for $|1-\gamma|$ (3σ detection)

Test with the Hipparcos results

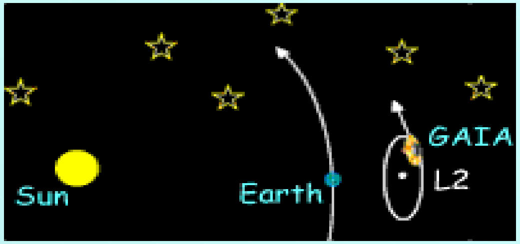
Reliability of the results by comparison with those of Hipparcos

(Froeschlé, Mignard & Arénou, 1997, ESA SP-402)

- Montecarlo experiment using the error budget of Hipparcos (n*=15000)
- For 1 yr of mission the result is $\sigma_{\gamma} \cong 2 \times 10^{-3}$
- Scaled it with the numbers used for the experiment with Hipparcos (n*=4400, 3 yr of mission) it gives $\sigma_{\gamma} \cong 1.1 \times 10^{-3}$

This is compatible with the value obtained by Hipparcos and validates the Gaia simulation

Detectable relativistic deflections at L2

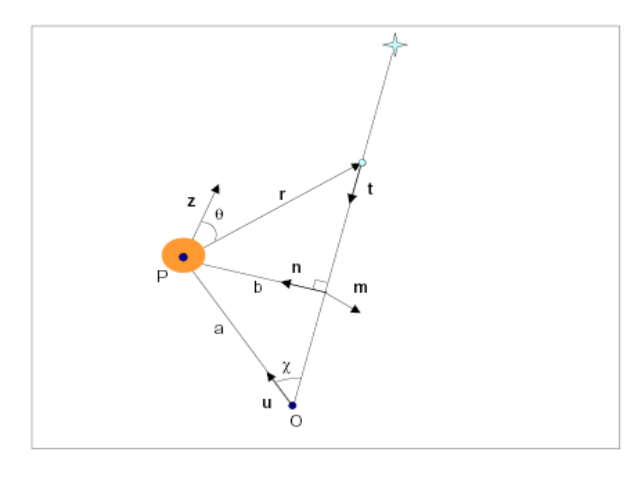


	$\delta\chi_{PN}$	$\delta\chi_{J_2}$	$\delta\chi_L$	χ_{max}
Sun	$1''.75$	$\sim 1\ \mu$as	$0.7\ \mu$as	$(180°)$
Mercury	$83\ \mu$as	–	–	$(7')$
Venus	493	–	–	$(4.0°)$
Earth	574	0.6	–	$(101°)$
Moon	26	–	–	$(2.3°)$
Mars	116	0.2	–	$(17')$
Jupiter	16290	240	0.2	$(87°/3')$
Saturn	5772	94	–	$(16°/51'')$
Uranus	2030	7	–	$(67'/4'')$
Neptune	2487	8	–	$(50'/3'')$
Pluto	7	–	–	$(0'.3)$

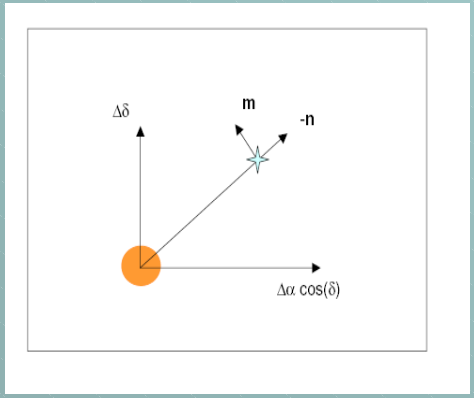
Light deflection-2: new relativistic test due to an axisymmetric body

- A planet will act as a lens on the grazing light from a distant source. The deflection angle can be computed then as a vector $\Delta\Phi$

$$\Delta\Phi = \frac{2}{c^2}\int \nabla U_\perp dl$$



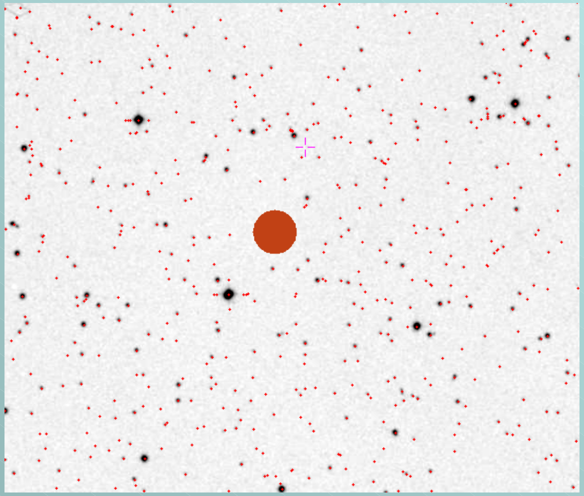
$$\Delta\Phi = \frac{4GM}{c^2 b}\left\{\left[1 + \frac{J_2 R^2}{b^2}\left(1 - 2(\mathbf{n}\cdot\mathbf{z})^2 - (\mathbf{t}\cdot\mathbf{z})^2\right)\right]\mathbf{n} + \left[2\frac{J_2 R^2}{b^2}(\mathbf{n}\cdot\mathbf{z})(\mathbf{m}\cdot\mathbf{z})\right]\mathbf{m}\right\}$$



Observer view. The position of the star is displaced both in the radial (-**n**) and orthoradial (**m**) directions. The spin axis of the planet lies somewhere out of plane

Jupiter on the background starfield during the Gaia mission

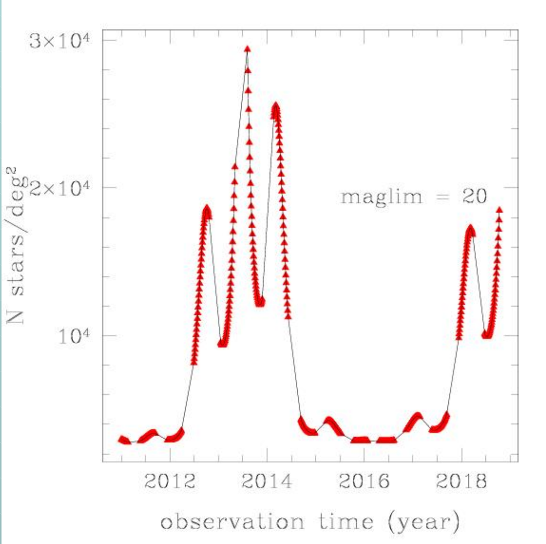
Visibility of Jupiter



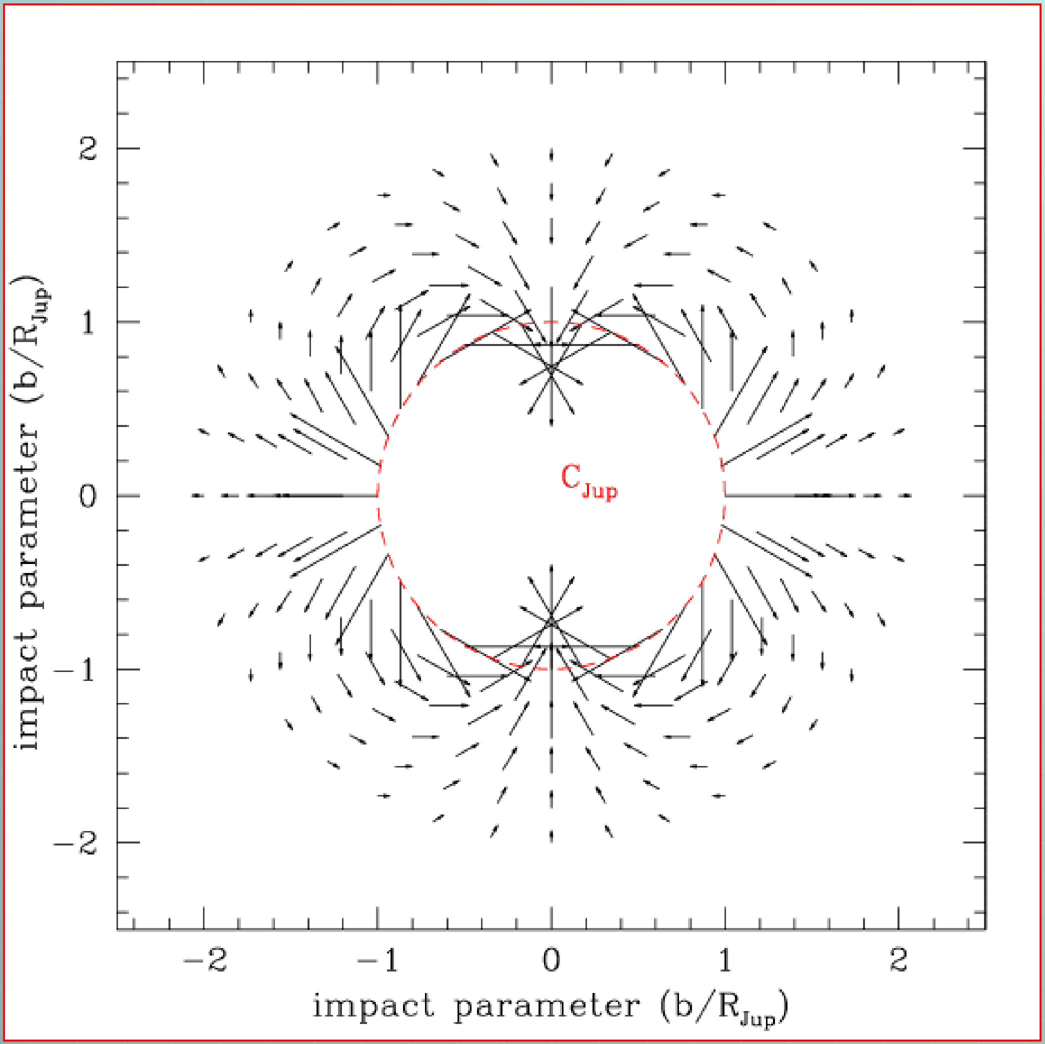
Jupiter in a real starfield in mid 2013 near the galactic plane (plate from the Palomar digitalized survey). The faintest stars are around V=18.The red spots (UNSO-B2) are stars around V=20.

Stellar density around Jupiter

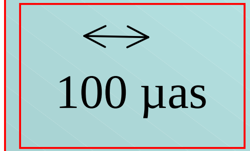
V < 20

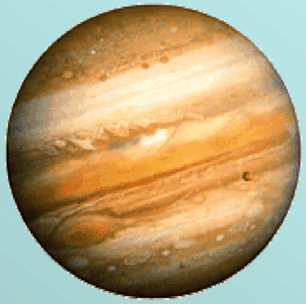


GAREX
GAia Relativistic EXperiment



Deflection vector field for the quadrupole contribution only.





β experiment

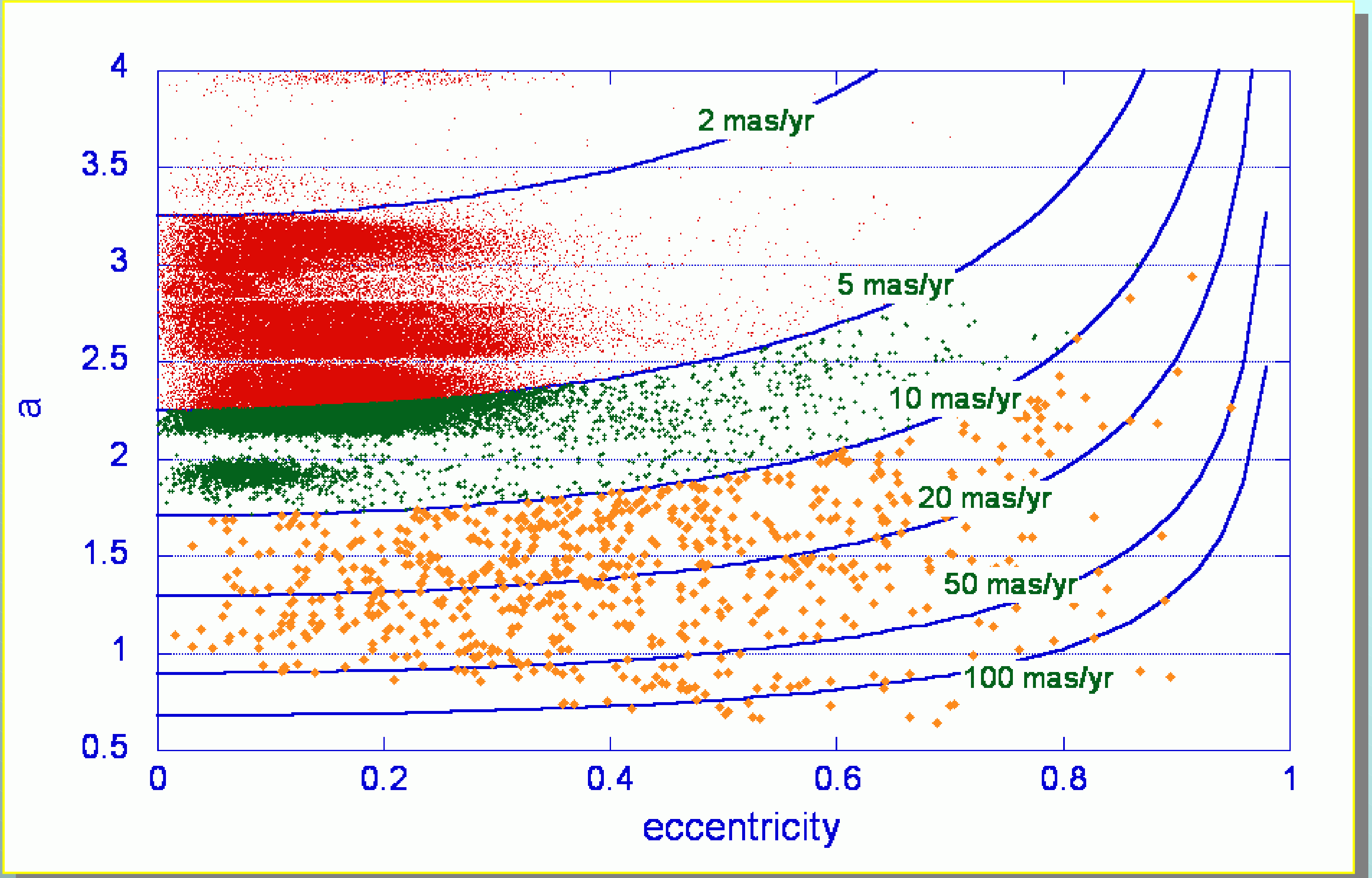
- Perihelion precession

$$\Delta\varpi = \frac{6\pi\lambda GM}{a(1-e^2)c^2} + \frac{3\pi J_2 R^2}{a^2(1-e^2)^2} \qquad \lambda = (2\gamma - \beta + 2)/3$$

PPN precession coefficient

			$\delta\varpi$: GR mas/yr	$J_2(=10^{-7})$ mas/yr
Mercury :			430	0.124
main belt :	a = 2.70 AU	e = 0.1	3.4	0.0001
3200 Phaeton	a = 1.27 AU	e = 0.83	102	0.040
1566 Icarus	a = 1.08 AU	e = 0.83	101	0.030
5786 Talos	a = 1.08 AU	e = 0.82	101	0.030

Distribution of the known minor planets in the plane a-e



(F.Mignard, in “Gaia: a European Space project”, pag 105-121, EAS,EDP Sciences 2002)

Constrains on β determination

- Sampling in *a* : separation of the GR and J2 effect
- There are three options to be decided later :
  - solve for β and J2 ➔ Correlation between the two
  - fix J2 from helioseismology and solve for β
  - fix β from, e.g LLR, and solve for a model independant J2
- Orbit determination including all other perturbations

Hard to decide on β without extensive simulations :

β to 10^{-3} - 10^{-4}

Conclusions

- *No other foreseen measurements of γ can challenge Gaia in the next decade:*
  - Eddington experiment by observing many times the same background field
  - First measurement of the quadrupole deflection due to a planet
  - Indirect determination of the center of gravity of the planet
  - Test alternative theories of gravity that claim to provide a route to quantization of gravity
  - The accuracy of 10^{-7} -> all the details of the Solar System gravitational field, instrumental characteristics must be included in the modelling
- *The determination of β will be improved by accurate determinations of the orbit of the Solar System objects*
- **Simulations are on-going to assess the capabilities and the accuracy of the measurements.**