Color pictures complementary to the paper: Dissipative and weakly–dissipative regimes in nearly–integrable mappings (Preprint 2005)

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1 Analysis of a dissipative 2-dimensional mapping

We recall the dissipative standard map described by the equations:

$$\begin{aligned} y' &= by + c + \frac{\varepsilon}{2\pi}\, s(2\pi x) \\ x' &= x + y' \ , \end{aligned} \tag{1}$$

where $y \in \mathbf{R}$, $x \in [0,1)$, c is a real constant and $s(x)$ is a regular periodic function; the mapping depends on two parameters: $b \in \mathbf{R}_+$ is the *dissipative* parameter, while $\varepsilon \in \mathbf{R}_+$ is the *perturbing* parameter. We recall that a widely studied mapping belonging to the class (1) is the paradigmatic standard map, where the function $s(x)$ is defined as

$$s(x) = \sin(2\pi x) \tag{2}$$

.

2 Study of the dynamic in the space of parameters: b, ϵ, $y(0)$, $x(0)$.

The following set of figures corresponds to grids of 500×500 initial conditions regularly spaced in some suited space of parameters as precised below. The colors in the figures are associated to the values:

$$sgn(FLI(T)) \log 10(|FLI(T)|) \ ,$$

where T is the number of iterations used in the computation. The FLI-values are reported with different coulors:

- The orange stands for the presence of curve attractor: the FLI values are in the interval $[-\log T : \log T]$, i.e. the largest Lyapunov exponent goes to zero (and the second is negative).
- The yellow color corresponds to strange attractors: the FLI values are larger than $\log T$, i.e. the largest Lyapunov exponent is positive.
- The colors going from purple to black reveal the presence of periodic attracting orbits: the plotted quantity has decreasing negative values from purple to black , i.e. the largest Lyapunov exponent has decreasing negative values.

In the following we shall consider the function $s(2\pi x)$ as:

- A) $s(2\pi x) = sin(2\pi x)$
- B) $s(2\pi x) = sin(2\pi x \cdot 3)$
- C) $s(2\pi x) = sin(2\pi x \cdot 5)$
- D) $s(2\pi x) = sin(2\pi x) + \frac{1}{3} sin(2\pi x \cdot 3)$
- E) $s(2\pi x) = sin(2\pi x) + \frac{1}{20} sin(2\pi x \cdot 5)$
- F) $s(2\pi x) = \frac{sin(2\pi x)}{cos(2\pi x)+1.4}$

For each of the above mappings, we consider the following rotation numbers:

- aa1) $\alpha = \frac{\sqrt{5}-1}{2}$
- aa2) $\alpha = \frac{2}{3}$
- aa3) $\alpha = [1, 3, 4, 1^{\infty}]$
- aa6) $\alpha = \frac{1}{2}$

3 DFLI-Charts

Analysis of: **Space $b - \epsilon$ and Space $b - y$** The set of figures (left) corresponds to grids of 500×500 initial conditions regularly spaced in b and ϵ, both taken in the interval $[0.01 : 1]$ for $y(0) = 5.$ and $x(0) = 0.$, for the 3 function $s(2\pi x)$ and for the 4 values of the parameter α. The time is always $T = 1000$. The set of figures (right) corresponds to grids of 500×500 initial conditions regularly spaced in b and y, taken respectively in the interval $[0.01 : 1]$ and $[0.01 : 10]$ for $\epsilon = 0.9$ and $x(0) = 0.$, for the 3 function $s(2\pi x)$ and for the 4 values of the parameter α. The time is always $T = 1000$ after a transition of 10000 iterations computed in order to go to the attractor.

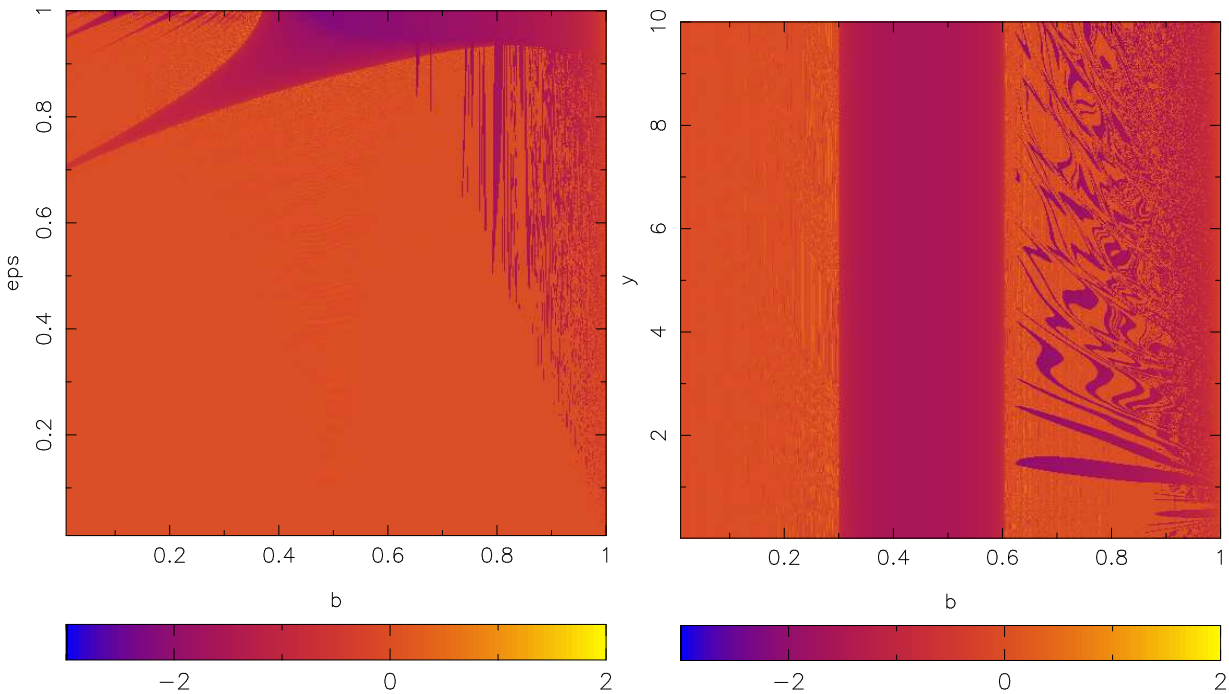

Figure 1: Map A) , aa1): $\alpha = \frac{\sqrt{5}-1}{2}$. (left) grid $b - \epsilon$, (right) grid $b - y$.

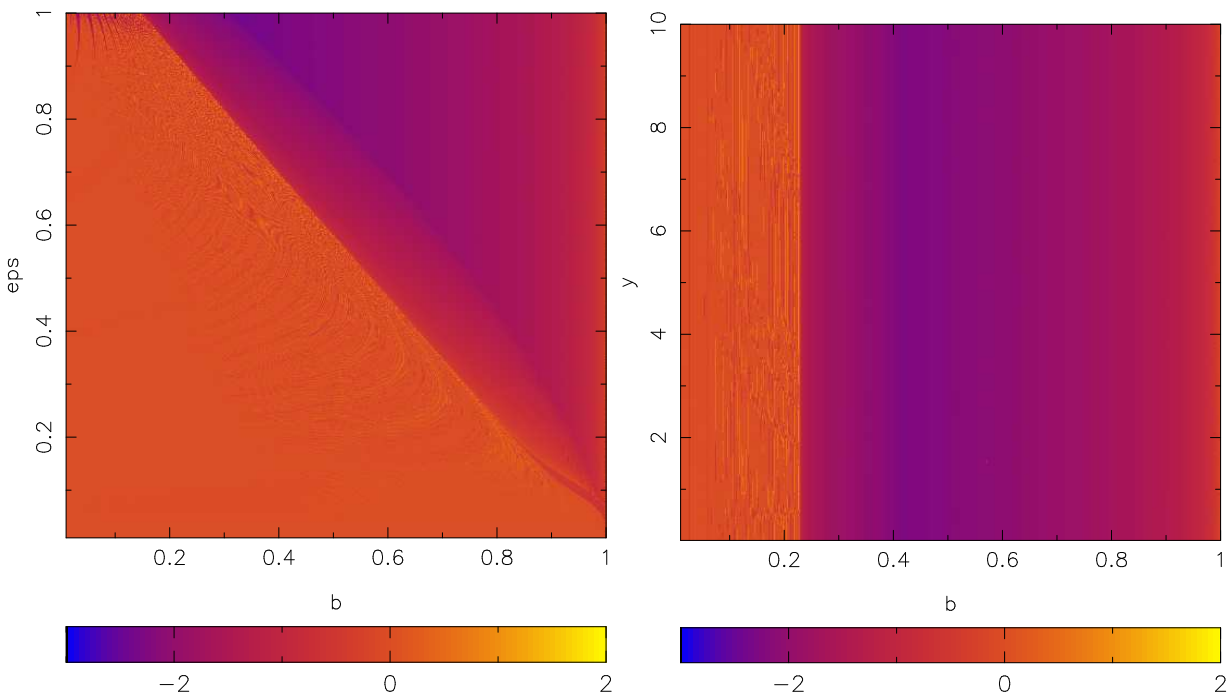

Figure 2: Map A) , aa2): $\alpha = \frac{2}{3}$. (left) grid $b - \epsilon$, (right) grid $b - y$.

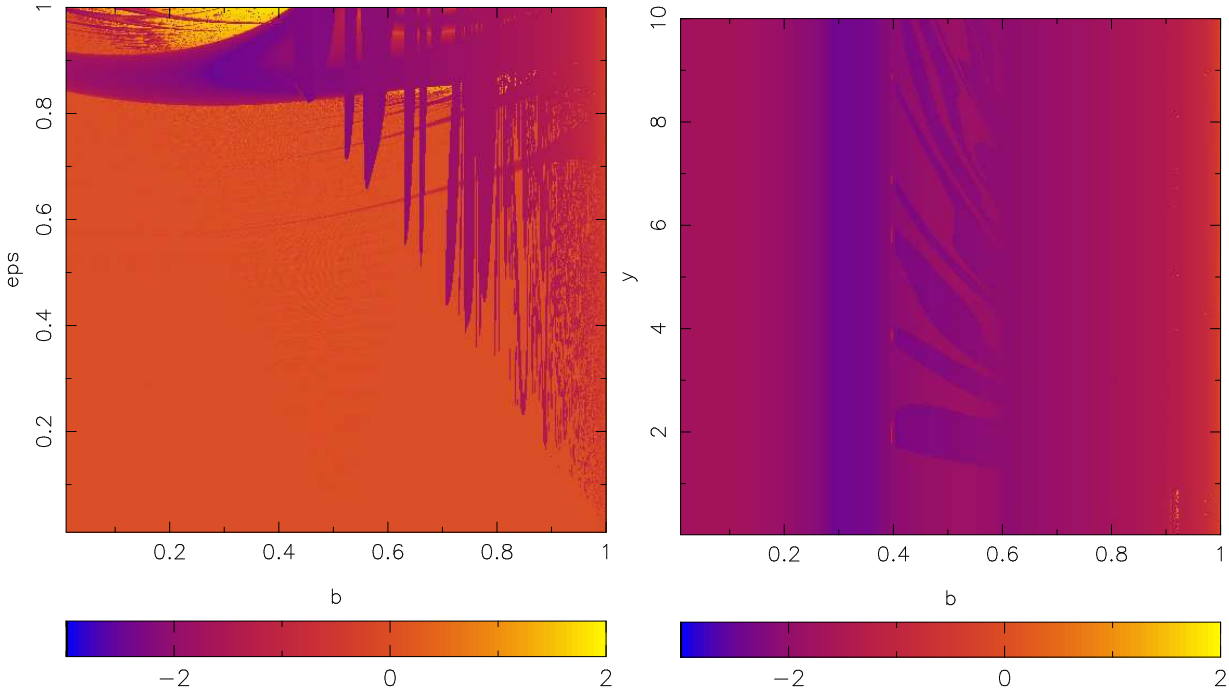

Figure 3: Map A) , aa3): $\alpha = [1, 3, 4, 1^{\infty}]$. (left) grid $b - \epsilon$, (right) grid $b - y$.

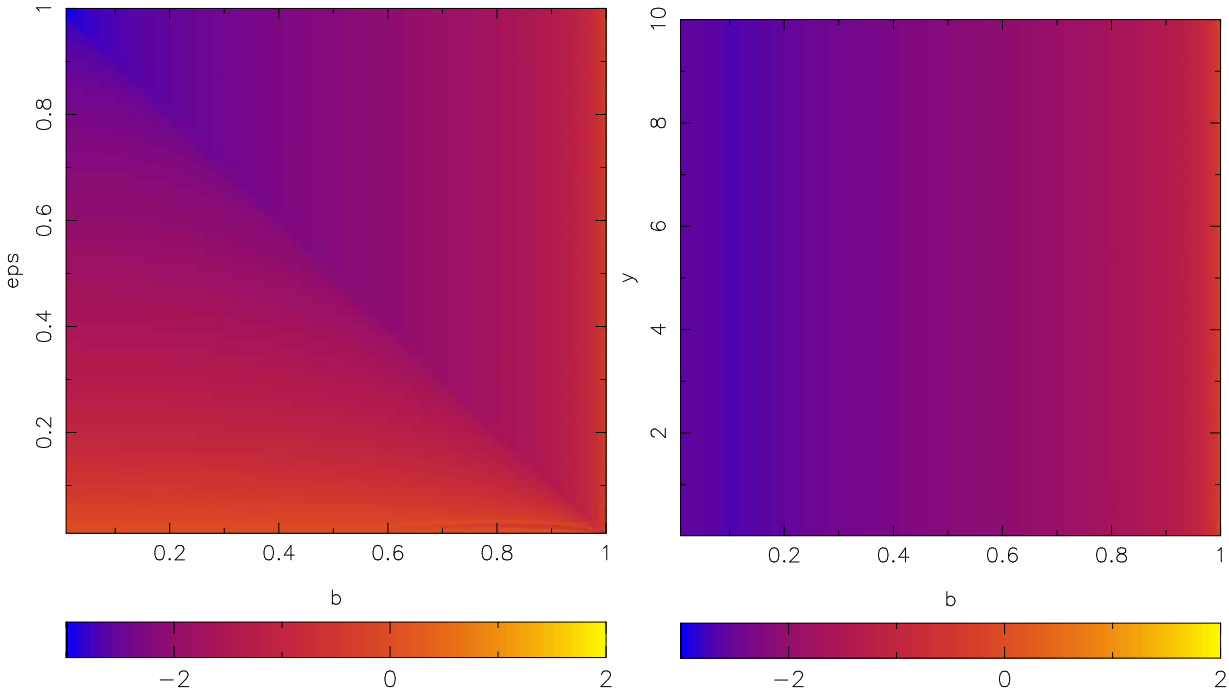

Figure 4: Map A) , aa4): $\alpha = \frac{1}{2}$. (left) grid $b - \epsilon$, (right) grid $b - y$.

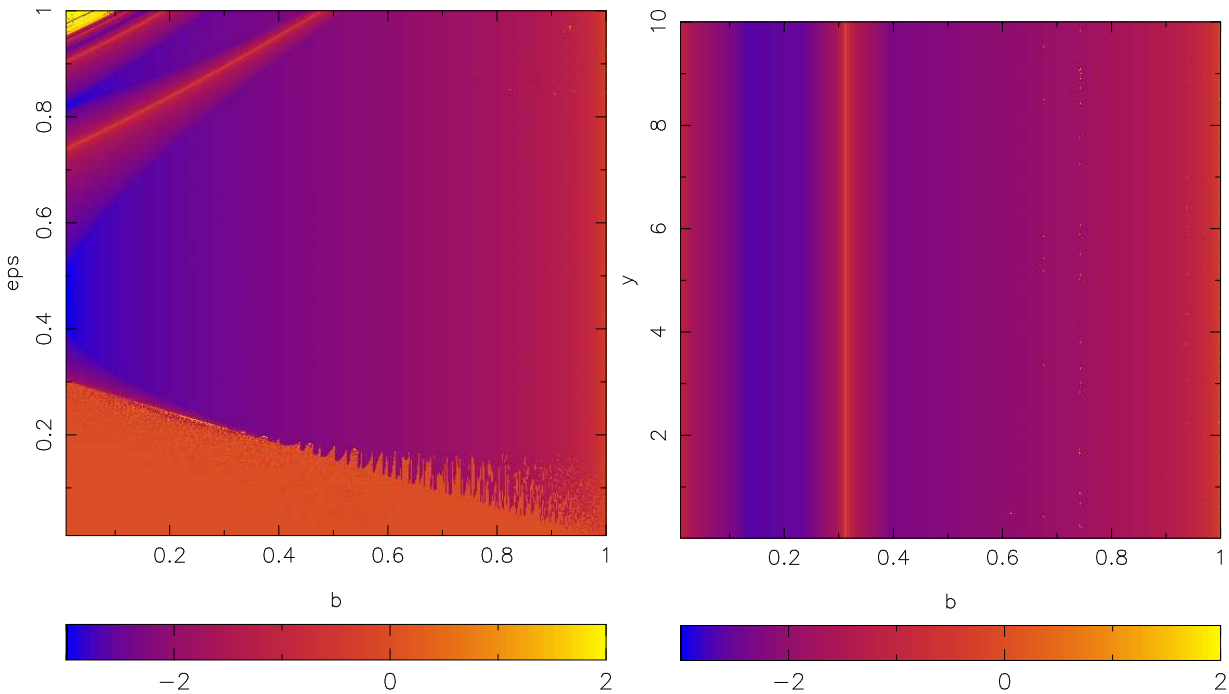

Figure 5: Map B) , aa1): $\alpha = \frac{\sqrt{5}-1}{2}$. (left) grid $b - \epsilon$, (right) grid $b - y$.

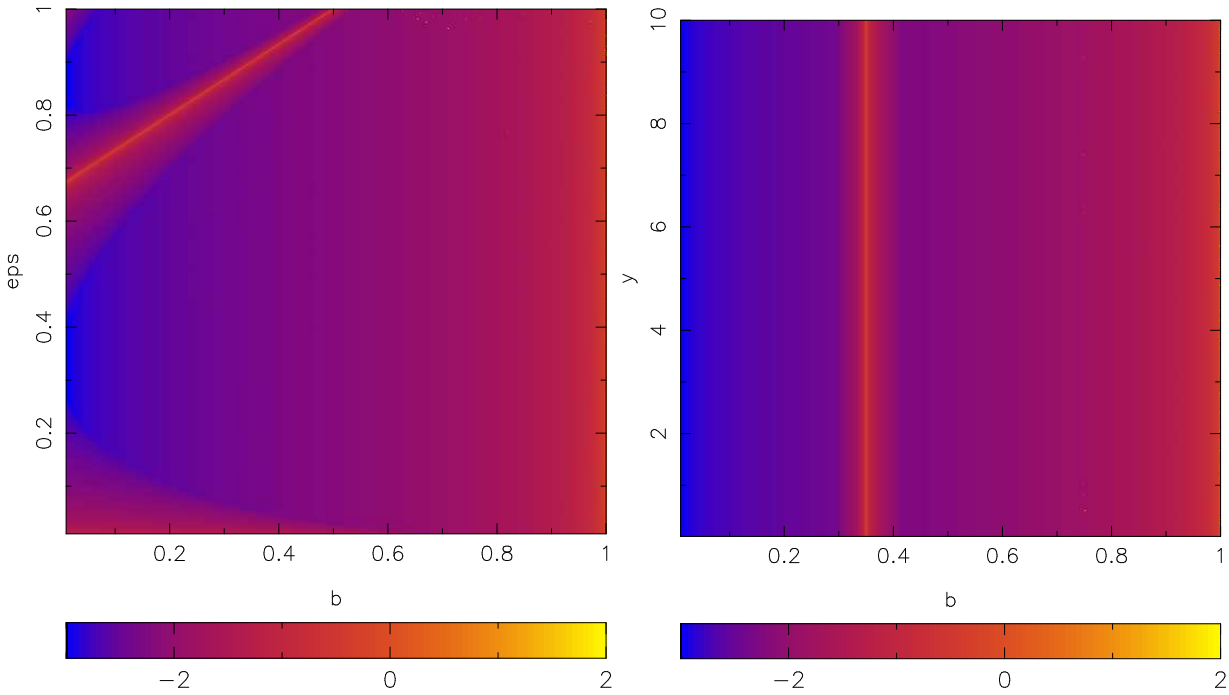

Figure 6: Map B) , aa2): $\alpha = \frac{2}{3}$. (left) grid $b - \epsilon$, (right) grid $b - y$.

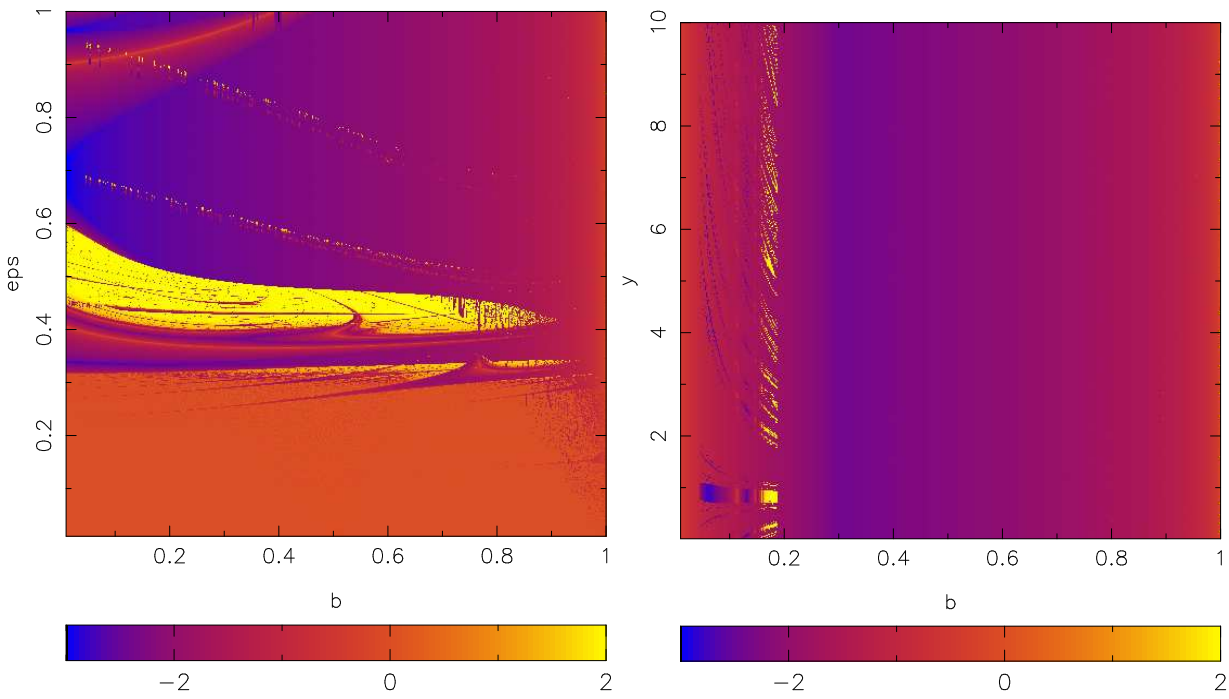

Figure 7: Map B) , aa3): $\alpha = [1, 3, 4, 1^{\infty}]$. (left) grid $b - \epsilon$, (right) grid $b - y$.

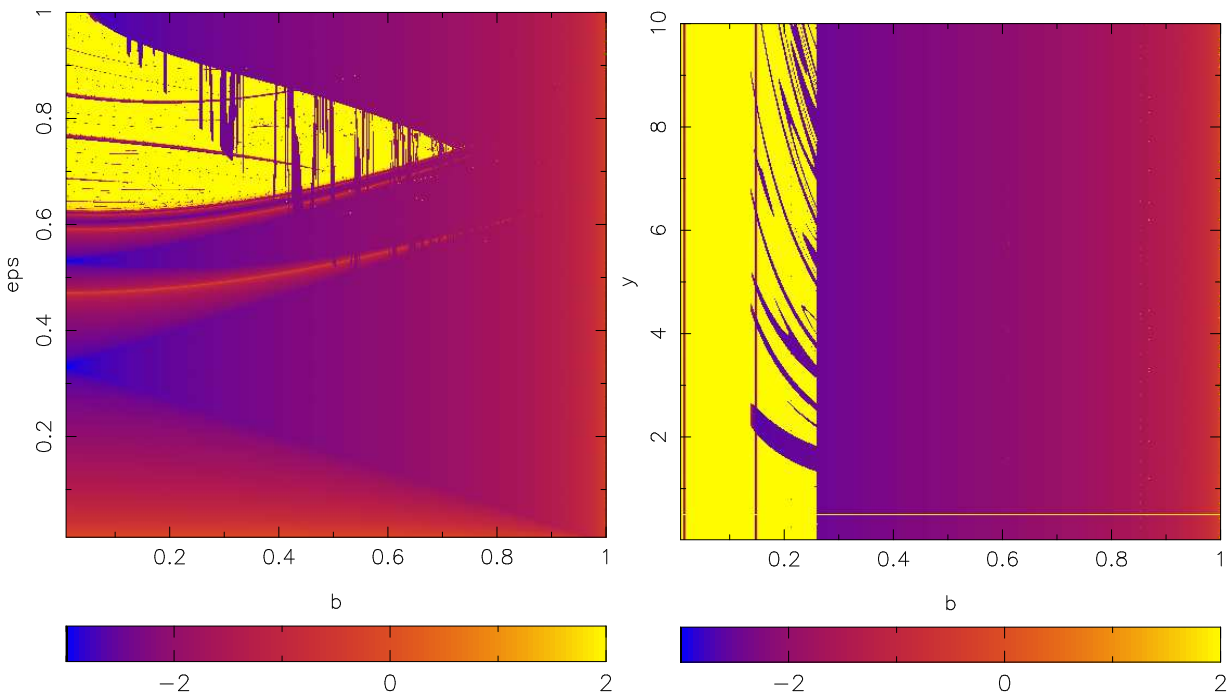

Figure 8: Map B) , aa4): $\alpha = \frac{1}{2}$. (left) grid $b - \epsilon$, (right) grid $b - y$.

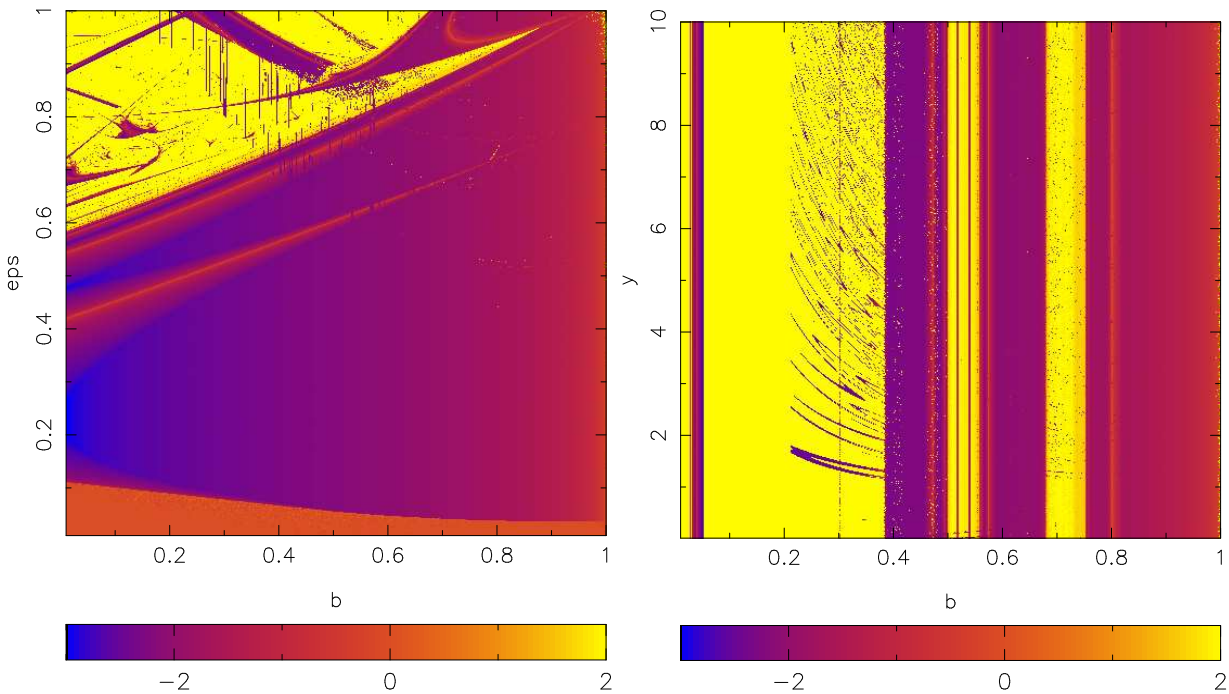

Figure 9: Map C) , aa1): $\alpha = \frac{\sqrt{5}-1}{2}$. (left) grid $b - \epsilon$, (right) grid $b - y$.

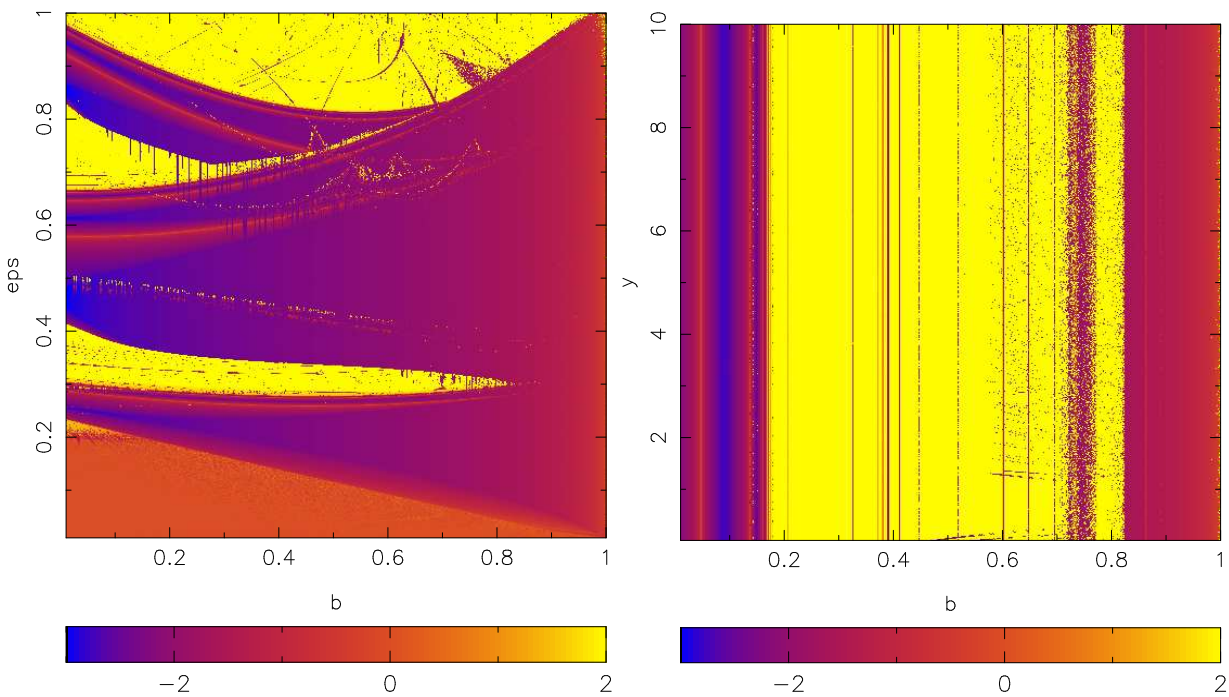

Figure 10: Map C) , aa2): $\alpha = \frac{2}{3}$. (left) grid $b - \epsilon$, (right) grid $b - y$.

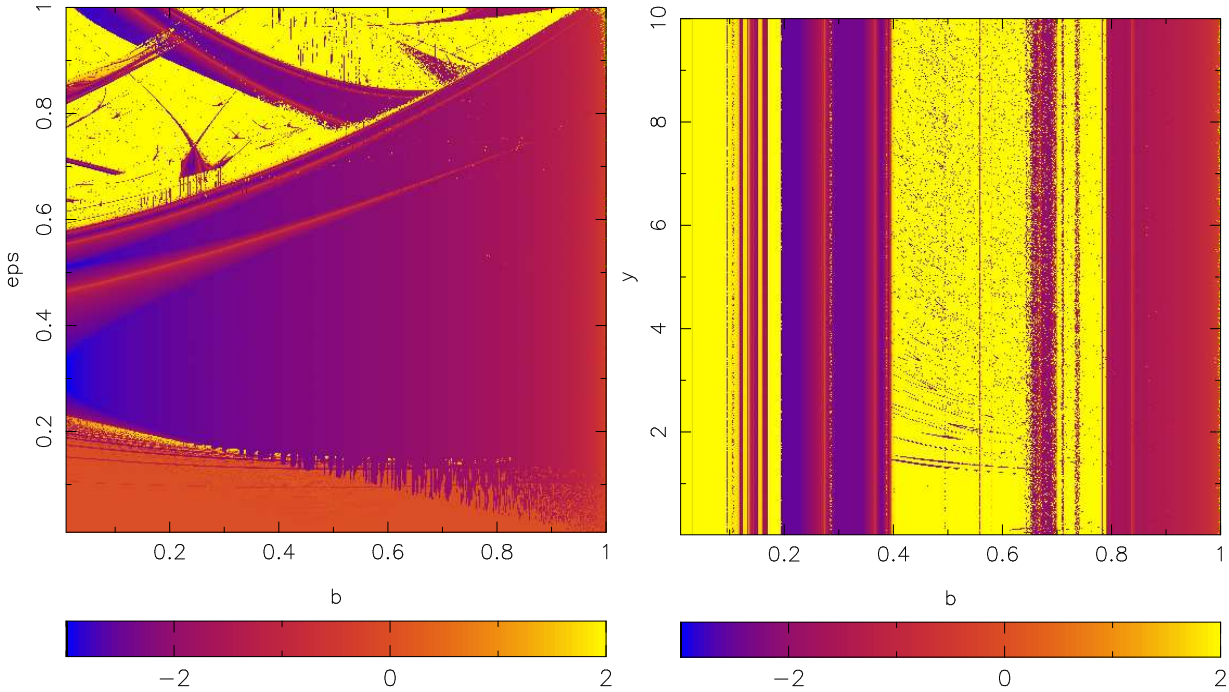

Figure 11: Map C) , aa3): $\alpha = [1, 3, 4, 1^{\infty}]$. (left) grid $b - \epsilon$, (right) grid $b - y$.

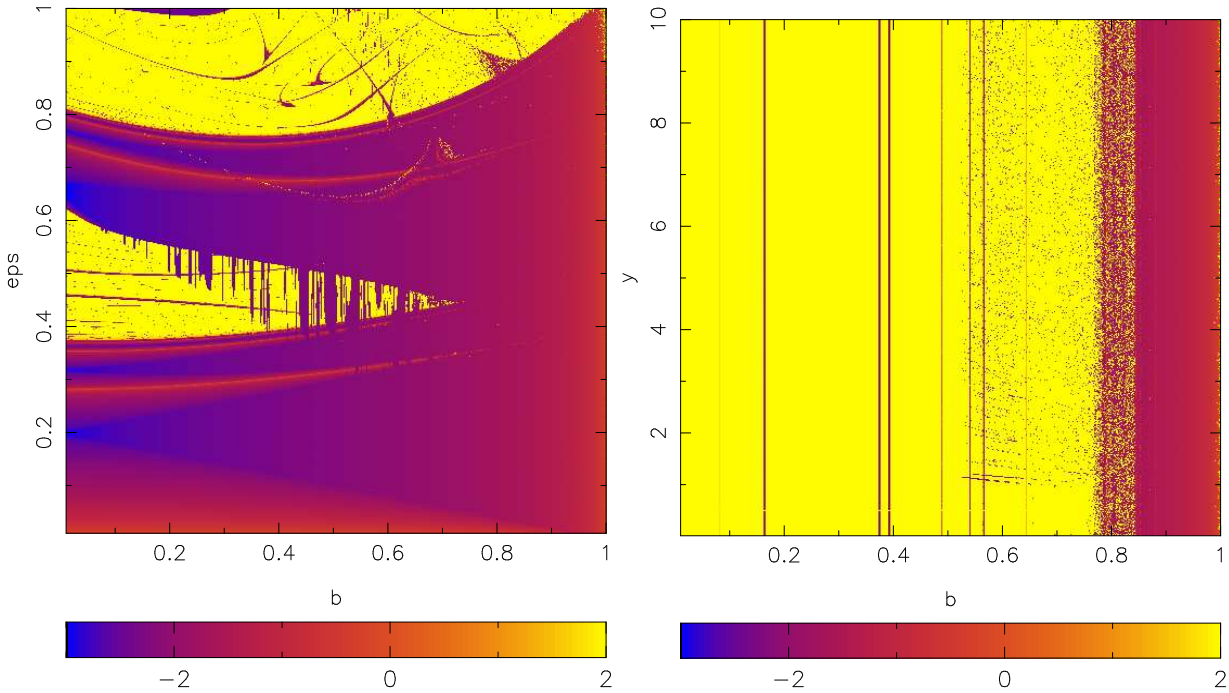

Figure 12: Map C) , aa4): $\alpha = \frac{1}{2}$. (left) grid $b - \epsilon$, (right) grid $b - y$.

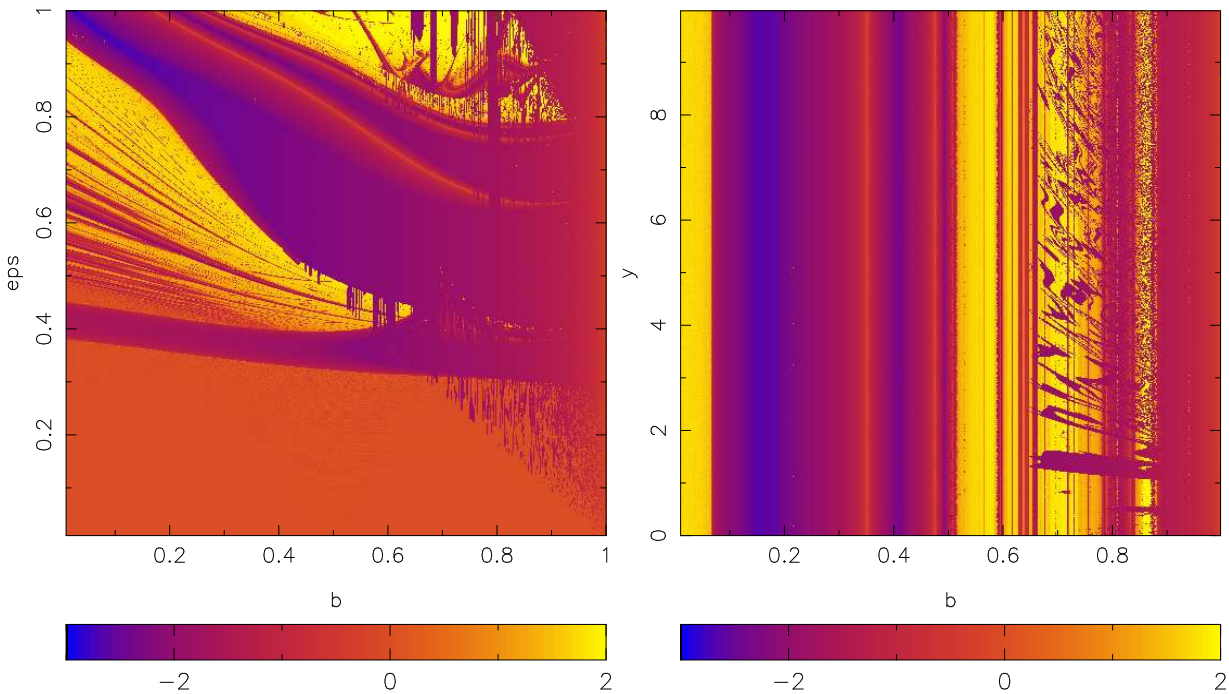

Figure 13: Map D) , aa1): $\alpha = \frac{\sqrt{5}-1}{2}$. (left) grid $b - \epsilon$, (right) grid $b - y$.

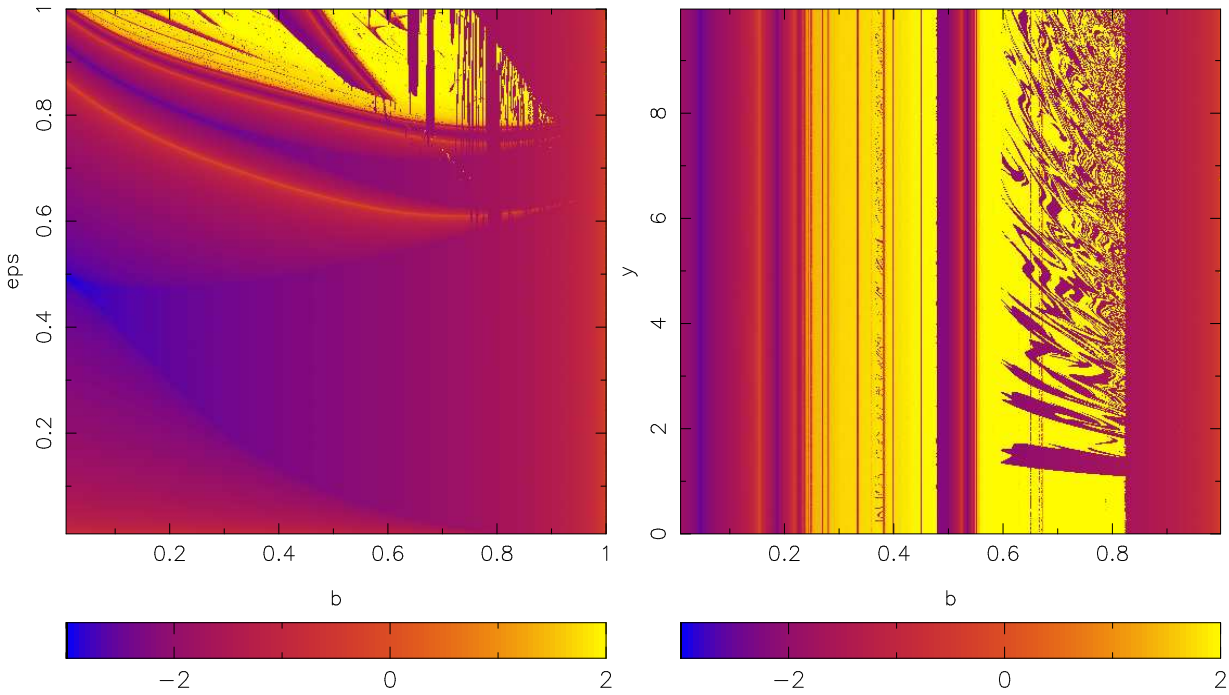

Figure 14: Map D) , aa2): $\alpha = \frac{2}{3}$. (left) grid $b - \epsilon$, (right) grid $b - y$.

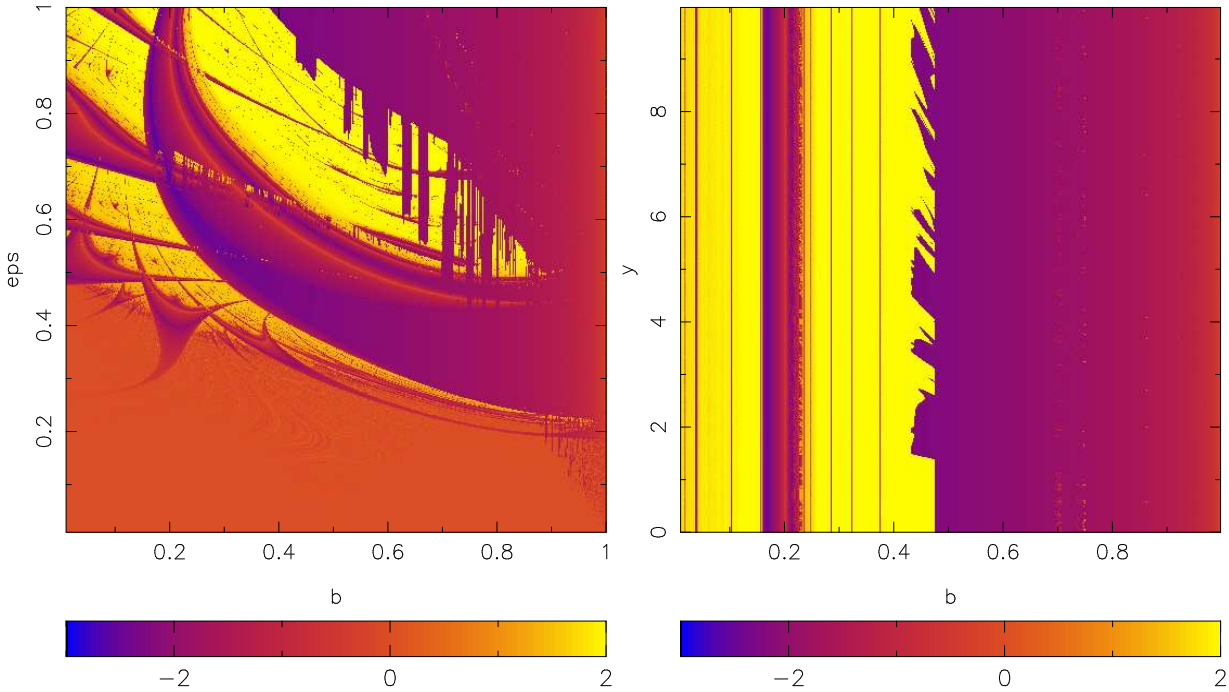

Figure 15: Map D) , aa3): $\alpha = [1, 3, 4, 1^{\infty}]$. (left) grid $b - \epsilon$, (right) grid $b - y$.

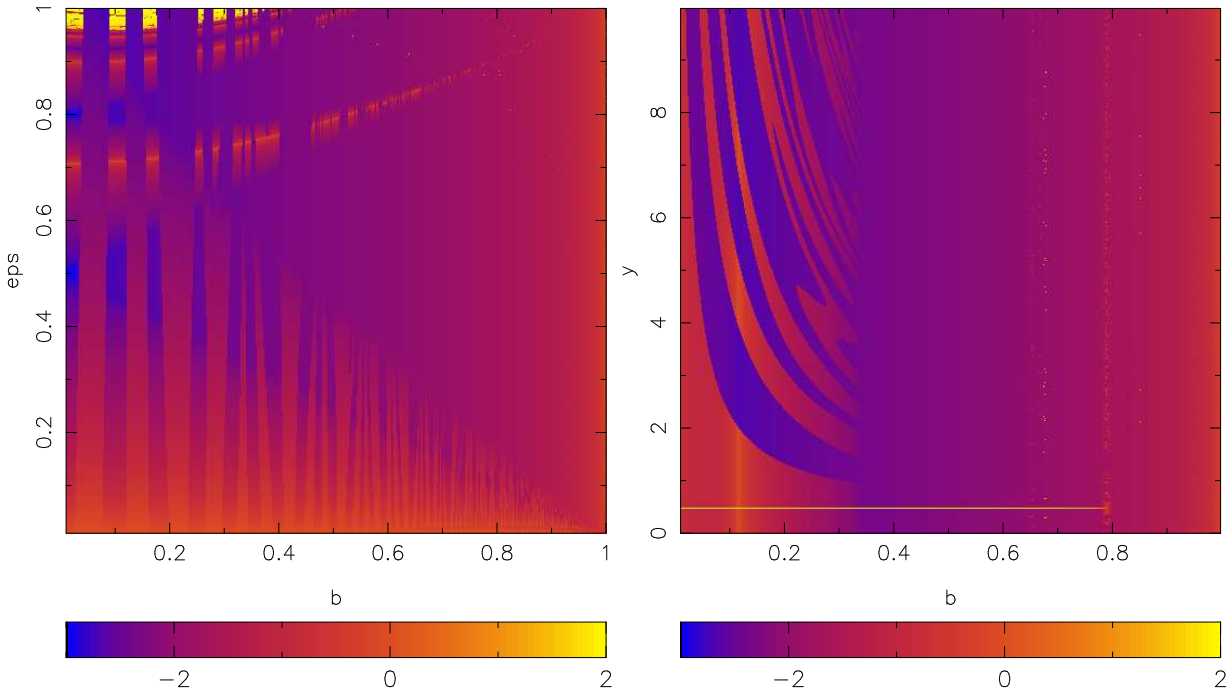

Figure 16: Map D) , aa4): $\alpha = \frac{1}{2}$. (left) grid $b - \epsilon$, (right) grid $b - y$.