THESE de DOCTORAT

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Spécialité Sciences Economiques.

Analyse de la dynamique du modèle néo-autrichien de croissance. Etudes d'économies hors de l'équilibre

par

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à mon mari Maurizio et à mes enfants
Alessandro et Cristina

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Ce travail interdisciplinaire a été également facilité par les interactions informels avec mes collegues de l'Observatoire de Nice que ce soit sur le plan purement scientifique ou humain.

Preface

Cette thèse est le résultat, sous forme de collection d'articles, de cinq ans de travail interdisciplinaire en qualité d'ingénieur de recherche, au sein de l'équipe de macro-économie de M. J.L.Gaffard, professeur à l'université de Nice-Sophia Antipolis, et de l'équipe de mécanique céléste et systèmes dynamiques de M. C.Froeschlé, astronome de 1ère classe à l'Observatoire de la Côte d'Azur.

L'origine lointaine de ce travail se trouve dans une collaboration commencée il y a vingt ans entre l'Observatoire de la Côte d'Azur et le département d'économie de l'Université de Nice-Sophia Antipolis. En particulier, au debut des années 90 le développement du modèle néo-autrichien de Hicks, realisé par M.Amendola et J.L.Gaffard et, a rencontré tout l'interêt de C.Froeschlé car il s'agit d'un modèle ou l'aspect dynamique, c'est-à -dire la dépendance du temps, rentre en jeu explicitement.

Mon poste d'ingénieur de recherche a eu pour objectif initial la programmation du modèle, à partir de la version écrite par C.Froeschlé. A l'aide des simulations numériques nous avons par la suite analysé les proprietés dynamiques du modèle néo-autrichien et étudié des économies caracterisées par des modèles ayant tous en commun une structure néo-autrichienne de la production.

Je tiens à preciser que cette thèse est interdisciplinaire avec tous les forces et les faiblesses d'une telle demarche. La recherche à l'intersection entre deux disciplines amène souvent à des résultats inattendus et dans ce cas à des nouvelles reflextions sur les fondements dynamiques et/ou économiques. Le caractère passionnant de cette démarche ne saurait occulter les faiblesses qui en découlent c'est-à -dire le fait que je ne suis pas une économiste (et j'en suis parfaitemente consciente) et des nombreuses maladresses en particulier dans la forme se retrouvent tout au long des articles non cosignés avec des économistes professionnels (chapitres 2 et 4).

Resumé

Le travail effectué concerne l'analyse du modèle néo autrichien, introduit par Hicks (1973) et reformulé par Amendola et Gaffard (1998). Une des idées de base du modèle est la dimension temporelle de la structure de la capacité productive, c'est- à -dire l'aspect sequentiel des évènements de construction et d'utilisation. Cet aspect rend non trivial le modèle d'un point de vue purement dynamique et le rend particulierement representatif de la problèmatique des phénomenes économiques hors de l'équilibre. Le modèle néo-autrichien est décrit dans les quatres articles présentés aux Chapitres 2,3,4 et 5. Dans chaque cas un choc spécifique est traité ou une différente modélisation de la politique économique ou du degré de rationnalité des agents sont introduits. Une description detaillée du modèle est donnée au Chapitre 1.

Le modèle de Hicks est écrit en temps discret, l'unité de temps est la semaine Hicksienne, mais lors de l'intégration numérique sur un certain intervalle de temps on peut en reduire la durée et étudier les consequences du changement du pas de discrétisation sur l'evolution de l'économie suite à un certain choc. Cette problématique du passage de la description du temps continu au temps discret et vice-versa est au coeur du prémier article, paru dans la Revue Economique et presenté au Chapitre 2.

Dans l'article reporté au Chapitre 3, nous etudions les effets d'un choc technologique sur les salaires réels et le taux d'emploi. Grâce à l'utilisation de la méthode Monte-Carlo nous avons pu montrer que la transition à la nouvelle technologie n'est pas lié à la flexibilité des salaires réels mais plutôt aux conditions d'accumulation du capital.

Dans l'article reporté dans le Chapitre 4 nous decrivons les possibilité d'utilisation des simulations numériques et nous introduisons une méthode non standard pour l'étude numérique des orbites d'un système dynamique. Les résultats nous montrent que la complexité du modèle n'est pas due seulement aux defauts de coordination des agents liés à leur connaissance imparfaite, mais aussi au caractere définitif de la structure de la capacité productive determiné par le decisions d'investissement et par les contraintes de ressources humaines.

La propagation d'un choc technologique et son influence en particulier sur la productivité et le taux de chômage dans des économies caracterisées par differentes choix de politique monétaire est présenté dans l'article au Chapitre 5.

Dans l'article exposé au Chapitre 6 nous passons en revue les méthodes existantes pour le calcul de l'indicateur de Lyapunov sur des séries expérimentales et nous introduisons une nouvelle méthode qui semble être bien adaptée aux modèles conservatifs tout en donnant des bons résultats pour le cas dissipatifs.

Dans le Chapitre 7 nous appliquons une des méthodes introduites à des séries chaotiques issues du modèle néo-autrichien. Nous retrouvons par cette méthode que les degrés de liberté importants pour la dynamique du modèle sont exactement ceux que nous avons appellé variables clés du modèle. Ce résultat nous semble ouvrir une piste pour l'analyse des séries économetriques.

Mots clés:: modèle néo-autrichien, méthodes d'analyse numérique, sentiers d'équilibre, analyse des chemins de traverse, indicateurs du chaos, séries temporelles.

" Knowledge once gained casts a faint light beyond its own immediate bouderies. There is no discovery so limited as not to illuminate something beyond itself" (Tyndall)

Sommaire

Introduction

Une des idées de base du modèle néo autrichien, introduit par Hicks (1973) et reformulé par Amendola et Gaffard (1998), est la dimension temporelle de la structure de la capacité productive, c'est- à -dire l'aspect sequentiel des évènements de construction et d'utilisation. Cet aspect rend non trivial le modèle d'un point de vue purement dynamique et le rend particulierement representatif de la problèmatique des phénomenes économiques hors de l'équilibre.

Le modèle néo-autrichien est décrit dans les quatres articles présentés aux Chapitres 2,3,4 et 5. Dans chaque cas un choc spécifique est traité ou une différente modélisation de la politique économique ou du degré de rationnalité des agents sont introduits. Pour essayer de faciliter la lecture, nous préférons donner une description detaillée du modèle au Chapitre 1.

Il existe deux grands classes de systèmes dynamiques: ceux dont les équations sont écrites en temps continu et ceux dont les équations sont écrites en temps discret. Les deux formulations ont été utilisés en économie: le modèle proie-predateur de Goodwin (1967) par example est écrit en temps continu alors que celui de Hicks (1973) est écrit en temps discret.

Un des avantages de la description en temps continu est, en général, la possibilité d'établir de théorèmes qui démontrent par exemple dans le cas Hamiltonien l'existence de varietés invariantes, c'est- à - dire des solutions quasi-régulières (Arnold 1963, Kolmogorov 1954, Moser 1958). Le désavantage majeur est la nécessité de discrétiser lors de l'intégration numérique, passage obligé pour l'exploration des systèmes dynamiques, mis à part le petit nombre de cas intégrables analytiquement. Or, il est bien connu que la discrétisation est une procédure non triviale qui nécessite une profonde connaissance de la structure du système.

Les systèmes discrets, sont beaucoup plus difficiles a traiter analytiquement, mais sont particulierement adaptés aux simulations numériques. [1]

Cette deuxième formulation a une motivation économique, car certaines variables ne changent pas de manière continue, par exemple les salaires ne sont pas renégociés tous les jours alors que

[1] Les experiences numériques, rendues de plus en plus accessibles par le developpement des ordinateurs, sont devenues un outil standard dans presque toutes les branches de la science. Naturellement c'est en physique que elle se sont imposées les premières et que s'est forgé, à la suite de ces que nous pourrions appeller les maladies enfantines de l'experimentation numérique (la tentation de tout mettre dans un modèle, la multiplication des experiences dans l'espoir de trouver la perle rare etc.. etc..), une déontologie de l'experimentateur numérique, tout du moins en ce qui concerne les travaux de recherche de base. Tout d'abord il faut définir le plus clairement possible la question posée, ensuite formuler un modèle non trivial mais simple, enfin, tout en s'appuyant au maximum sur les résultats analytiques qui peuvent exister pour certains cas particuliers du modèle, il faut testér la fiabilité des résultats numériques à l'interieur de l'approche statistique (par example en utilisant la méthode Monte Carlo). Les résultats nous semblent devoir être publiés avec suffisament de précision pour que toute personne "courageuse" puisse les reproduire.

les decisions d'investissement ont lieu de façon continue. En effet, des intervalles de temps specifiques à chaque variable pourraient être envisagés, au prix evidemment d'une complexification ultérieure de la modélisation.

Le modèle de Hicks est écrit en temps discret, l'unité de temps est la semaine Hicksienne, mais lors de l'intégration numérique sur un certain intervalle de temps on peut en reduire la durée et étudier les consequences du changement du pas de discrétisation sur l'evolution de l'économie suite à un certain choc.

Cette problématique du passage de la description du temps continu au temps discret et vice-versa est au coeur du prémier article, paru dans la Revue Economique et presenté au Chapitre 2.

A partir du travail pionnier de M.Hénon (Hénon and Heiles 1964) l'utilisation des simulations numériques pour l'étude des systèmes non linéaires a été largement accepté dans la communauté des dynamiciens et plus en général des physiciens. Bien que ne constituant pas une demonstration mathématique, cet outil s'est revelé trés important pour l'extension des domaines de validité des théorèmes fondamentaux en mécanique céleste. Parallèlement, l'introduction par le même auteur de modèles simples mais non triviaux a constitué une vraie percée dans la comprehension des systèmes non linéaires. Fameux est le cas de l'attracteur étrange (Hénon 1976), modèle discret défini par deux équations de récurrence qui sont de simples polynomes d'ordre inférieur ou égal à deux. Malgré sa simplicité, ce système a les propriètés de celui décrit par les équations de Lorentz, modèle continu défini par trois équations differentielles lui même issu des équations aux derivées partielles de Navier-Stokes. Notons au passage l'abandon de l'ambition de décrire le monde réel à fin d'arriver à une meilleure comprehénsion de la base même des phénomènes non linéaires.

C'est dans cet esprit que nous considerons le modèle néo-autrichien: comme étant une description simple mais à notre avis non triviale du processus de production macro-économique. La prédiction quantitative des observables macro économiques va bien au delà de possibilités du modèle, alors que la comprehénsion des phénomènes macro est à sa portée.

Dans l'article reporté au Chapitre 3 nous etudions les effets d'un choc technologique sur les salaires réels et le taux d'emploi. Puisque les résultats des simulations numériques peuvent dépendre du choix des paramètres nous avons testé la robustesse de nos résultats à l'aide de la méthode Monte-Carlo. Il s'agit d'un test classique, pour les études qui impliquent des simulation numérique, effectué sur un grand nombre de simulations en tirant au hazard les valeurs d'un certain nombre de paramètres. C'est grâce à ce test que nous avons pu montrer que la transition à la nouvelle technologie n'est pas lié à la flexibilité des salaires réels mais plutôt aux conditions d'accumulation du capital.

Outre la mise en evidence de l'esprit du modèle et l'étude analytique du régime d'equilibre, dans l'article reporté dans le Chapitre 4 nous decrivons les possibilité d'utilisation des simulations numériques et nous introduisons une méthode non standard pour l'étude numérique des orbites d'un système dynamique. L'approche habituelle consiste à se donner les conditions initiales, à fixer les paramètres et à decrire l'évolution de l'orbite correspondante. Dans l'approche introduite étant donné les conditions initiales on cherche une orbite menant à des conditions finales correspondant par exemple à un nouveau type d'équilibre et on observe les variations necessaires des paramètres pour assurer la viabilité d'un tel chemin de traverse.

La complexité du chemin de traverse de ces variables nous donne des renseignements importants sur la possibilité d'atteindre l' équilibre desiré. Un travail ultérieur pourrait être fait sur la caractérisation de la notion de complexité par exemple en utilisant la largeur du spectre de Fourier d'une telle orbite.

Notons que, dans ce même travail, contrairement aux expériences précédentes, nous avons accordé aux agents une connaissance parfaite du modèle. Les résultats nous montrent que la complexité n'est pas due seulement aux defauts de coordination des agents liés à leur connaissance imparfaite du modèle, mais aussi au caractere définitif de la structure de la capacité productive determiné par le decisions d'investissement et par les contraintes de ressources humaines.

C'est dans ce cadre que nous avons étudié la propagation d'un choc technologique et son influence en particulier sur la productivité et le taux de chômage dans des économies caracterisées par differentes choix de politique monétaire. Les résultats sont présentés dans l'article au Chapitre 5.

Une autre question qui nous semble interessante concerne l'étude du modèle du point de vue de la caractérisation dynamique des orbites. Puisque il s'agit d'un modèle non-linéaire (la non-linéarité derivant du coté court de certaines variables) on se trouve face à la problématique de l'ordre et du chaos. Dans le cas d'orbites chaotiques il est importante de pouvoir quantifier la force du chaos. La méthode classique pour cela est le calcul des exposants de Lyapunov, c'est à dire la mesure de la divergence au cours du temps de deux orbites trés proches. Dans le domaines des systèmes dynamiques d'autres méthodes ont été developpés au cours des dérniers dix ans pour caracteriser la dynamique des orbites en particulier pour pouvoir differencier, parmi les orbites régulieres entre îles de libration et tors de circulation, les deux ayant les exposants de Lyapunov égals à zéro. Nous citons parmi eux l'analyse en fréquence par Laskar et al (1992) et Lega et Froeschlé (1996), l'analyse de l'extremum de l'action (Laskar 1990, Froeschlé and Lega 1996) ou de l'angle d'helicité (Contopoulos and Voglis 1997, Froeschlé and Lega 1998). La necessité d'effectuer des analyse globales sur beaucoup d'orbites de l'espace de phases a aussi poussé la recherche d'indicateurs rapides comme l'Indicateur Rapide de Lyapunov (Froeschlé et al. 1997, Froeschlé et al. 2000).

Mais toutes ces méthodes nécessitent la connaissance du modèle, alors que souvent on est face à cette problematique pour des séries experimentales pour lesquels on ne connait pas le système d'équations sous-jacent. Beaucoup d'efforts ont été faits dans de nombreuses disciplines y compris l'économétrie pour l'étude de telles séries temporelles. Dans le cas de séries chaotiques la prémière question est liée à la nature du chaos, à savoir s' il s'agit du chaos deterministe ou stochastique. Dans le cas deterministe il faut pouvoir à partir d'une seule série établir le nombre de degrés de libertés du système pour mesurer ensuite la valeur des indicateurs de Lyapunov.

Dans l'article presenté dans le Chapitre 6 nous passons en revue les méthodes existantes pour le calcul de l'indicateur de Lyapunov sur des séries expérimentales et nous introduisons une nouvelle méthode qui semble être bien adaptée aux modèles conservatifs tout en donnant des bons résultats pour le cas dissipatifs. Notons que les méthodes introduites s'appliquent à des systèmes à un petit nombre de degrés de liberté (inférieur ou égale à 5) ou à des systèmes à un grand nombre de degrés de liberté dans lesquels seul un petit nombre de degrés de liberté joue un role important dans la dynamique.

Dans le Chapitre 7 nous appliquons une des méthodes introduites à des séries chaotiques issues du modèle néo-autrichien. Nous retrouvons par cette méthode que les degrés de liberté importants pour la dynamique du modèle sont exactement ceux que nous avons appellé variables clés du modèle. Ce résultat nous semble ouvrir une piste pour l'analyse des séries économetriques.

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Chapitre 1

Un modèle dynamique à plusieurs degrés de liberte: le modèle néo-autrichien.

1.1 Le modèle néo-Autrichien

Le modèle néo-autrichien de J.R. Hicks (1973) est un modèle de croissance exogène où la croissance du produit est déterminée par celle de la population (et pourrait l'être par une croissance exogène de la productivité)[1].

Les modèles néo-autrichiens ont pour spécificité de mettre en évidence la dimension temporelle de la production en proposant une représentation de celle-ci qui distingue la phase de construction de la capacité de production de sa phase d'utilisation.

1.2 Un modèle séquentiel

Un modèle séquentiel, qui étend le modèle néo-autrichiens de Hicks (1973) pour prendre en considération la possibilité de défauts de coordination de marché, est présenté dans Amendola et Gaffard (1998). La structure générale est, reprise ci-après.

Ce modèle est constitué d'un ensemble d'équations qui, certes, peuvent être qualifiées d'*ad hoc*, mais dont l'objet est bien de rendre compte du caractère séquentiel à la fois des processus de production et des processus de décision. Il est, essentiellement, destiné à effectuer des simulations numériques susceptibles d'éclairer certaines propriétés de l'évolution économique.

La capacité de production

Il s'agit d'un modèle dans lequel un seul bien est produit à travers le travail. Chaque processus de production est représenté par une matrice d'inputs de travail et par un vecteur de d'output de bien final qui s'écrivent respectivement:

$\mathbf{A} = [\mathbf{A^c}, \mathbf{A^u}]$

avec $\mathbf{A^c} = \begin{bmatrix} a_{1,1} & ... & a_{n^c,1} \\ ... & ... & ... \\ a_{1,s} & ... & a_{n^c,s} \end{bmatrix}$ et $\mathbf{A^u} = \begin{bmatrix} a_{n+1,1} & ... & a_{n^c+n^u,1} \\ ... & ... & ... \\ a_{n+1,s} & ... & a_{n^c+n^u,s} \end{bmatrix}$

où h=1...s désignent les différentes qualifications de la main d'oeuvre engagée dans des processus dont la durée de construction est n^cet la durée d'utilisation est n^u,

$\mathbf{b} = [\mathbf{b_{n^c+1}}, ..., \mathbf{b_{n^c+n^u}}]$

La capacité productive au temps t s'écrit:

$\mathbf{x^c(t)} = [x_1(t), x_2(t), ..., x_{n^c}(t)]$

[1]Ce modèle pourrait être formulé de telle manière à accueillir un mécanisme d'apprentissage par la pratique à la Arrow, impliquant les mêmes restrictions formelles pour obtenir une croissance endogène (R. Solow 1997).

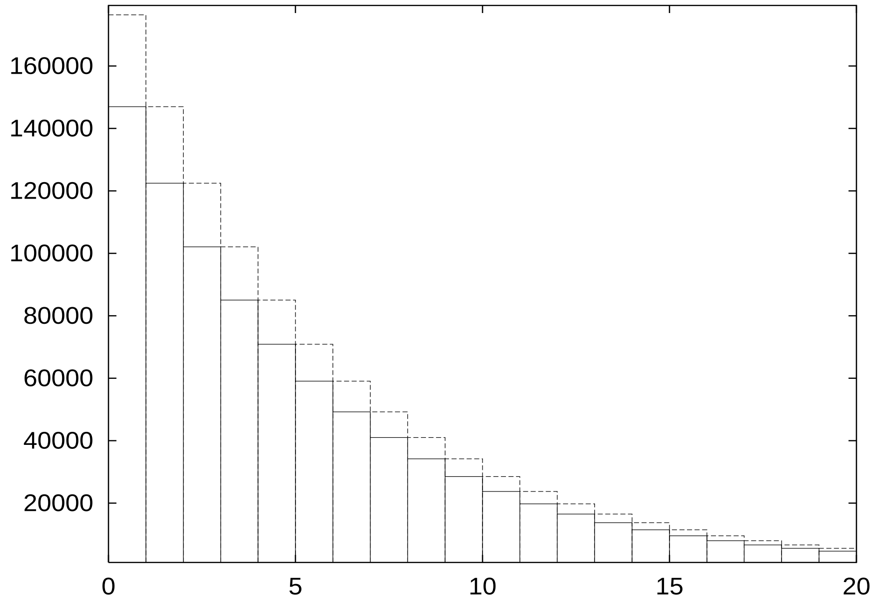

Figure 1.1: Profil de la structure de processus de production à un temps t donné (ligne continue), pour un régime régulier avec $n^c = 10$, $n^u = 10$ et un taux de croissance $g = 2\%$. Evolution, au temps $t+1$, de la structure de la capacité de production par le seul effet du viellissement (ligne pointillé).

$$\mathbf{x^u(t)} = [x_{n^c+1}(t), x_{n^c+2}(t), ..., x_{n^c+n^u}(t)]$$

où $\mathbf{x^c(t)}$ et $\mathbf{x^u(t)}$ sont les vecteurs des processus de production respectivement en phase de construction et en phase d'utilisation.

En régime régulier:

$$x_j(t) = x_{j-1}(t-1) = x_{j+1}(t)G$$

$$x_j(t) = x_N(t)G^{N-j}; N = n^c + n^u; j = 1, ..., N$$

où G=1+g est le facteur de croissance et g le taux de croissance naturel.

La Figure 1.1 montre le profil de la structure de la capacité de production à un temps t donné. Dans le modèle la dynamique de l'évolution est le viellissement naturel de processus, c'est-à -dire que les processus d'âge j au temps t sont donnés par le processus d'âge $j-1$ à la période precedente $t-1$. Des modifications de cette structure sont réalisées, comme on verra par la suite par le processus de decision des agents. Deux sortes d'agents sont considerés: les firmes et les ménages. Les échanges entre eux se font par l'intermediaire de la monnaie.

Le Fonds des Salaires

Les ressources requises pour mettre en oeuvre les processus de production et soutenir la consommation sont des ressources financières. Ce sont des ressources internes et externes.

$m(t) = \min\left[p(t)s(t), p(t)d(t)\right]$ est le montant des ventes (les ressources financières internes), où s et d sont l'offre et la demande du bien final et p son prix.

$f(t)$ est l'offre de monnaie (les ressources financières externes).

Dans un modèle néo-Autrichien, le Fonds des Salaires représente les ressources qui soutiennent le processus d'accumulation du capital. Il est déterminé par le minimum entre les ressources financières disponibles et un fonds de salaires contraint par les ressources humaines disponibles:

$\omega(t) = \min\left[F(t), w(t)L^s(t)\right]$

où $w(t)$ est le taux de salaire, $L^s(t)$ l'offre de travail donnée par:

$L^S(t) = L^S(0)\left[1+g\right]^t$.

et $F(t) = m(t-1) + h^f(t-1) + f(t) - c(t)$

$c(t)$ est le prélèvement ou take out (les ressources retirées du financement des processus de production) et $h^f(t)$ les encaisses oisives des firmes qui s'accumulent quand la contrainte de ressources humaines devient plus forte que la contrainte financière:

$h^f(t) = F(t) - w(t)L^s(t)$

Les décisions sont prises comme suit:

Les décisions de production

La production courante $q(t)$ est la difference entre l'offre courante $s(t)$ et le montant des stocks $o(t-1)$, réellement remis sur le marché: $q(t) = s(t) - o(t-1)$

où $o(t) = s(t) - d(t)$

La valeur monétaire de l'offre courante est determinée sur la base des recettes monétaires anticipées:

$p(t)s(t) = p(t-1)d(t-1)(1+g)$

La production courante determine le vecteur des processus de production en phase d'utilisation $\mathbf{x}^u\mathbf{(t)}$ de la façon suivante:

- si $q(t)$ s'avère être plus élevé ou égal à la la capacité de production héritée du passé alors la structure de production au temps t sera exactement celle du temps $t-1$ vieillie d'une période, c'est à dire que les plus vieux processus de la période précédente sont detruits et que de nouveaux processus (ceux d'âge n^c à la période $t-1$) deviennent productifs.

- en revanche si $q(t)$ est moins elevé que la capacité de production héritée du passé alors on detruit (ou bien on reduit la production) des processus en cours d'utilisation en commencant par les plus vieux.

Les décisions de consommation

La valeur monétaire de la demande finale des ménages est déterminée par leur contrainte financière:

$y(t) = p(t)d(t) = \omega(t) + c(t) + h^h(t-1)$

où $h^h(t) = \max\left[p(t)\left(d(t) - s(t)\right), 0\right]$ sont les encaisses oisives des ménages qui s'accumulent quand la valeur de la demande finale excède la valeur de l'offre courante.

Les décisions d'investissement

Toutes les ressources financières disponibles peuvent être investies:

$i(t) = \omega(t) - \omega^u(t)$

où $\omega^u(t) = w(t) < \mathbf{a^u(t)x^u\ (t)} >$

Alternativement l'investissement peut être déterminé comme suit:

$$i(t) = \min\left[\omega(t) - \omega^u(t), w(t) < \mathbf{a^c(t)x^c(t)} >\right] \tag{1.1}$$

où le taux de natalité $x_1(t)$ est determiné par les firmes de façon à prévenir des distorsions dans la structure par âges de la capacité de production.

Dans ce dernier cas de figure, une demande de financement est déterminée, qui sera confrontée à l'offre de financement assimilée ici à un offre de monnaie centrale. Elle s'écrit:

$$f^d(t) = \omega^*(t) - m(t-1) - h^f(t-1) \tag{1.2}$$

où $\omega^*(t)$ est le fonds des salaires requis pour satisfaire la demande de financement.

Le financement effectif est, alors, tel que:

$f(t) = \min\left[f^d(t), f^s(t)\right]$

Il détermine le fonds des salaires effectif $\omega(t)$ et l'investissement effectif $i(t)$.

Les variables de contrôle

Etant donné le régime régulier (c'est-à -dire étant fixés tous les paramètres du modèle: le nombre des processus, la durée des phases de constructions et utilisation, le taux de croissance de la population etc..) et étant donnée les règles pour l'évolution temporelle du processus de production, la dynamique du modèle dépend de 4 variables de contrôle ou variables-clés: les prix, les salaires, le financement externe et le prélèvement.

Prix et salaires changent d'une période à l'autre en réaction aux déséquilibres de marché:

$$g_p(t) = \kappa\Phi(t-1) \tag{1.3}$$

$$g_w(t) = \nu\Psi(t-1) \tag{1.4}$$

où $g_p(t)$ est le taux de variation du prix, $\Phi(t-1)$ le taux de demande excédentaire de produit final, $g_w(t)$ le taux de variation des salaires,$\Psi(t-1)$ le taux de demande excédentaire de travail, κ et ν sont les coefficients de réaction aux déséquilibres.

L'offre de monnaie est, alternativement determinée sur la base du taux de croissance courant:

$$f^s(t) = f^s(t-1)\left[1 + g_m(t-1\right] \tag{1.5}$$

ou de telle manière à garder le niveau des prix stable:

$$f^s(t) = f^s(t-1)\left[1 + g_f(t-1) - \varsigma g_p(t)\right] \quad \varsigma > 0 \tag{1.6}$$

ou sur la base d'un taux de croissance cible:

$$f^s(t) = f^s(t-1)\left[1 + g^\star(t-1\right] \tag{1.7}$$

Le take out est alternativement determiné sur la base du taux de croissance courant

$$c(t) = c(t-1)\left[1 + g_m(t-1\right] \tag{1.8}$$

ou de telle manière à augmenter graduellement le montant des ressources dédiées au financement de l'investissement:

$$c(t) = c(t-1)\left[1 + g_m(t-1)\epsilon^{t-t_c}\right] \tag{1.9}$$

où t$_c$ est la date à laquelle le taux de prélèvement commence à décroître.

1.2.1 L'algorithme

Pour résumer le contenu du modèle et en preciser la dynamique la Fig.1.2 montre le diagramme à blocs utilisé pour les simulations numériques. La sequence des instructions est la suivante:

1. Initialisation du régime régulier: on fixe tous les paramètres du modèle: le nombre des processus, la durée des phases de construction et utilisation, le taux de croissance de la population, la technologie etc.., ainsi que l'économie que l'on veut étudier en fixant les équations qui donnent l'évolution temporelle des variables-clés.

2. On avance le temps d'une période

3. En début de période on détermine la valeur des prix et des salaires, l'offre de monnaie et le prélèvement. La valeur des quattre variables-clé permet de calculer la masse salariale à la période courante.

4. Ensuite les firmes anticipent la production pour la période. En cas d'anticipations adaptatives un déclassement des processus en cours d'utlisation peut avoir lieu.

5. A ce stade on a determiné la structure de la capacité productive en utilisation. Le côut correspondant permet de trouver, par différence avec la masse salariale totale, le montant des ressources disponibles pour la phase de construction et donc le nombre des processus d'âge zéro. Si les firmes ont un comportement rationnel une demande de monnaie est calculée pour pouvoir construire le nombre desiré de nuveaux processus. Le coté court entre l'offre et la demande de monnaie permet de calculer le nombre effectif des nouveaux processus.

6. En cas de contrainte de ressources humaines le nombre total de processus est adapté à l'offre de travail et des éventuelles encaisses non desirées par les firmes sont calculées.

7. On calcule la demande de bien formulée par les menages.

8. Le bien produit est vendu en fin de période et le montant des recettes est donné par le minimum entre l'offre et la demande.

9. Plusieurs variables sont calculées à ce stade comme le taux de croissance, le taux de chômage etc..

10. les données sont enregistrées et la simulation boucle sur l'instruction avancement du temps. La boucle est repetée pour le nombre de périodes desirés.

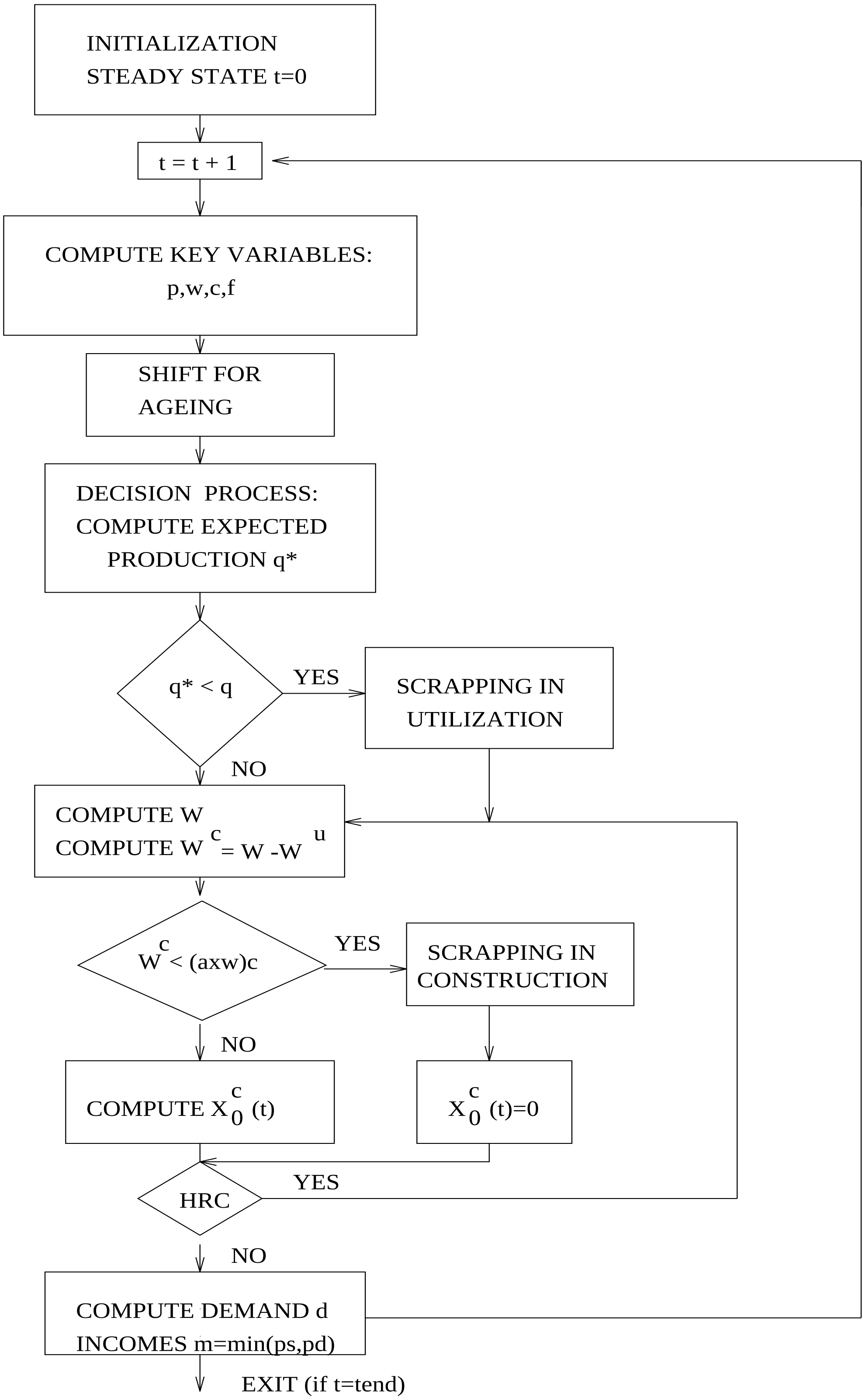

Figure 1.2: Diagramme à blocs du modèle néo-autrichien

1.3 Chocs et modélisation spécifique aux différents travaux presentés dans la thèse

Etant donnée la structure générale du modèle, à chaque problématique correspond un choix particulier de l'économie étudiée, du choc analysé et de différentes politiques économiques ainsi que des stratégies des agents pour faire face au choc. Ces choix donne au modèle une specificité propre à chaque analyse effectuée.

Pour faciliter la lecture du modèle dans le differents articles de la thèse nous allons alors schematiser les differences de modélisation propres à chaque chapitre.

1.3.1 Le choc financier et le choc energetique

Dans l'article presenté au chapitre 2 on analyse les effets de la discrétisation en simulant un choc financier et un choc sur le prix de l'énergie. Le premier type de choc modelisé dans les équations 2.29,2.30 represente une manque de confiance des firmes et des ménages qui épargnent de l'argent sous forme d'encaisses desirées. Ceci engendre une diminution de la demande et de la masse salariale et donc une distorsion dans la structure de la capacité productive qui n'est pas toujours résorbée (Fig.2.6). Prix et salaires changent en réponse aux signaux du marché (Eq.1.3 et 1.4), l'offre de monnaie et le prélèvement sont determinés sur la base du taux de croissance courant (Eq.1.5 et 1.8).

Le choc sur le prix de l'energie induit par une augmentation du volume de l'activité productive est modelisé dans l'équation 2.18. Dans ce cas on suppose que les agents ont un comportement rationnel au sens qu'ils sont capables de calculer les valeurs de la demande de monnaie et du prélèvement necessaires pour repositionner l'économie sur le regime régulier. Toutefois on suppose que des limites existent sur les variations requises de f et de c, c'est-à -dire que la demande de financement et le prélèvement peuvent être differents des ceux desirés. Nous suggérons au lecteur qui connait le modèle de ne pas lire la section 2.3.1 qui en donne l'explication. En revanche la section 2.3.2 est importante car elle concerne la procédure de normalisation nécessaire pour l'étude des effets de discretisation ainsi que l'application aux deux chocs.

1.3.2 Le choc technologique

Quand une économie subit un choc technologique la construction et l'utilisation ne sont plus en synchronisation, autrement dit la croissance est irrégulière. Ce type de choc a été étudié aux Chapitres 3,4,5. Le modèle décrit une économie dont la source fondamentale de croissance est une augmentation de la population. Des gains de productivité réguliers et exogènes ne sont pas ici envisagés. Les chocs technologiques sont, en revanche, susceptibles d'accroître définitivement le produit par tête à chaque période et de provoquer une augmentation temporaire du taux de croissance.

Dans le cas général, tout changement de technologie, qui appelle la mise en oeuvre de nouveaux processus de production[2], comporte un coût de construction supplémentaire qui a pour effet de réduire au moins temporairement l'investissement en capacité, c'est-à-dire, la taille de la capacité mise en construction. Ce phénomène introduit une distorsion dans la structure par âges de la capacité productive génératrice des irrégularités de la croissance. Les premiers effets de cette distorsion se manifestent au terme d'une période de temps qui correspond à la durée de construction des nouveaux processus. Ils prennent la forme d'un recul temporaire du niveau relatif d'activité[3].

L'accent mis sur la dimension temporelle de la production interfère avec la conception même de la technologie et du développement technologique. L'approche néo-autrichienne, parce qu'elle permet de révéler les propriétés dynamiques d'une économie qui évolue hors de l'équilibre, donne, en effet, un éclairage du développement des technologies qui lui confère une dimension essentiellement économique. Elle permet, en effet, de mettre en évidence, que le gain retiré d'un changement de technologie dépend, certes, des propriétés intrinsèques de la nouvelle technologie, telles qu'elles peuvent être mesurées quand l'économie est en régime régulier, mais aussi et surtout des conditions de coordination qui prévalent au cours du processus de changement lui même. Quand des retards ou des ralentissements sont observés, ils ne traduisent pas nécessairement, ou pas seulement, des problèmes de délais de diffusion ou d'apprentissage qui renverraient aux propriétés intrinsèques d'une technologie particulière, définie en dehors de la sphère économique. Ils traduisent des défauts de coordination de l'activité qui finissent par structurer la technologie elle même.

Comme l'on verra dans les articles presentés aux Chapitres 3 et 5, les 'fondamentaux' déterminent le potentiel de croissance de l'économie, mais les mécanismes de coordination, qui mettent en jeu le mode de formation des prix et des salaires, la politique monétaire, la politique budgétaire, déterminent la capacité de l'économie à réaliser ce potentiel. Tout dépend de la solution apportée aux problèmes de coordination. Naturellement, des anticipations rationnelles qui correspondent à une pleine coordination garantissent de réaliser le potentiel de croissance. Ce résultat bien que tautologique est interessant d'un point de vue dynamique car il montre que la complexité du modèle n'est pas completement liée aux defauts de coordination mais aussi au caractère definitif de la structure de la capacité productive (Chapitre 4). Il convient, donc,

[2]Ce sera, évidement, le cas avec l'irruption de technologies génériques, mais aussi, plus généralement, à chaque fois qu'il faut construire de nouvelles usines ou constituer de nouvelles équipes de recherche-développement.

[3]Le phénomène ainsi mis en évidence ne procède pas comme dans le modèle de E. Helpman et M. Trajtenberg (1998), d'un déplacement des ressources vers l'amont des processus, mais au contraire d'une contrainte imposée sur les ressources affectées à la construction. En outre, il ne se manifeste pas immédiatement, au moment de l'apparition de la nouvelle technologie, mais au terme d'une période qui peut être relativement longue.

d'explorer différents scénarios correspondant à de formes imparfaites de coordination. Dans les articles presentés aux Chapitres 4 et 5 on a supposé que les firmes sont rationnelles dans le sens où elles décident de l'investissement, non pas de manière mécanique en suivant un principe d'accélérateur (comme dans les cas étudiés au Chapitre 3), mais en ayant pour objectif de maintenir équilibrée la structure temporelle de la capacité de production[4]. Les écarts enregistrés par rapport à cet objectif résultent de l'existence de contraintes de ressources financières ou humaines. En revanche (Chapitres 3 et 5), on a considéré que l'offre courante est déterminée par une demande finale anticipée de manière adaptative. Ce sont les informations fournies par les déséquilibres perçus sur le marché qui sont déterminantes. Elles conduisent, le cas échéant, à des déclassements de processus de production en cours d'utilisation ou à une diminution de leur taux d'utilisation.

Ici aussi, nous suggerons au lecteur qui connais le modèle de ne pas lire les sections 3.3, 4.2 et 5.3 qui en donnent la description.

1.3.3 L'analyse des series expérimentales

Au Chapitre 6 nous traitons la question de l'analyse des proprietés dynamiques d'une orbite à partir de la connaissance des donnés experimentales dans l'hypothèse que les équations qui gouvernent la dynamique sont inconnues. Cette question nous semble importante pour l'analyse économique, car on dispose des beaucoup des données experimentales. En particulier, et c'est le rêve pour le physicien, des séries financieres trés longues sont disponibles. En effet, pour l'analyse des séries longues (de l'ordre de 10 000 données ou plus) plusieurs methodes ont été developpés (parmi les plus importantes celle de Eckmann et al. 1986 et de Grassberger et Procaccia 1983).

En revanche, nous avons focalisé l'attention sur les méthodes d'analyse des séries courtes (maximum 1000 données), méthodes qui pourraient se reveler interessant pour l'analyse des séries macro-économique (ou on peut avoir trente ou quarante ans de données mensuels). Pour étudier et comparer les méthodes nous avons utilisés, comme cas d'école, des systèmes dynamiques très bien connus comme le mapping standard et le mapping d'Hénon. De la meme façon, dans le chapitre 7, nous utilisons des series chaotiques obtenues avec le modèle néo-autrichien comme cas d'école pour un eventuel travail futur sur des séries macro.

Le choc consideré est un choc financier qui n'a pas de caractère permanent comme dans le cas traité au chapitre 2 (mais ceci n'est pas fondamentale afin d'obtenir une série chaotique). Les firmes pour une seule période ont un comportement pessimiste (Eq.7.2) c'est-à -dire que le nombre des nouveaux processus et moindre que celui du régime régulier à la période t et cet effet

[4]Elles connaissent le poids des complémentarités intertemporelles et se comportent en conséquence. Si l'objectif est atteint, alors il existe une relation stable de type accélérateur qui commande l'investissement.

est compensé par l'augmentation du nombre des nouveaux processus à la période $t+1$. Malgré le fait que à la période $t+1$ le nombre totale des processus corresponde a celui du régime régulier (on fait l'hypothèse de croissance zéro) le choc introduit suffit pour casser l'équilibre inter-temporel de la structure productive et pour donner, sous le conditions ue l'on verra ci-dessous, une orbite complexe.

Les agents ont un comportement rationnel en ce qui concerne l'offre (on ne fait pas de déclassement de processus) et en ce qui concerne l'objectif de construction. Le financement est demandé comme dans l'équation 1.2. Afin d'obtenir des séries chaotiques bornées nous avons rajouté des limites à la demande de financement, c'est à dire que le taux de natalité évolue dans un domaine limité autour du taux de natalité du régime régulier (Eq.7.4).

Les prix et les salaires varient selon les déséquilibres du marché et le prélèvement évolue selon le taux de croissance de l'économie.

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Chapitre 2

Le problème de la description du modèle en temps discret et en temps continu

Le papier presenté dans ce chapitre traite des relations entre les solutions d'un système dynamique dont les équations, discretes ou continues, concernent le même modèle. Nous avons introduit le problème pour un simple cas physique: le pendule. Nous montrons que dans le cas de la représentation néo-Autrichienne du processus de production, comme pour le cas de tous systèmes dynamiques il existe un seuil sur le pas de discrétisation à partir duquel le modèle continu est bien approximé du point de vue numérique (les résultats deviennent independants du pas de discretisation). Pour un pas de temps superieur au seuil seul des considérations économiques justifient le modèle. On presente enfin une méthode numérique générale pour la transformation d'un problème continu dans un problème discret.

FROM DISCRETE TO CONTINUOUS DYNAMICAL SYSTEMS AND VICE-VERSA. APPLICATION TO A NEO-AUSTRIAN REPRESENTATION OF THE PRODUCTION PROCESS

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abstract

This paper deals with the relations between the solution of dynamical systems whose equations, either discrete or continuous originate from the same model. We introduce the problem for a simple physical model: the pendulum. We discuss then a neo-Austrian representation of the production process. Finally we present a general numerical method which allows to transform a continuous problem into a discrete one.

Key-words: Dynamical Systems, Neo-Austrian model

Ce papier traite des relations entre les solutions d'un système dynamique dont les equations, discretes ou continues, concernent le même modèle. Nous avons introduit le problème pour un simple cas physique: le pendule. Nous avons, par la suite, discuté la representation neo-Autrichienne du processus de production. On presente enfin une methode numérique generale pour la transformation d'un problème continu dans un problème discret.

2.1 Introduction

There are many arguments in favor of continuous modeling of economy. However even continuous models, if not trivially simple, implies discretization as for any method used to solve numerically ordinary differential equations.

Using a very simple pendulum differential equation we will first show that going from such equation to a mapping can be extremely misleading and can give rise, even qualitatively to different behaviour depending on the method used to discretize. Indeed in the case of an undamped pendulum the symplectic structure plays a fundamental role. For a damped pendulum the situation is less critical.

The second section deals with a neo austrian model with a large number of degrees of freedom. Going to a continuous system, i.e. decreasing the time interval increases proportionally the number of degrees of freedom and nevertheless increases also the stability of the system. However again a close analysis of the structure of the system can lead to a kind of re-normalization on the parameters leading to the same behaviour independently of the discretization. Finally we will come back to a more general problem: given a continuous system find, even for Hamiltonian systems, a computable Poincaré map. Our approach allows to obtain the true correspondence between the two kinds of modelization.

2.2 A simple example: the pendulum

Let us consider as a simple problem the Hamiltonian pendulum:

$$H = \frac{p^2}{2} - \cos\theta \tag{2.1}$$

where p and θ are conjugate variables. We have the system:

$$A = \begin{cases} \dot{\theta} & = & \frac{\partial H}{\partial p} = & p \\ \dot{p} & = & -\frac{\partial H}{\partial \theta} = & -\sin\theta \end{cases} \tag{2.2}$$

it is well known that this conservative system with one degree of freedom is integrable and the phase space diagram is shown on fig.2.1.

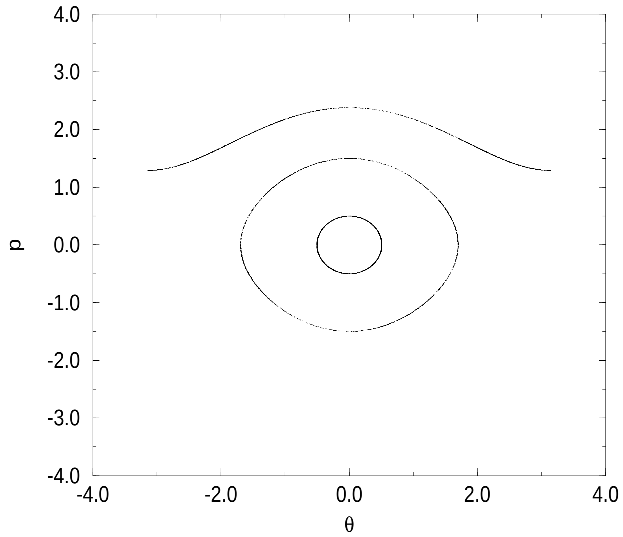

Figure 2.1: The phase portrait of the pendulum

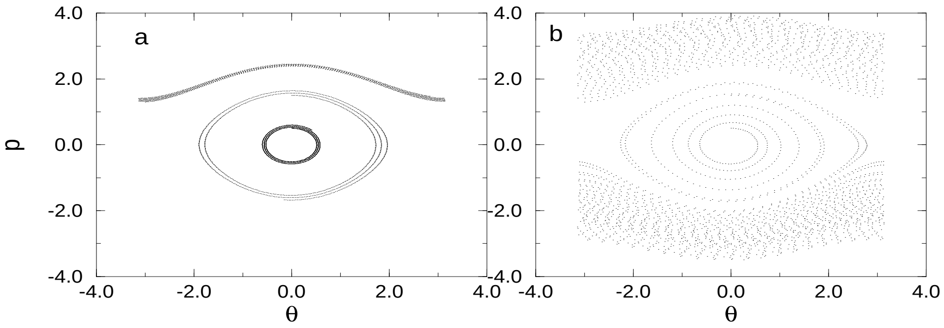

Figure 2.2: Same as figure 1 but obtained with the mapping T_1 (see text) for the following steps of discretization: a) $\Delta t = 0.02$, b)$\Delta t = 0.1$

The mapping T_1:

$$T_1 = \begin{cases} \theta_1 &= \theta_0 + p_0 \Delta t \\ p_1 &= p_0 - \Delta t \sin\theta_0 \end{cases} \tag{2.3}$$

is nothing but the Euler method to compute orbits of ordinary differential equations. Fig.2.2(a) shows clearly that even with a small value of $\Delta t = 0.02$ the orbits are no more closed and the system appears slowly expanding. For $\Delta t = 0.1$ the expansion of the orbits is indeed drastic (fig.2.2b).

If we compute the determinant of the Jacobian matrix J we obtain:

$$|J| = \begin{vmatrix} 1 & \Delta t \\ -\Delta t \cos\theta_0 & 1 \end{vmatrix} = 1 + \Delta t^2 \cos\theta_0 \tag{2.4}$$

which is not equal to one and therefore the mapping is not an area preserving one, as we would expect since the flow under study is Hamiltonian.

Let us make a slight change in the mapping and consider instead the mapping T_2:

$$T_2 = \begin{cases} \theta_1 &= \theta_0 + p_0 \Delta t \\ p_1 &= p_0 - \Delta t \sin\theta_1 \end{cases} \tag{2.5}$$

Again if $\Delta t \to 0$, $\theta_1 \to \theta_0$, $p_1 \to p_0$ and we obtain dividing by Δt the system of equations A i.e. $\lim \frac{\theta_1 - \theta_0}{\Delta t} = p_0$ gives $\dot{\theta} = p$ etc.

However the determinant of the Jacobian matrix is now equal to 1 and fig.2.3 shows an astonishing agreement of the orbits of the mapping T_2, even for $\Delta t = 0.7$, with the solutions of system A (fig.2.1) without a significant increase of the complexity, i.e. of the dimension of the mapping, like using a more sophisticated method. Indeed the only difference is that the underlying mathematical structure (symplectic character) of equation 1 is preserved. Of course for $\Delta t = 1$ all the well known features of non-linear area preserving mapping (i.e. islands, chaotic zones etc.) appear. The slight and powerful change of T_1 in order to obtain T_2 is not a miracle, but the result of a search for a symplectic map. The Hamiltonian H can be considered as $H = H_1 + H_2$ with $H_1 = p^2/2$ and $H_2 = -\cos\theta$. Alternatively we solve H_1 and H_2 with $H_1 = p^2/2$ we have:

$$B = \begin{cases} \frac{d\theta}{dt} &= p \\ \frac{dp}{dt} &= 0 \end{cases} \tag{2.6}$$

which gives rise to the symplectic (area preserving mapping)

$$T_2' = \begin{cases} \theta_1 &= \theta_0 + p_0 \Delta t \\ p_1 &= p_0 \end{cases} \tag{2.7}$$

For the Hamiltonian: $H_2 = -\cos\theta$ we have

$$C = \begin{cases} \frac{d\theta}{dt} &= 0 \\ \frac{dp}{dt} &= -\sin\theta \end{cases} \tag{2.8}$$

and consequently:

$$T_2'' = \begin{cases} \theta_2 &= \theta_1 \\ p_2 &= p_1 - \sin\theta_1 \Delta t \end{cases} \tag{2.9}$$

Therefore $T_2 = T_2' \circ T_2''$ is also area preserving. This simple examples show clearly that the discretization of a continuous system can be misleading and should reflect the deep structure of the equations.

In the case of a damped pendulum we have a dissipation term $-\beta\dot{\theta}$ and we get the system:

$$D = \begin{cases} \dot{\theta} &= p \\ \dot{p} &= -\sin\theta - \beta\dot{\theta} \end{cases} \tag{2.10}$$

and the two corresponding mappings:

$$\tau_1 = \begin{cases} \theta_1 &= \theta_0 + p_0 \Delta t \\ p_1 &= p_0 - \sin\theta_0 \Delta t - \beta p_0 \Delta t \end{cases} \tag{2.11}$$

and:

$$\tau_2 = \begin{cases} \theta_1 &= \theta_0 + p_0 \Delta t \\ p_1 &= p_0 - \sin\theta_1 \Delta t - \beta p_0 \Delta t \end{cases} \tag{2.12}$$

The true orbit shown on fig 2.4(a) spiral down to the origin. With the mapping τ_1 in the same laps of time the spiral has not yet reached the origin conversely to the orbit of τ_2. It seems that

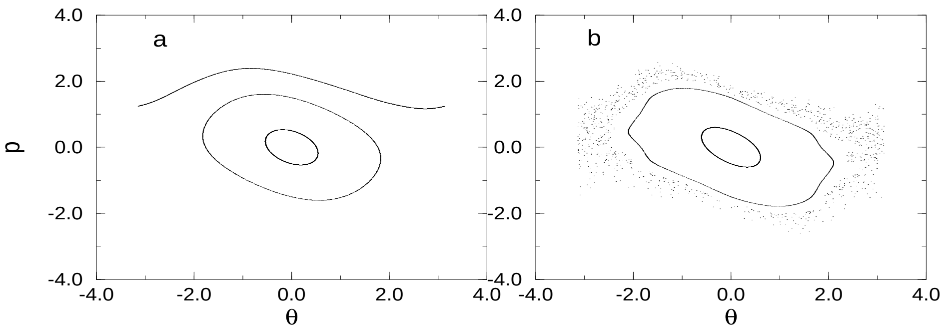

Figure 2.3: Same as fig.1 but obtained with the mapping T_2 (see text) for the following steps of discretization: a) $\Delta t = 0.7$, b)$\Delta t = 0.1$

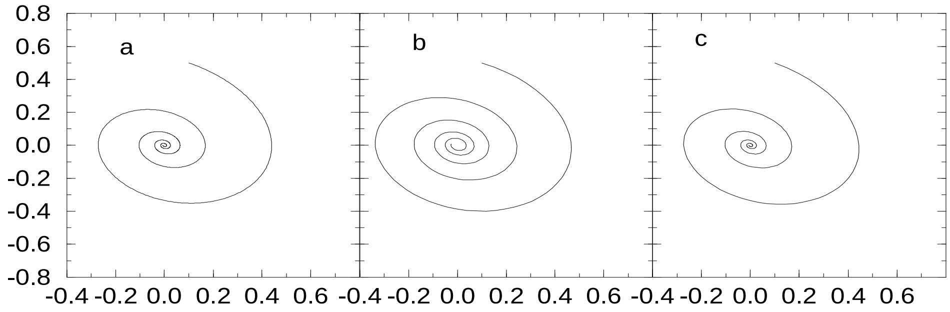

Figure 2.4: Phase portrait for the damped pendulum for a value of the damping coefficient: $\beta =$. a) True portrait. b) Phase portrait obtained with the mapping τ_1 (see text). c) Same as b) for the mapping τ_2 (see text).

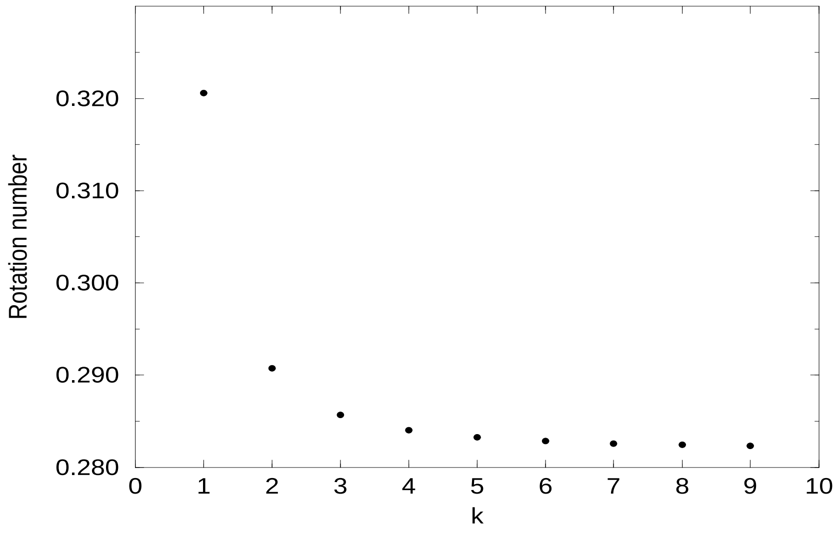

Figure 2.5: Variation of the rotation number as a function of the step size: $\Delta t_k = \Delta t_1/k$ for the mapping T_2.

even in the case of non Hamiltonian systems it is better to discretize the Hamiltonian part in a symplectic manner.

In order to have a better inside on the system and on the quality of the discretization for the mapping T_2 we measured a quantity, called the rotation number, for 9 values of Δt given by: $\Delta t_k = \Delta_1/k$ for $k = 1, ..9$ and $\Delta t_1 = 1.8$. The rotation number (see Herman 1983) used numerically to characterize an invariant curve (Laskar et al. 1992) , gives the frequency at which a point belonging to a given orbit rotates around the orbit itself. The computation has been done with a fast method introduced by M.Henon (Celletti and Froeschlé 1995). We can see on fig.2.5 that the rotation number tends to a limit when k increases, corresponding to the value obtained for the continuous case. The value $k = 5$ corresponds to a threshold on Δt, for a greater step size we do not reproduce adequately the original system.

2.3 From a discrete to a continuous model

2.3.1 The neo-austrian model

We consider a neo-austrian model (see Hicks 1973 , Amendola & Gaffard 1988 and in this volume, Amendola et al. 1993) where a homogeneous output is obtained from two primary inputs: labour and energy (oil).

Let us recall the set of discrete equations governing the dynamic. The elementary production is defined by the set of matrices:

$$a^c = [\mathbf{a}_0, \mathbf{a}_1, ...\mathbf{a}_n] \tag{2.13}$$

$$a^u = [\mathbf{a}_{n+1}, \mathbf{a}_{n+2}, ...\mathbf{a}_{n+N}] \tag{2.14}$$

$$b = [\mathbf{b}_{n+1}, \mathbf{b}_{n+2}, ...\mathbf{b}_{n+N}] \tag{2.15}$$

whose elements represent the quantities of labour and energy required by the process itself in the different periods of the phase of construction (0 to n) and utilization ($n+1$ to $n+N$) and the amount of final output obtained in the different periods of the phase of utilization. In the following we set: $\mathbf{b}_i = [b]$ for $i = n+1, ...n+N$ and $\mathbf{a}_i = [a, \gamma]$ for $i = 0, 1, ..n+N$ where a is the quantity of labour required and γ that of energy. The structure of the economy is represented by the vector $\mathbf{x}(t)$ whose elements are the numbers of production processes of a particular age at a given moment of time:

$$\mathbf{x}(t) = [x_0(t), x_1(t), ...x_{n+N}(t)] \tag{2.16}$$

Hence at time t the quantity of labour required is $A(t) = aX(t)$ with $X(t) = \sum_0^{n+N} x_i(t)$, the production is $B = b\sum_{n+1}^{n+N} x_i(t)$ and, if γ is the amount of the consumption of energy by process then the total consumption of energy is:

$$E(t) = \gamma X(t) \tag{2.17}$$

The price of unit of energy is given by a step shape function which modelizes a sudden increase of the price of energy due to an excess of demand as the volume of activity increases:

$$p_\gamma(t) = p_\gamma(0)\{1 + \eta_1[\tanh(\eta_2(\eta_3(t) - \eta_4)) - \tanh(-\eta_4\eta_2)]\} \tag{2.18}$$

where the $\{\eta_i\}_{i=1,4}$ are respectively:

1. η_1: the amplitude of the step.

2. η_2: the slope of the perturbation.

3. $\eta_3(t) = \frac{x(t)-x(t_0)}{x(t_0)}$ if $\eta_3 > -\eta_4$ and $\eta_3 = 0$ otherwise: it is the sensibility of the price of energy to the rate of the volume of activity.

4. η_4 measure the minimum value of the rate of activity which influences the price of energy.

Let us remark that the price of energy depends only on the volume of activity that is only on the demand. The value of the total consumption of energy is:

$$z(t) = p_\gamma(t)E(t) \tag{2.19}$$

The structure of the economy given by the set of $\{x_j(t)\}_{j=0,..n+N}$ changes with time according to the following rules:

1. - **Ageing** - as a consequence of the ageing we have:

$$x_j(t) = x_{j-1}(t-1) \text{ for } j = 1, ...n+N \tag{2.20}$$

$$x_0(t) = 0 \tag{2.21}$$

2. - **Scrapping** - the real number of production processes carried out is such that:

$$\tilde{x}_j(t) = x_j(t) - u_j(t) \text{ for } j = 1, ...n+N \tag{2.22}$$

$$\tilde{x}_0(t) = \tilde{u}_0(t) \tag{2.23}$$

   where $\tilde{u}_0(t)$ is the number of new processes and $u_j(t)$ is the number of processes which have been scrapped according to the rules described below.

The variables u_0, $\mathbf{x}^c$ and $\mathbf{x}^u$, that is, the number of new processes and the vector of the processes still in the phase of construction and already in the phase of utilization respectively, are determined, at each moment, as solutions of the model. Current production decisions determine $\mathbf{x}^u$ and, given the resources constraints and the scrapping rules (see below), investment decisions determine $\mathbf{x}^c$ and u_0. Scrapping of production processes $(u_j(t))$ occurs when human or financial resources constraints are so stringent as not to allow all the processes inherited from the past to be carried out. The scrapping rules taken into account are the following:

- when there is a final demand constraint the first processes in the utilization phase to be cut are the older;

- when there is a financial constraint on the amount of investment, the first to be cut is the rate of starts of new processes, then the processes still in the construction phase following their age.

Of course if a shortage of money induces a scrapping of processes in construction then $\tilde{u}_0(t)$ will be equal to zero. Let us also remark that in the case of regular regime $u_j(t) = 0$ for any $j \geq 1$ and $\tilde{u}_0(t) = \tilde{u}_0(0)(1+g)^t$ where g is the growth rate of the economy.

Finally the labour supply is given by:

$$l^s(t) = l^s(0)(1+g^*)^t \tag{2.24}$$

where g^* is the growth rate of labour supply. In the case of regular regime with full employment $g = g^*$.

Two classes of economics agents are considered: firms and households. All exchanges between them are intermediated by a financial asset, call it "money", which is the only one considered in the model. The resources required to carry out production, and to sustain consumption, are therefore financial resources, not physical output. These resources, for both producers and consumers, come from their participation to current productive activity. External financial resources are required when productive activity is expanding, and at the rate at which the expansion takes place.

In each period the firms level of activity is constrained by available financial resources or, alternatively, by labour supply. Let $m(t-1)$ be the money proceeds from the sales of the previous period, $f(t)$ the external financial resources, $w(t)$ the wage fund, $c(t)$ the resources devoted to consumption by producers, $z(t)$ the value of the total consumption of energy (eq.2.19), $h^f(t)$ the monetary idle balances held by firms (desired: $h^f_d(t)$, and undesired: $h^f_{nd}(t)$, i.e. $h^f = h^f_d + h^f_{nd}$). Then, the firms finance constraint which determines $w(t)$ can be written:

$$m(t-1) + h^f(t-1) + f(t) = w(t) + c(t) + z(t) + h^f_d(t) \text{ for } w(t) \leq \tilde{w}(t) \tag{2.25}$$

or alternatively

$$m(t-1) + h^f(t-1) + f(t) = \tilde{w}(t) + c(t) + z(t) + h^f(t) \text{ for } w(t) \geq \tilde{w}(t) \tag{2.26}$$

where $\tilde{w}(t)$ is the wage fund "constrained" by the available human resources given the current wage rate $\omega(t)$. $\tilde{w}(t) = \omega(t)l^s(t)$ if $l^s(t)\langle l^d(t)$, where l^s and l^d are respectively the supply of and the demand for labour.

When $w(t)\rangle\tilde{w}(t)$, that is, when firms are constrained in their activity by human resources, the shortage of labours creates a constraint upon investment decisions which results in a fall in the rate of starts and eventually in the scrapping of production processes. The available financial resources not used up result in turn non desired idle balances which are a sign of disequilibrium:

$$h^f_{nd}(t) = w(t) - \tilde{w}(t) \tag{2.27}$$

We assume that producers are not willing to hold these balances and hence try to use them since the following period.

In the model $h^f_d(t)$, $f(t)$ and $c(t)$ are explained either by exogenous functions (and then appear among parameters) or by endogenous mechanisms (see below).

Total expenditure by households also depends on financial resources available to them. Let $y(t)$ be the money value of current demand for final output, $h^h(t)$ the households' idle balances (desired: h^h_d, and undesired: h^h_{nd}), as defined in what follows. Then the households' finance constraint can be written:

$$y(t) = w(t) + c(t) + z(t) + h^h(t-1) - h^h_d(t) \tag{2.28}$$

since $z(t)$ appears in demand equation 2.28 it means that in some sense in this model the price of the energy is considered, as far as demand is concerned, as a salary which is reinjected in the economy. Owners of oil field are renters.

The desired idle balances for householders and firms are given respectively by:

$$h^h_d(t) = h^h_d(t-1) + \sigma(t)(w(t-1) + c(t-1)) \tag{2.29}$$

$$h^f_d(t) = h^f_d(t-1) + \rho(t)(m(t-1) + f(t-1)) \tag{2.30}$$

The coefficients σ and ρ change with time in a exogenous manner modelizing a pessimistic mood for firms and household. The resources devoted to consumption and the financial resources are explicited below (section 3.2) for the particular experiences studied.

Prices, and wages, are set at the beginning of each period and cannot be changed during the period; thus changes can only take place at the junction from one period to the next. This implies that the firms' money proceeds are given by:

$$m(t-1) = min[p(t-1)s(t-1); p(t-1)d(t-1)] \tag{2.31}$$

where d is the current demand for final output, s its current supply (which includes non desired stocks that the firms put back on the market) and p its price.

Real stock changes are then a substitute for price changes within each period. Excess supply of final output results in an accumulation of undesired stocks for the firms:

$$o(t) = s(t) - d(t)\rangle 0 \tag{2.32}$$

while excess demand results in undesired idle balances households:

$$h^h_{nd}(t) = p(t)[d(t) - s(t)]\rangle 0 \tag{2.33}$$

Households do not want to keep idle balances, and hence try to use them for consumption purposes since the following period.

Let the operator F be defined by $F_z(t) = \frac{(z(t)-z(t-1))}{z(t-1)}$ for any $z(t)$. Then prices and wages changes, which can only take place between periods and which we assume to be market determined, are given by:

$$F_p(t) = k\xi(t-1) \tag{2.34}$$

where

$$\xi(t-1) = \frac{[d(t-1) - s(t-1)]}{s(t-1)} \tag{2.35}$$

is the rate of excess demand of final output, and by:

$$F_\omega(t) = \nu\phi(t-1) \tag{2.36}$$

where $\omega(t)$ is the wage rate, and

$$\phi(t-1) = \frac{[l^d(t-1) - l^s(t-1)]}{l^s(t-1)} \tag{2.37}$$

is the rate of excess demand for labour. k and ν are given reaction coefficients. Different values of these coefficients stand for different regulating mechanisms which can be made to represent alternative market organizations.

Demand for final output in each period, in real terms, is given by:

$$d(t) = \frac{y(t)}{p(t)} \tag{2.38}$$

Supply of final output in each period, in real terms, is given by:

$$s(t) = \frac{m^*(t)}{p(t)} \tag{2.39}$$

where m^* is the money value of final demand expected by the firms in the same period according to different rules of expectation which will be explicited for the two sets of numerical experiments considered in section (3.2). In the case of energy crisis $m^*(t)$ is computed with the assumption of full information about the production of the considered period taking into account the ageing process and without any scrapping of processes in utilization. In the case of financial shock $m^*(t) = (1+g) * m(t-1)$.

Current final production will then be:

$$q(t) = s(t) - \eta[o(t-1)] \text{ for } 0 \leq \eta \leq 1 \tag{2.40}$$

where η is a given parameter which takes into account some "volatility" of the stocks. In the following stocks don't induce any cost and $\eta = 1$.

Thus we determine the vector of production processes in the phase of utilization $\tilde{\mathbf{x}}^u(t)$ given the scrapping rules in order to adjust the productive capacity with the current final production. $\tilde{\mathbf{x}}^u(t)$ will satisfy:

$$q(t) = b_{n+1}\tilde{x}_{n+1}(t) + ...b_{n+N}\tilde{x}_{n+N}(t) \tag{2.41}$$

Hence investment will be:

$$i(t) = w(t) - w^u(t) \tag{2.42}$$

where $w^u(t)$ is the fraction of the wage fund spent to carry out the production processes in the utilization phase, that is:

$$w^u(t) = \omega(t) < \mathbf{a}^u, \tilde{\mathbf{x}}^u(t) > \tag{2.43}$$

We determine then the vector of production processes in the phase of construction $\mathbf{x}^c(t)$ given the scrapping rules.

Total employment at time t is then:

$$L(t) =< \mathbf{a}^u, \mathbf{x}^u(t) > + < \mathbf{a}^c, \mathbf{x}^c(t) > \tag{2.44}$$

which results from current production and investment.

Given a perturbation in the economy there are several forces which could lead to a new equilibrium path. These are related to four keys variables: $p(t)$, $\omega(t)$, $f(t)$ and $c(t)$ which react to disequilibrium. We give in section 3.2 two examples of disequilibrium: one due to a financial shock, the other due to a sudden increase of the price of energy.

In the first case all the keys variables will be taken to react according to some blind market law: that is the agents change prices and wages according to the disequilibrium observed at the previous period, $c(t)$ and $f(t)$ change according to the growth factor of the previous period.

In the second one if we consider that the information of the agents about the model is more complete it can be shown that it is always possible to reestablish equilibrium path as far as the following policy is followed:

1. It should be no scrapping of any processes in order to avoid an increase of unemployment.
2. $\tilde{u}_0(t)$ should be such that a regular growth equal to g^* will ensure after $n + N$ iteration full employment.

The two key equations in order to achieve this program are equations 2.25 and 2.28 and among the four control variables $p(t)$, $\omega(t)$, $f(t)$ and $c(t)$ two of them ($p(t)$ and $\omega(t)$ change according to the market law (see eq. 34 35 36 37) while $f(t)$ and $c(t)$ adjusted in order to satisfy equations 2.25 and 2.28. Actually in equation 2.25 $w(t)$ and $y(t)$ are computed in order to satisfy requirements 1) and 2). Social and international constraints may prevent to adjust at once these control parameters therefore a threshold have been putted on their variations.

2.3.2 Applications

In order to test the passage from discrete to continuous we have studied the evolution of the system described above for two sets (A et B) of numerical experiments characterized by a particular perturbation in the economy. For the set A we have introduced a financial shock while for the set B we have simulated an oil crisis.

We studied the behaviour of the system for a large number M of paths, i.e. a number M of different initial conditions, measuring for each one, at each time t, economic quantities like prices, wages, total number of processes, production ...., whose value should remain economically plausible along the path. If not we stock the value of the time $\bar{t}$ at which the dynamical process is stopped. Of course if the simulation is run over a time T, the economy is viable for all orbits for which $\bar{t} = T$.

We have taken the distribution of $\bar{t}$ as a global indicator of viability of the economy, measuring the percentage of viable paths.

Regular regime

The numerical experiments start with an economy in the regular regime. In this case the following equation has to be satisfied:

$$w(t) = (1-\alpha)m(t) \tag{2.45}$$

where w is the wage fund: $w(t) = \omega(t) < \mathbf{a}, \mathbf{x} >$, and αm is the fraction of money m devoted to consumption. If $l^c n$ is the time for construction and $l^u n$ is that for utilization equation 2.45 becomes:

$$\omega(t)\bar{a} \sum_{i=0}^{(l^c+l^u)n-1} (1+g)^i = (1-\alpha)b(t)p(t) \sum_{i=0}^{l^u n-1} (1+g)^i \tag{2.46}$$

where $\bar{a} = a + \gamma$, for $\mathbf{a} = [a, \gamma]$ as defined in section 3.1. Considering $g \neq 0$ we obtain:

$$\frac{1-u^{l^c+l^u}}{1-u^{l^u}} = (1-\alpha)\frac{b(t)p(t)}{w(t)\bar{a}} \tag{2.47}$$

where $u = (1+g)^n$. In the particular case $l^c = jl^u$ the first term of equation 2.47 can be simplified giving: $\sum_{i=1}^{j} v^i$ where $v = u^{l^u}$. In our numerical experience we have considered $l^c = l^u$, that is $j = 1$ meaning for equation 2.47:

$$(1+g)^n = (1-\alpha)\frac{b(t)p(t)}{\omega(t)\bar{a}} - 1 \tag{2.48}$$

Let us remark that the amplitude of the discretization step on time does not appear explicitly as for the pendulum example (section 2). Here given a physical time T (one year for example) the step Δt is given by:

$$\Delta t = t_{i+1} - t_i = \frac{T}{(N_{iter})} \tag{2.49}$$

where N_{iter} is the total number of iterations considered, in general an integer multiple of the time $(l^c + l^u)n$ required for construction and utilization. For a given value of N_{iter} we analysed the effect of discretization for different values of Δt:

$$\Delta t_k = \frac{T}{kN_{iter}} \text{ for } k = 1, 2, ...9 \tag{2.50}$$

Of course if we change Δt some characteristic parameters have to be renormalized if we want to represent the same system: the daily production b or the rate of growth g are not the same as the weekly one. For both perturbations we made the following normalizations on final output and on the unitary cost of processes:

$$b_k = b_1/k \tag{2.51}$$

$$\omega_k a_k = \omega_1 a_1/k \tag{2.52}$$

Let us remark that as a consequence of this normalization, the right term of equation 2.45 is constant for any time step Δt. Therefore the normalization for the rate of growth is given by:

$$(1+g_k)^{kn} = (1-\alpha)\frac{b_1(t)p(t)}{\omega_1 a_1} = (1+g_1)^n \tag{2.53}$$

Depending on the perturbation some further normalization could be done.

Financial shock

The first case considered is a special case of the neo-austrian model for which the role of energy is not taken into account, i.e. $\gamma = 0$. According to equations 2.29 and 2.30, governing the desired balance for households and firms, the system undergoes a shock which last nc iterations after which ρ and σ go to zero.

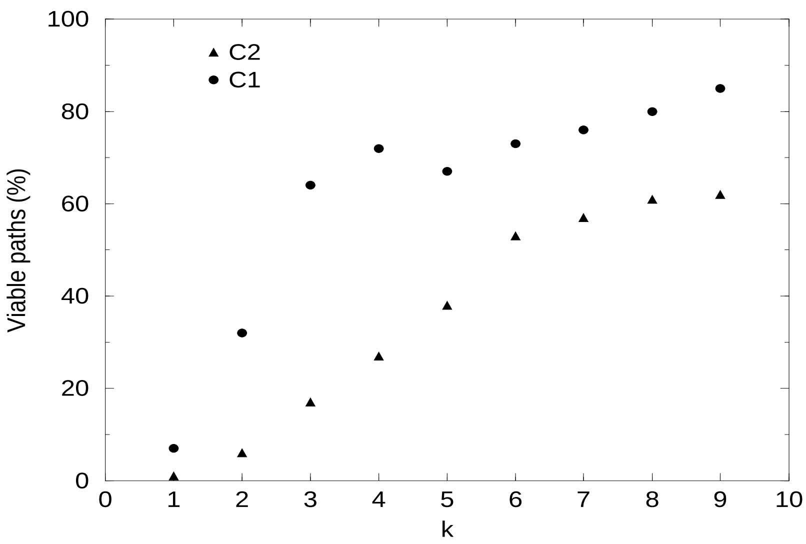

Figure 2.6: Percentage of viable processes for the case of financial shock (perturbation A, see text) as a function of the discretization step $\Delta t_k = T/kN_{iter}$. $C_2 = 2C_1$ with $C_1 = 0.0025$

For each economy the coefficients of reaction to disequilibrium between supply and demand (k and ν on equations 2.34 and 2.36) are taken respectively equal to 1 for the price (i.e. the market is cleared with a lag of one period) and to 0.5 for wages respectively.

For the set of 9 experiences done, the shock disturb the economy for the same interval of real time: $nc(k) = k(nc(1))$. For each value of k from 1 to 9, we computed, with all the renormalization described above, a number $M = 20 \times 20$ of paths, each one corresponding to a particular value of ρ and σ:

$$\begin{cases} \rho_k(j) = j\Delta\rho_k & forj = 1, ...20 \\ \sigma_k(j) = j\Delta\sigma_k & forj = 1, ...20 \end{cases} \tag{2.54}$$

where $\Delta\rho_k = \Delta\sigma_k = C/k$ with C constant. We stress the fact that once again in order to compare equivalent systems the step size of the magnitude of the perturbation is reduced as the inverse of the duration of the perturbation itself.

Fig.2.6 shows for two different values of C: $C_2 = 2C_1$ ($C_1 = 0.0025$) a non linear increase of the number of viable economies. It appears clearly, especially on the lower curve, that the number of viable processes tend to a limit as the discretization becomes finer. In other words, for $k > 5$ the discretized system tends to the continuous one. Let us remark that we got the same phenomena using the leap frog mapping (T_2) of the pendulum equations (fig.2.5).

The increase of viable paths with k is due to the well known fact which governs auto-regulating systems: actually wages and prices react smoothly to shocks which are themselves smooth. It is clear that for this kind of perturbation, when k is smaller than 5 the problem treated is different. In this case we don't have equivalence between the discretized and the continuous cases and only economic considerations allow to choose the model to be used.

It appear on fig.2.7 that either economies are viable within a time T, or they are quickly non viable.

Energy (oil) crisis

We introduce now a second factor of production, the energy ($\gamma \neq 0$), whose price undergoes an abrupt change around given volume of activity. We assume that agents make rational conjectures, i.e. know that by a fine tuning on the take out and on credits a regular path of the economy can be reestablished. However we suppose that there exists some limitations to the change of this key variables from one period to the next one. In other words bounds prevent the desired variations of $c(t)$ and $f(t)$.

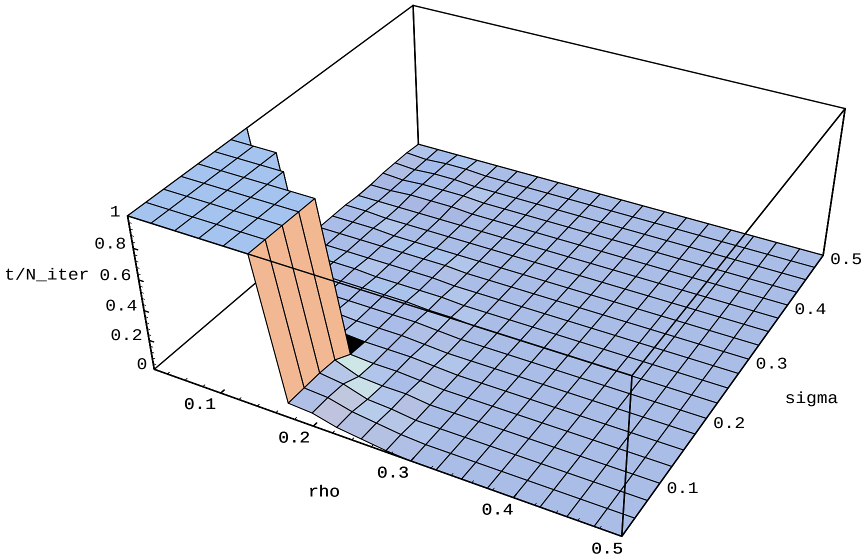

Figure 2.7: Distribution of the $\bar{t}/N_iter$ for the M=400 paths corresponding to the financial perturbation (A) for the case $k = 5$.

In this case we make the hypothesis that wages are entirely spent on consumption goods: $h_d^h(t) = 0$ $(\rho = \sigma = 0)$. Like for the financial shock we explored the viability of the economy for a number $M = 50 \times 50$ of paths for different values of the bounds:

$$\begin{cases} cmax_k(j) = \alpha m(1+g)(1+j\Delta c_k) & for j = 1, ...50 \\ cmin_k(j) = \alpha m(1+g)(1-j\Delta c_k) & for j = 1, ...50 \end{cases} \tag{2.55}$$

where Δc_k is either a fixed value or a normalized one. The normalization factor: $\Delta c_k = \Delta c_1(g_k/g_1)$ is based on the assumption that in spite of the energy crisis, the rate of growth remains positive and therefore the desired variations of the value of the take out will be monotone with g. Therefore in order to get, for the same amount of real time, the same value for the variation of the bounds as the discretization step decreases (k increases) we introduce the same renormalization factor of the rate of growth. The same holds for credits where an upper bound has been introduced:

$$fmax_k(j) = m(1+g)j\Delta f_k \text{ for } j = 1, ..50 \tag{2.56}$$

Prices and wages react like in the case A. Let us remark that if the bounds are sufficiently large all paths are viable.

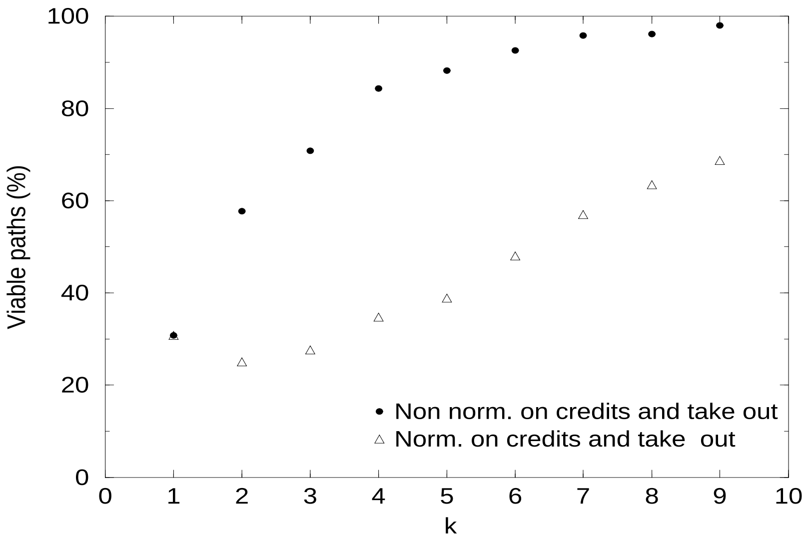

Figure 2.8: Same as fig.1 for the case of the oil crisis (perturbation B, see text). The two curves refer respectively to the non-normalization and to the normalization on both bounds on take out and credits.

The bounds introduce a discontinuity in the path which makes this set of experiences intrinsically different from the set A. Indeed in the case of the financial shock, by increasing the value of k we tend to a set of regular differential equations. This is no more the case here. However we observe the same general tendency to an increase of viability with the decreasing of Δt. Moreover a kind of renormalization appears when changing adequately the bounds values (fig.2.8).

2.4 From continuous to discrete models

In the last two examples it would have been possible to construct, for any $k \rangle 1$, a mapping with the same time step than for $k = 1$, by taking the values of the economical quantities at regular intervals of time: $\tau = k * \Delta t_k$. Of course this mapping will be different of the original one $(k = 1)$.

Different numerical methods have been introduced to transform continuous system, meaning flows associated with differential equations, into mapping close to the exact one, i.e. for transforming a nonlinear differential system, and more particularly a Hamiltonian one, into a map without having to integrate the whole orbit as in the well known Poincaré map technique. Here we just introduce such methods, whose detailed explication is out of the purpose of this paper (reader can refer to Froeschlé & Petit 1990, Froeschlé & Petit 1993 and Froeschlé & Lega 1995

).

Let us recall that the Poincaré map is nothing but the map induced by the successive intersection of the orbit with a given manifold or surface of section (often a plane) of the phase space. In the case of differential equations with a periodical dependence on time (with period T) a peculiar, but often used, surface of section is the set of solutions at times $t_k = t_0 + kT$. In the case of differential equations which do not depend explicitly on time we have a mapping for any T taking the solutions at $t_k = kt_0 + kT$. actually this is a stroboscopic view of the continuous orbit.

Only very peculiar sets of differential equations are analytically treatable in order to get explicit equations of the Poincaré map. We have introduced two methods which provide in a controlled manner a mapping closed to the Poincaré map. We construct piecewise polynomial maps by coarse-graining the phase surface of section into parallelograms using values of the Poincaré maps at the vertices to define a polynomial approximation within each cell. The numerical experiments are in good agreement with the standard map taken as a model problem. The agreement is better when the number of vertices and the order of the polynomial fit increase. The synthetic mapping obtained is not symplectic even if at vertices there is an exact interpolation. A second method is based on a global fitting . The polynomial are obtained using at once all the vertices and fitting by least square polynomials but in such a way that the symplectic character is not lost.

2.5 Conclusion

We have shown that in order to obtain an equivalent modelization it is necessary to know the structures of a system as much as possible in order to obtain an equivalent modelization.

We have shown, in the case of the financial shock, that the effect of the discretization is nothing but the well known effect of replacing a continuous system by a discrete one through the numerical procedures used to solve differential equations. As we have shown for the pendulum case, a threshold on the discretization step indicates the maximum step size for which the discrete system is a good approximation of the underlying continuous one as far as numerical simulations are concerned. For the other cases, meaning Δt greater then the threshold or the introduction of bounds, i.e. inequalities to be respected, only economics considerations can justify the model.

However the variation of some variables, like salaries or prices, is correctly modelized using a discrete system with a precise time step (salaries are not negotiated every hour) while the variation of some others may be represented by differential equations (investment decision occur all along the period). An homogeneous representation of the system associated can be realized through the Poincaré map technique. Of course the mapping obtained in such a way is not the exact one but it is very close to the exact one, within a controlled error.

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Chapitre 3

Etude par la méthode habituelle de l'effet d'un choc technologique

Dans ce papier on se pose le problème des conséquences sur l'économie d'un choc technologique. En particulier, on essaie de comprendre les relations entre salaire réel et chômage en étudiant les effet de la flexibilité des salaires sur le chômage. L'analyse néo-autrichienne nous montre que les gain potentiel de productivité de la nouvelle technologie ne sont pas acquis par le simple mécanisme de flexibilité des salaires à cause des défauts de coordination des agents. En utilisant la méthode Monte-Carlo nous montrons que ce résultat ne depend pas du choix des paramètres de flexibilité de prix et de salaires. Nous montrons que l'équilibre correspondant à la nouvelle technologie (augmentation du produit par tête) est atteint grâce à des mesures opportunes de politique économique.

ROUND-ABOUT PRODUCTION, CO-ORDINATION FAILURE, TECHNOLOGICAL CHANGE AND THE WAGE-EMPLOYMENT DILEMMA

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Abstract
The contrasting experiences of European and the US labour markets suggest to most analysts the conclusion that technological shocks would be the cause of the appearance of unemployment, and institutional settings, that generate real wage rigidity, would be the cause of its persistence. In this paper we intend to invalidate this conjecture. In order to do so we shall turn to an analytical framework which differs from the standard neo-classical one in that it allows to enlighten what happens step by step along a way that begins with the breaking of the existing steady state due to an attempt to bring about a change in technology.

Key words: Wage, Employment, Unemployment, Technological Change

JEL numbers E 24 O 42

3.1 Introduction

What happens to real wages and employment in an economy facing technological change was first asked in the early days of the Industrial Revolution. It remains a question of great importance in the modern world where the strength of innovation processes seems to point to a kind of trade-off between real wage deflation and unemployment. In particular, the contrasting experiences of European and the US labour markets suggest to most analysts the conclusion that low employment is associated with high real wages and vice-versa. The obvious next step is to look at downward wage flexibility as a means to re-establish full employment. Technological shocks would be the cause of the appearance of unemployment, and institutional settings, that generate real wage rigidity, would be the cause of its persistence.

In this paper we intend to invalidate this conjecture. In order to do so we shall turn to an analytical framework which differs from the standard neo-classical one in that it allows to enlighten what happens step by step along a way that begins with the breaking of the existing steady state due to an attempt to bring about a change in technology (section 2). The sequential model presented in section 3 is a heuristic device for exploring by means of numerical simulations different scenarios corresponding to different conditions of co-ordination of the economy undergoing a change in technology (section 4). The results obtained help to understand crucial aspects of the unemployment crisis in Europe in our days (section 5).

3.2 The analytical framework

We propose to consider an analytical framework that makes it possible to posit the transition between two different techniques as a process in real time and to analyze this process by following it in its sequential development. This requires in the first place to focus on the time structure of the production processes, which is the distinctive feature of the neo-Austrian model (Hicks 1970, 1973). This model assumes that the activity of production is divisible into operations or processes each of which contains a construction stage in which a plant (a machine) is produced starting from scratch (that is, with the only help of 'primary' resources - labour for short), and an utilization phase in which the machine is worked to produce final output. There is no equivalence between an old machine and a new one. As a consequence, the substitution of a new technique for the old one implies to build an entirely new productive capacity.

In this context Hicks has originally analyzed the case of a new technique characterized by an increase in the construction cost (in terms of labour) more than compensated later by a decrease in the utilization cost. With given resources to sustain labour in production, the same labour force applied to machine making will produce less machines (capacity units). Thus the capacity to produce final output is to fall as the new more expensive machines come into utilization.. A stage in which final output is less than it would have been had the innovation not occurred is inevitable. Under a fix wage assumption, this results in a temporary unemployment, a Ricardo's 'machinery effect' (J.R. Hicks 1973). Of course a reduction of the real wage is an alternative to the fall in employment. The same trade-off as in the neo-classical framework occurs, although in the different meaning that the level of employment reflects the ratio of the resources available to sustain labour in production (the 'wage fund') to the wage rate . But here "the condition which is necessary for this to happen is not particularly stringent, or unrealistic. It does not look like being an 'empty economic box' " (Hicks 1977 p. 189). As a matter of fact it can be proved that the effect confined by Hicks to a specific case of technical change is instead a very general one. It takes place whatever the change in technology considered (M. Amendola 1972), or when, with a heterogeneous labour force, a human resource constraint arising from a change in the structure of the labour demand imposes to build less capacity units (M. Amendola and J-L Gaffard 1988, 1998).

In this analysis what matters is no longer the nature of technological change, but the process of change itself. During this process problems of co-ordination arise, which concern not only the labour market. Co-ordination failures that concern the market of final output or the financial market also influence what happens to unemployment. When these failures occur adjustment processes set in which engender irregular evolution paths. We need therefore to focus not so much on what happens to nominal and real wages at the precise moment when, as a consequence of a technological shock, a fall in final output is registered, but rather on what happens to these variables along the way, during the out-of -equilibrium process stimulated by the co-ordination failures.

3.3 The sequential model

The model that gives structure to the above argument is a sequential model (M. Amendola and J-L Gaffard 1988, 1998), which is an extension of the Neo-Austrian model (Hicks (1970, 1973). The extension consists essentially in adding the consideration of sequential decision processes to that of the temporal dimension of the production processes originally proposed by Hicks. Further differences are represented by the substitution of a heterogeneous labour for the homogeneous one and by the consideration of financial resources to sustain labour in production (Hicks' is a barter model).

3.3.1 The production process

Production is carried out by means of production processes which use a heterogeneous human resource characterized by s different skills. An elementary process of production embodying a given technology j is defined by the input matrix:

$\mathbf{A_j} = [\mathbf{a_{i,h}}]_\mathbf{j}$ $j = 1, 2, ..., m;$ $i = 1, 2, ..., n + N; h = 1, ..., r.$

whose elements represent the quantities of the different types of labour (skills) required in the successive periods of the phase of construction (from 0 to n) and, following it, of the phase of utilization (from $n + 1$ to $n + N$) of the productive capacity of commodity j, so that:

$\mathbf{A_j} = \left[\mathbf{A^c_j}, \mathbf{A^u_j}\right]$

and by the output vector:

$\mathbf{b_j} = [\mathbf{b_{j,n+1}}, ..., \mathbf{b_{j,n+N}}]$

At each given moment t the productive capacity embodying the technology j is represented by the intensity vectors:

$\mathbf{x_j(t)} = \left[\mathbf{x^c_j(t)}, \mathbf{x^u_j(t)}\right]$

$\mathbf{x^c_j(t)} = [x_{j1}(t), x_{j2}(t), ..., x_{jn}(t)]$

$\mathbf{x^u_j(t)} = [x_{jn+1}(t), x_{jn+2}(t), ..., x_{jn+N}(t)]$

each element of which is a number of elementary production processes of a particular age, still in the construction phase or already in the utilization phase.

When production processes embodying different technologies successively introduced are in use at the same time t, $(y = 1, 2, ...m)$, the productive capacity is represented by a vector $\mathbf{x(t)}$ each element of which is the number of elementary processes embodying the technology prevailing at the moment of their start. In the same way the matrix $\mathbf{A(t)}$ represents the quantities of the different types of labour required by the production processes embodying different technologies at time t.

The "productive structure" of the economy can then be defined in terms of the age structure of its productive capacity. It has a horizontal dimension, expressed at time (t) by the vectors of elementary production processes of the various commodities at different stages of their life - still in the phase of construction or already in the phase of utilization being carried on; which also implies a vertical dimension : the time pattern production associated with this age structure of productive capacity. These, in equilibrium, must be consistent with each other then, together with construction and utilization, also investment and consumption and supply and demand of final output are harmonized, at each given moment of time and over time. An equilibrium age structure of productive capacity also implies a given and stable relation between the relevant economic magnitudes.

3.3.2 The financial constraint

We consider two classes of agents: firms and households. All exchanges between them are intermediated by money which constitutes a financial resource required to carry out production and to sustain consumption. This resource, for both producers and consumers, comes from participation in current productive activity. It is required when productive activity is expanding, and at the rate at which this expansion takes place. In reality, this expansion depends on the choices of the monetary authority and/or the banks, but in the model they are considered as exogenously determined and actually dealt with as a control variable.

The financial constraint is the reflection of a theory of money that focuses on transaction and precautionary motives. This theory must not be confused with cash in advance models. Money - cash in hand or at the bank - matters because it is a complement of other running assets including real assets. Liquidity appears then as a bridge linking inputs and output over time. It is brought about by productive activity, and hence generated within the economy, but also furnished by the bank system or the running down of existing idle balances.

In each period the firms' level of activity is constrained by available financial resources or, alternatively, by labour supply. Let $m(t-1)$ be the money proceeds from the sales of final output of the previous period, $f(t)$ the external financial resources, $w(t)$ the wage fund - that is, the resources devoted to finance labour employed on production processes of all kind - $c(t)$ the resources devoted to consumption by producers and hence held from the financing of production (also dealt with as a control variable in the model), $h^f(t)$ the monetary idle balances held by firms (desired $h^f_d(t)$), and undesired $h^f_{nd}(t)$. Then the firms' finance constraint which determines $w(t)$ can be written:

$$\hat{w}(t) = m(t-1) + h^f(t-1) + f(t) - c(t) - h^f_d(t)$$

with $w(t) \equiv \hat{w}(t)$ if firms are not constrained in their activity by human resources. Otherwise $w(t) \equiv \tilde{w}(t)$ where $\tilde{w}(t)$ is the wage fund thus "constrained" by lack of human resources.. In this case the amount of available financial resources which is impossible to actually use up in production because of the existence of a human constraint, i.e. the difference between $\hat{w}(t) - \tilde{w}(t)$, gives rise to non desired idle balances, h^f_{nd}.

Desired balances are defined as:

$$h^f_d(t) = \rho(t)\left[m(t-1) + h^f(t-1) + f(t)\right]$$

and appear when the function $\rho(t)$, which represents the fraction of available resources that firms intend to withhold from production because of a loss of confidence in the existing state of affairs, takes a positive value.

Total expenditure by households in period (t) also depends on financial resources available to them, that is, on wages received at the beginning of the period $w(t)$, and on the idle balances h^h (desired h^h_d and non desired h^h_{nd}) carried over from the preceding period. If we add to this the resources devoted to consumption by producers $c(t)$ we obtain the money value of current demand for final output $y(t)$, that is:

$$y(t) = w(t) + c(t) + h^h(t-1)$$

Desired idle balances, as with producers, pile up when there is a loss of confidence in the existing state of affairs and the function $\sigma(t)$ takes a positive value in the expression:

$$h^h_d(t) = \sigma(t)\left[w(t) + h^h(t-1) + c(t)\right]$$

where we assume that producers behave like households when acting as consumers, and hence include $c(t)$ in the right hand side of the equation. In this case the money value of current demand becomes:

$$y(t) = (1-\sigma(t))\left[w(t) + c(t) + h^h(t-1)\right]$$

Within the sequential setting considered all prices, including wage rates, are fixed within each given period and can only change at the junction of one period to the next one.

As a consequence we have:

$$m(t) = \sum_j \min\left[p_j(t)s_j(t), p_j(t)d_j(t)\right]$$

that is, money proceeds are determined by the minimum between the value of current demand d and that of current supply s of final output, which can differ since prices (p_j) in this sequential model are not allowed to change within each given period, so as to bring demand and supply back into balance, but only at the junction of one period to the next one.

In the same way real and monetary stock changes are a substitute for the price changes which cannot take place within each period. Thus excess demand for final output (if any) results in the appearance of undesired idle balances for households:

$$h^h_{nd}(t) = \sum_j \max\left[p_j(t)\left(d_j(t) - s_j(t)\right), 0\right]$$

while excess supply results in an accumulation of undesired stocks for the firms:

$$o_j(t) = \max\left[0, \left(s_j(t) - d_j(t)\right)\right]$$

On the one hand, undesired accumulated stocks (real or financial) reveal the existence of market disequilibria. On the other hand, they are the bridge that links successive periods and, hence, they are the channel of transmission of these disequilibria down the sequence.

3.3.3 The human constraint and learning

Total labour supply, in each given period t, can be written:

$$\mathbf{l^s(t)} = [\mathbf{l^s_1(t)}, \mathbf{l^s_2(t)}, ..., \mathbf{l^s_r(t)}]$$

where $\mathbf{l^s(t)}$ is a vector whose elements represent the different skills, inherited from the past, which are the expression of the specific character of the existing human resources. With a homogeneous labour this vector, of course, becomes a scalar.

Total demand for labour, on the other hand, will be:

$$\mathbf{l^d(t)} = \left[\mathbf{l^d_1(t)}, \mathbf{l^d_2(t)}, ..., \mathbf{l^d_r(t)}\right]$$

where the demand for labour employed on production processes of commodity $\mathbf{l^d_j(t)}$ is determined by the activity of construction and utilization of productive capacity of type j, that is:

$$l^d_j(t) = \sum_i x_{i,j}(t) \sum_h a_{i,h,j}$$

The intensity of productive activity, in turn, depends on available financial resources (the wage fund $w(t)$) and the wage rates vector $\omega\mathbf{(t)} = (\omega_\mathbf{1}\mathbf{(t)}, \omega_\mathbf{2}\mathbf{(t)}, ...\omega_\mathbf{r}\mathbf{(t)})$ whose elements are fixed for the whole current period, as all prices. When, due to full employment in the case of homogeneous labour or to scarcity of at least one essential skill (that is when $l^s_h < l^d_h$ for some $h = 1, 2, ...r$) with a heterogeneous human resource, labour demand is limited by labour supply, a human constraint stronger than the financial constraint can appear. In this case productive activity is determined by a "constrained" wage fund which is given by:

$w(t) =< \omega(\mathbf{t})\tilde{\mathbf{l}}^{\mathbf{d}}(\mathbf{t}) >$

where $\tilde{\mathbf{l}}^{\mathbf{d}}(\mathbf{t})$ is the demand for labour "constrained". The human constraint, due essentially to the specific character of the human resources, is released by the learning process which affects the labour supply vector $\mathbf{l}^{\mathbf{s}}(\mathbf{t})$. This can come through an upgrading - the passage from a lower to a higher skill out of the existing ones - as the result of some activity of formation or of an on the job training. In fact we have implicitly considered that the scarcity of labour supply affects only high level skills. The supply of each given skill $\tilde{\mathbf{l}}^{\mathbf{s}}(\mathbf{t})$[5] at time (t) can then be written:

$\tilde{l}_h^S(t) = l_h^D(t)(1 + \psi(t))$ if $l_h^S(t) \geq l_h^D(t)$

$\tilde{l}_h^S(t) = l_h^D(t)(1 - \psi(t))$ if $l_h^S(t) \leq l_h^D(t)$

where the function of learning $\psi(t)$ $(\lim_{\to\infty} \psi(t) = 0)$[6] gives the labour of skill h accruing in period (t) including the result of an activity of formation and/or the result of on the job training in the preceding period[7].

3.3.4 Consumption, Production and Investment decisions

The demand for commodity j, in real terms, can be written:

$d_j(t) = \frac{y_j(t)}{p_j(t)}$

where

$y_j(t) = \delta_j y(t)$; $\sum_j \delta_j = 1$

where δ_j is an utility index. Supply of commodity j in real terms, at time (t), is determined as follows:

where $y_j^*(t) = y_j(t-1)[1 + g_m(t-1)]$ is the money value of final demand expected in the current period, obtained by extrapolating the trend of money proceeds from the sales of final output in the previous periods[8].

Current final production of commodity j will then be:

[5] Of course, the new vector is scaled in order to have a global labour supply as it is determined by the natural growth rate.

[6] The $\psi(t)$ is exogeneously choosen, therefore if $\tilde{l}_h^s(t)$ results to be greater than $l_h^s(t)$ in the case of an excess of supply for skill h we set $\tilde{l}_h^s(t) = l_h^s(t)$ and the same we do if $\tilde{l}_h^s(t) \leq l_h^s(t)$ in the case of scarcity of skill h.

[7] In the simulations carried out in what follows, only a change in the proportion between labour construction and utilization costs will be considered, so as no specific learning process will be required.

[8] In the simulations carried out in what follows the consumption side is supposed to be fully adapted to the production side: this is a situation strictly equivalent to that in which the same good is produced with the different successive techniques

$$q_j(t) = s_j(t) - \eta[o_j(t-1)] \text{ with } 0 \leq \eta \leq 1$$

where η represents the fraction of existing real stocks (if any) actually put back on the market. Moreover:

$$q_j(t) \leq \sum_{i=n+1}^{n+N} b_{i,j} x_{i-1,j}(t-1)$$

Thus we determine the vectors of production processes in the phase of utilization $\mathbf{x^u}$ given the scrapping rule, and the fraction of the wage fund required to finance the carrying out of these processes, that is:

$$w^u(t) = \sum_j < \omega(\mathbf{t})\mathbf{A^u_j}(\mathbf{t}), \mathbf{x^u_j}(\mathbf{t}) >$$

The resources available for financing construction processes will then be:

$$i(t) \equiv w^c(t) = w(t) - w^u(t)$$

and these resources, given the scrapping rule, will determine the vectors of production processes in the phase of construction:

$$w^c(t) = \sum_j < \omega(\mathbf{t})\mathbf{A^c_j}(\mathbf{t}), \mathbf{x^c_j}(\mathbf{t}) >$$

In other words, the rate of starts is at the maximum permitted by the share of final output that is not absorbed by the consumption out of wages paid to labour engaged on old processes and by consumption out of profits (the take-out). As we shall see this assumption of full performance can explain why, in certain circumstances, unemployment which appears as a consequence of technological changes, is reabsorbed.

In determining $\mathbf{x^c}$ and $\mathbf{x^u}$ the processes scrapped must also be taken into account. As a matter of fact, as a consequence of ageing and of scrapping, the actual number of production processes of type j of different ages carried out at the same moment t is such that:

$$x_{i,j}(t) = x_{i-1,j}(t-1) - u_{i,j}(t) \text{ for } i = 1, ..., n+N$$

where $u_{i,j}$ is the number of processes of type j which have been scrapped.

Scrapping of production processes occurs when human or financial resources constraints are so stringent as not to allow all the processes inherited from the past to be carried on. This scrapping, in each period, depends on the magnitude of final demand as concerns the utilization processes and on the available financial resources as regards construction processes. When there is a final demand constraint the first processes in the utilization phase to be cut are the older ones ; when there is a financial constraint the first to be cut is the rate of starts of new processes, then the processes still in the construction phase (first the younger, then the older). This reflects a flexibility criterion that focuses on expected final output (both in its amount and in its nearness in time), and hence on expected profits, as an index of a less stringent expected finance constraint.

As an alternative to scrapping we can consider a partial use of utilization processes : which, however, also implies a cost.

3.3.5 Key variables

Given a perturbation in the economy the path followed by the economy will be determined by the evolution of four keys variables: $p(t)$, $\omega(t)$, $f(t)$ and $c(t)$. The evolution laws actually considered in the simulations carried out will be expounded when discussing the simulations themselves in Section 3.

Prices and wages

Let the operator g be defined by:

$g(t) = \frac{z(t)-z(t-1)}{z(t-1)}$ for any z(t)

Then market determined wage rates changes can be written:

$g_{w_h}(t) = \nu_h \phi_h(t-1)$

where $\phi_h(t) = \frac{l_h^d(t)-l_h^s(t)}{l_h^s(t)}$ is the rate of excess demand for the h type of labour, and ν_h a reaction coefficient which can be different for different skills. As an alternative, we can consider an indexation rule, e.g. of the kind:

$g_{w_h}(t) = \alpha g_{\bar{p}}(t); 0 \leq \alpha \leq 1$

where $\bar{p}$ is the general price level.

On the other hand market determined prices changes can be written

$g_{p_j}(t) = \kappa_j \xi_j(t-1)$

where $\xi_j(t) = \frac{d_j(t)-s_j(t)}{s_j(t)}$ is the rate of excess demand for the j commodity, and κ_j is a reaction coefficient that can differ for the different commodities. Different values of the coefficients κ and ν stand for different regulating mechanisms which can be made to represent alternative market organizations..

External financial resources

External financial resources or money supply are given by one of the following equations:

$f^s(t) = f^s(t-1)\left[1 + g(t-1\right]$

$$f^s(t) = f^s(t-1)\,[1 + g^\star(t-1]$$

The money supply increases at a growth rate which can be the current growth rate of the economy or a different rate discretionary chosen (e.g., the growth rate of the initial steady state). These rates can be interpreted to stand for different institutional settings and/or different policies.

Take out

Take out, or consumption by producers, is given by one of the following equations:

$$c(t) = c(t-1)\,[1 + g_m(t-1)]$$

$$c(t) = c(t-1)\,[1 + g_m(0)]$$

$$c(t) = c(t-1)\,\left[1 + g(t-1)\epsilon^{t-t_c}\right]$$

The take out increases: at the current growth rate of the economy, or at the growth rate of the initial steady state, or in the aim to gradually increase the amount of internal resources devoted to the construction of the production processes.

3.3.6 The working of the model

With this modelling, it becomes possible to characterize "the way economies work when they work out of equilibrium" (R. Day 1993 p. 21). This means to sort out the conditions which, step by step, make the evolution path of the economy viable. In this perspective adaptive behaviours prevail. There is no predetermined target and hence there cannot be an objective function.

Given the quantity constraints inherited from the past, adaptive behaviours determine, in each period, the relevant variables of the economy, namely, the productive capacity, the employment level, the price and wage levels, and so forth. Under given conditions an equilibrium over time implying an intertemporal coherence of the plans of the agents may be established. More generally, an out-of-equilibrium dynamics is engendered, when the reactions to a given disequilibrium carries the latter down a sequence of periods. In this analytical context, any technological change causes a distortion in the structure of productive capacity. This distortion, once produced, has persistent effects. This is due to the interaction of the intertemporal complementarities of production and of decision processes, that is, to the fact that the working of price and wage reaction mechanisms does not succeed in eliminating instantaneously the distortion, and may tend itself to reinforce it over time.

The algorithm illustrated in Figure 3.1 shows the actual working of the model.

At the end of every period we have determined the productive capacity of the economy, the existing stocks of goods and money, whether desired or undesired, the money proceeds held by the firms, and a set of prices (p and w).

At the junction between periods the existing productive capacity gets one period older, and wages and/or prices change if unbalances appear in the labour and/goods markets.

In the new period there is the following ordering of events. On the firms' side the behavioural equations determine the new rate of 'take out' and the desired idle balances, while the supply of external financial resources is determined exogenously by the monetary authority. We can thus determine the desired wage fund. On the other hand, by extrapolation and considering the stocks left from the previous period, the desired quantity of final output to be produced is determined. At the same time also the supply of labour is determined exogenously.

In the successive step desired final production is compared with existing productive capacity. This leads to a reduction of production if this is constrained by productive capacity or to a scrapping of production processes in the phase of utilization if there is excess capacity. The fraction of the wage fund necessary to carry on final production is thus determined. Whatever is left is devoted to the production processes still in the phase of construction. A further residual is devoted to the starting of new production processes. If the fund is not large enough no new processes will be started or there may even be scrapping of construction processes.

This makes it possible to determine the actual structure of productive capacity and the demand for labour associated with it. This is matched with the existing labour supply. If a labour constraint appears the wage fund to be used in production is correspondingly reduced and there is an undesired money holding on the part of the firms.

At this point wages are paid and production takes place. Supply is determined and, given the effective wage bill and the desired money balances of households, also demand is determined. The matching of supply and demand of final output determines unsold stocks (if any) or non desired money holdings by households.

The same sequence of events is repeated in the successive period(s).

3.4 The real wage - employment dilemma: simulation results.

The analysis carried out by making use of our model makes it possible to sketch out the out-of-equilibrium paths followed by the economy under different conditions. It allows to bring to light the various effects of changes in technology - on wages, on prices, on employment - not dissociating them but taking into account the feedback mechanisms at work.

We shall consider the case of a single technological shock. A new, more costly (in terms of labour) technique in the phase of construction but implying a more than proportional saving of labour in the phase of utilization (the same case of technical advance considered by Hicks), has been assumed to take place at a given moment in an economy at full employment at that very moment[9].

In the first set of simulations a proportional take-out (that is, a take out which increases at the current growth rate of the economy) , and flexible prices and wages have been assumed. A *temporary* fall in employment associated with a fall in final output usually occurs after the end of the phase of construction of the first round of the new productive capacity. The initial reduction of the rate of starts due to the increase in the labour construction costs results in a reduction of the gross product of the economy when the new production processes reach the phase of utilization and hence in a reduction of the labour demand (the Ricardo machinery effect).

Nominal wages flexibility makes it possible to reduce the time necessary to reabsorb unemployment, but cannot hamper it from appearing, as wages cannot adjust instantaneously to the arising disequilibrium in the labour market, but only with a one period lag (Fig 3.2). This result seems to be robust. By means of a Monte Carlo method, 900 simulations corresponding to different values of the prices and wages reaction coefficients randomly chosen in the interval [0 - 1.5] have been run. In all the viable cases[10] (93% of the simulations), there is an unemployment period, but most often (40% of the viable cases) it is less than 10% of the length of the corresponding period in the fix wage case (Fig 3.3a). The cases in which the effect on employment can be considered as negligible, because its length is very short, are those corresponding to a very low wage reaction coefficient to a labour excess demand, that is, to an upward rigidity of wages, and, simultaneously to a downward flexibility of wages (Fig.3.3d)]. In most of these simulations (89% of the viable cases) real wages fall despite the fact that the new technique is more efficient than the old one(Fig.3.3b). The new stationary value of real wages, which is compatible with full employment, is less that it would have been had the technological change not occurred. On the one hand, the nominal price is moving slowly towards a level higher than the initial equilibrium value (which is itself higher than the new equilibrium value). On the other hand, nominal wages are decreasing as a consequence of the existence of a Ricardo's 'machinery effect'. A recurrent scrapping of production processes in the utilization phase is the signal of the existence of a persistent excess capacity on the product market. Due to these co-ordination failures, the productivity level of the economy decreases together with real wages (Fig.3.3c). Sticky nominal wages do not prevent unemployment from being re-absorbed (Fig.3.4). As we have

[9]It must be clear that this new technique is chosen because it is superior to the old one in the sense that, in a steady state, it is less costly.

[10]Economy is no longer viable when all processes in the construction phase must be scrapped.

seen, adaptive expectations concerning final demand create the conditions for a recurrent excess capacity which results in the scrapping of production processes in the utilization phase. The truncation of production processes is due to an insufficient effective demand. In this context, downward nominal wages flexibility cannot be a solution. On the contrary, sticky wages may appear as a regulation factor involving less scrapping, less productivity losses and real wages falls, as suggested by a close inspection of the figures 3.3 and 3.4.

Briefly. Capital shortage is the true reason for the fall in employment. This fall cannot be but transient because, on the one hand, the reduction in the utilization labour costs results in an increasing gross product, and, on the other hand, all the available resources are supposed to be invested [11] In this context, wage flexibility helps to re-absorb unemployment. Dampened oscillations registered in the rate of unemployment result from the tension between the effect of a (decreasing) capital shortage, and that of downward wage flexibility. But, given the co-ordination conditions, wage flexibility does not allow the economy to go back to a full equilibrium. Scrapping of production processes signals the persistence of co-ordination problems that concern the balance between investment and consumption. These problems cannot be solved by making nominal wages more flexible. On the contrary, they are aggravated by a stronger wage flexibility.

The co-ordination problem involved may concern the rigidity of real wages, not that of nominal wages. If wages are fully indexed on prices, that is, with constant real wages from the beginning, a pseudo-equilibrium prevails which is associated with a persistent unemployment (Fig 3.5). In other words, there can be a problem of real wage flexibility rather than a problem of nominal wage flexibility.

A second set of simulations has been carried out with an increasing saving (decreasing take-out) and flexible price and wages. A temporary fall in employment is registered (Fig. 3.6a). Once again, the employment effect is the more negligible the lower the wage reaction coefficient to an excess labour demand (Fig. 3.6d). However, an increase in real wages is obtained for a large number of viable paths (40%) corresponding to different values of the price and wages reaction coefficients randomly chosen in the interval [0 - 1.5] (Fig. 3.6b). A sort of trade-off exists between real wages and the take-out or social consumption. A decreasing social consumption (increasing saving) allows real wages to increase. The stronger the decrease in social consumption the more numerous the cases in which real wages are increasing. Nevertheless, a decreasing social consumption does not prevent the same kind of co-ordination failures as in the previous set of simulations from occurring. Once again the productivity level of the economy is decreasing (in the 82% of the viable cases, Fig. 3.6c). As mentioned, in the model increasing

[11] It is worth mentioning that, when investment is determined in such a way as to maintain the balance between the two phases of the production processes, a positive unemployment persists over time as a consequence of the persistence of the capital shortage(see Amendola and Gaffard 1998 pp. 159-60).

saving is the same as decreasing the take-out, which includes not only private consumption (consumption out of profits) but also public consumption. Therefore, in the case just examined, higher (direct) real wages are actually a substitute for a lower social spending. Of course, that does not affect the final demand and a recurrent excess capacity continues to be registered along the way. Nevertheless if wages are characterized by a relative upward rigidity there can be a decrease in final demand which makes the evolution of the economy non viable (for the set of simulations represented in Fig.3.6 only 60% of the 900 simulations are viable, against 93% of the simulations represented in Fig. 3.3). In this context it is not surprising to observe that for the set of simulations in which we end up with higher real wages, prices adjustments to market disequilibria are characterized by a relative upward rigidity while wages adjustments are, on the contrary, characterized by a relative downward rigidity (Figs. 3.7a,b). The persistence of market co-ordination failures renders the values of reaction coefficients crucial.

In a third set of simulations a proportional saving has been associated with a higher growth rate of money supply. We always observe the existence of a Ricardo's 'machinery effect' (Fig. 3.8a). For a significant number of simulations we also observe an increase in real wages (55% of the viable cases, Fig. 3.8b); and an end to the scrapping of the production processes (simulations end with no scrapping in the 46% of the viable cases, as opposed to the 15% and 8% in the simulations represented in Fig.3.6 and Fig. 3.3, see Fig. 3.8d), respectively; a once-and-for-all increase in productivity (in 54% of viable cases, Fig. 3.8c). In other words if an easy monetary policy is carried out, the consistency over time between the construction of productive capacity and its utilization is re-established. This result supports the following statement by Tobin: "monetary policy cannot be expected to lift the long-term sustainable growth rate (but) the Fed's role is to make sure that any productivity gains that occur spontaneously or as a result of supply-side policies are realized in jobs and output and do not go to waste in recessions and unemployment " (J.Tobin 1996, p. 8). In this context, whether prices and wages are more or less flexible does not really matter (Fig. 3.9).

3.5 Summing -up: what do we learn about unemployment from simulating out-of-equilibrium paths.

The main results obtained by our simulation analysis can be summarized as follows:

- A weak flexibility of nominal wages and of the price makes it possible to reduce the time necessary to reabsorb technological unemployment under a full performance assumption according to which all the financial resources are used up in production.

- A reduction in real wages may be a permanent effect of a technological change. This reduction appears as a consequence of the shortening of the length of the production processes and of the decrease of the level of productivity due to out-of-equilibrium co-ordination failures. In other words, a trade-off between real wage deflation and unemployment exists as an effect of co-ordination failures, which do not concern the labour market.

- An increase in real wages may be obtained at the cost of a decrease in the take-out (an increase in savings). But this does not prevent the shortening of production processes. In this context, co-ordination failures determine a trade-off between real wage inflation and public consumption.

- Only a better co-ordination over time makes it possible an increase in real wages without any forced shortening of the length of the production processes.

These results do not depend on the degree of flexibility of price and wages, but rather on the monetary policy carried out. They have an essentially heuristic character, which might help to enlighten one of the main aspects of the unemployment crisis in Europe. European countries faced successive supply shocks involving technological changes, and hence successive Ricardo's 'machinery effects'. Different kinds of co-ordination failures can explain why unemployment has increased. The first one consisted, during the seventies, in the quasi-indexation of nominal wages (and/or an increasing rate of take-out); the second one, during the nineties, in a tight monetary policy. Too high real wages may be, and possibly have been, responsible for a persistent unemployment. But reductions in real wages are not a solution. Co-ordination problems, as we have shown, are not confined to the labour market.

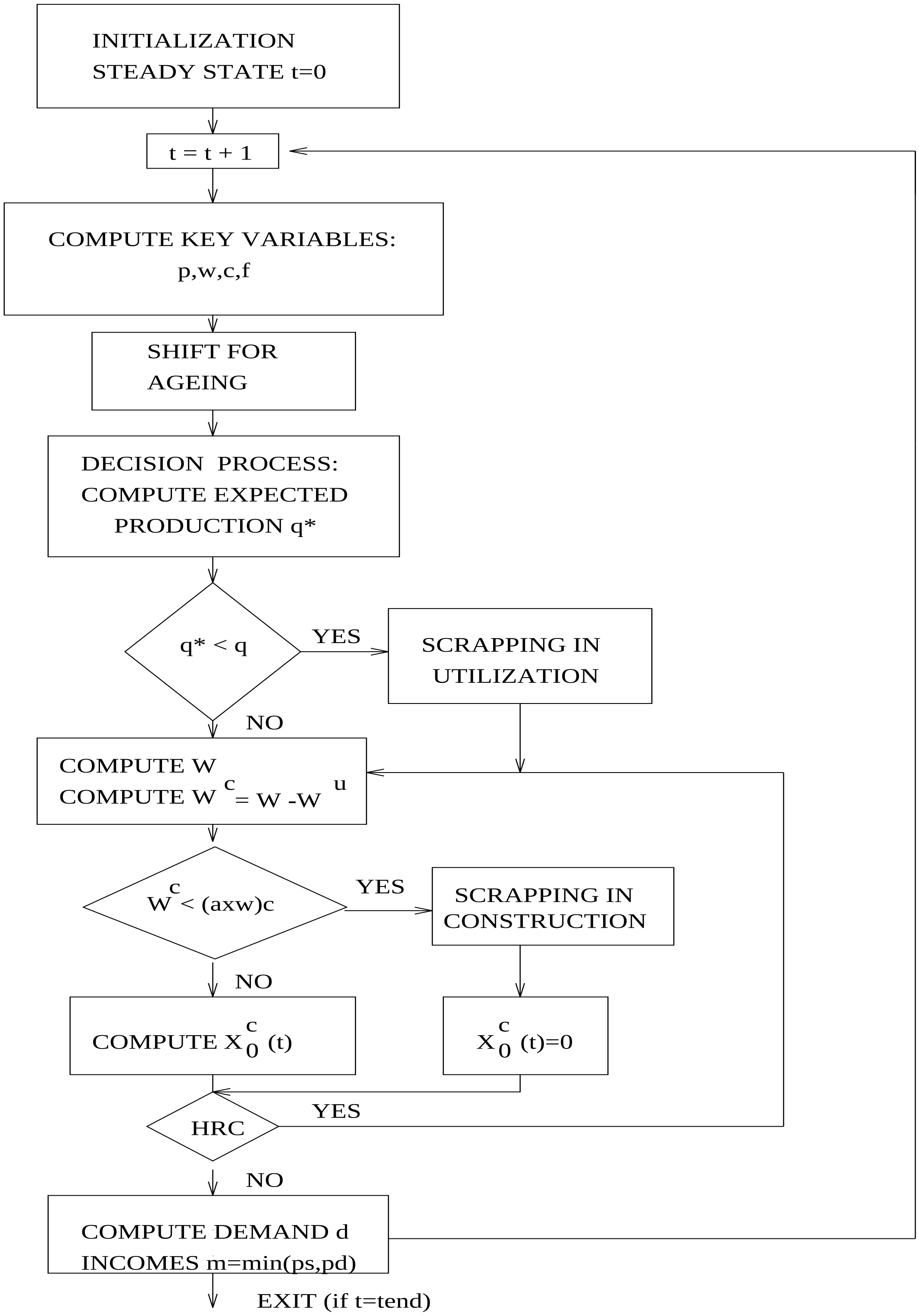


Figure 3.1: Algorithm of the neo-austrian model.

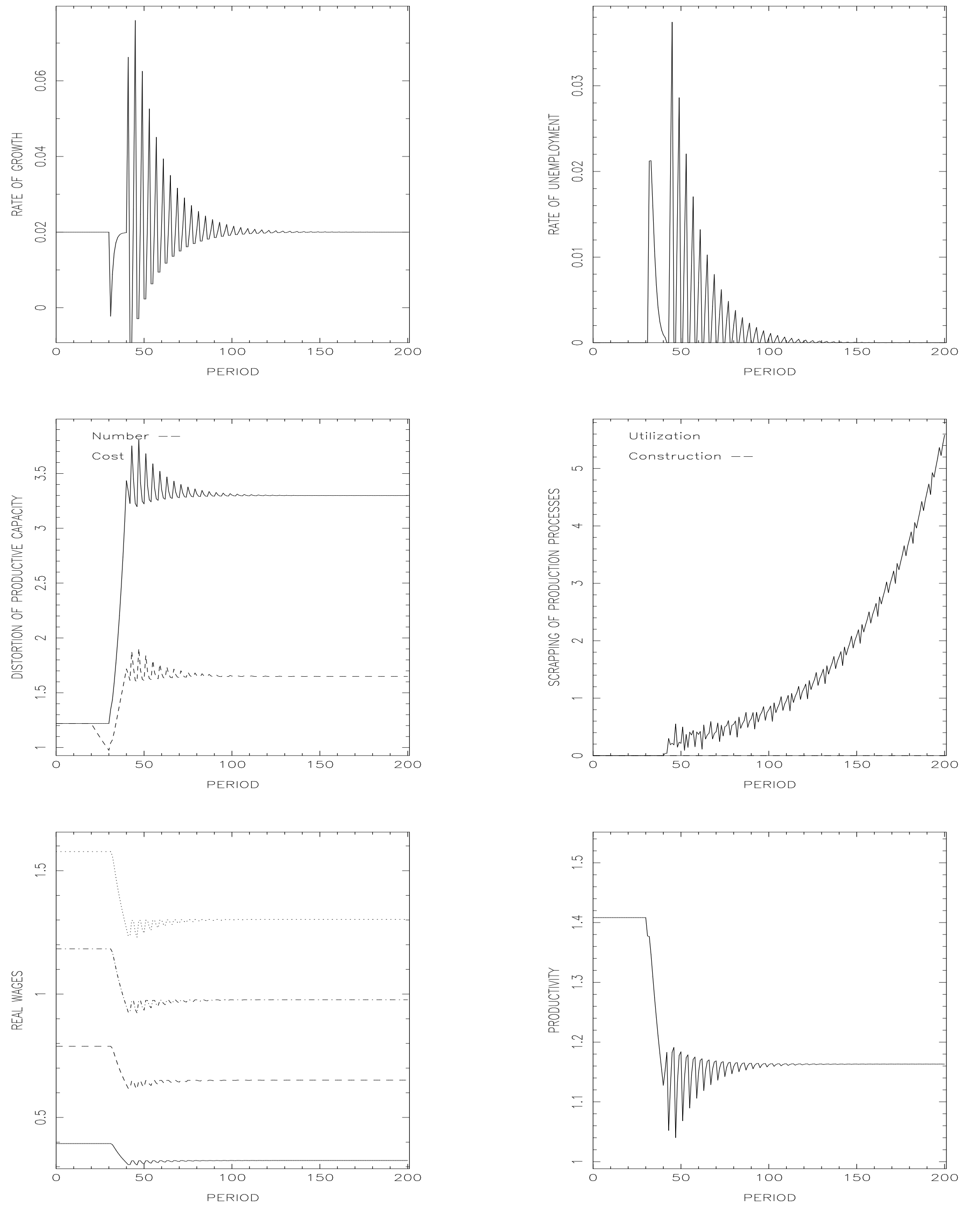


Figure 3.2: Analysis of a technological change. The initial steady state is characterised by: 10 periods of construction, 10 periods of utilization and a rate of growth of 2%. The shock occurs at time $t = 20$. The labour coefficients are changed from $[A^c, A^u] = [8, 8]$ to $[\hat{A}^c, \hat{A}^u] = [10, 5]$. Prices and wages have reaction coefficients $k = \nu = 0.5$ and external financial resources and take out increase respectively at the rate of growth of the initial steady state and at the current rate of growth of the economy.

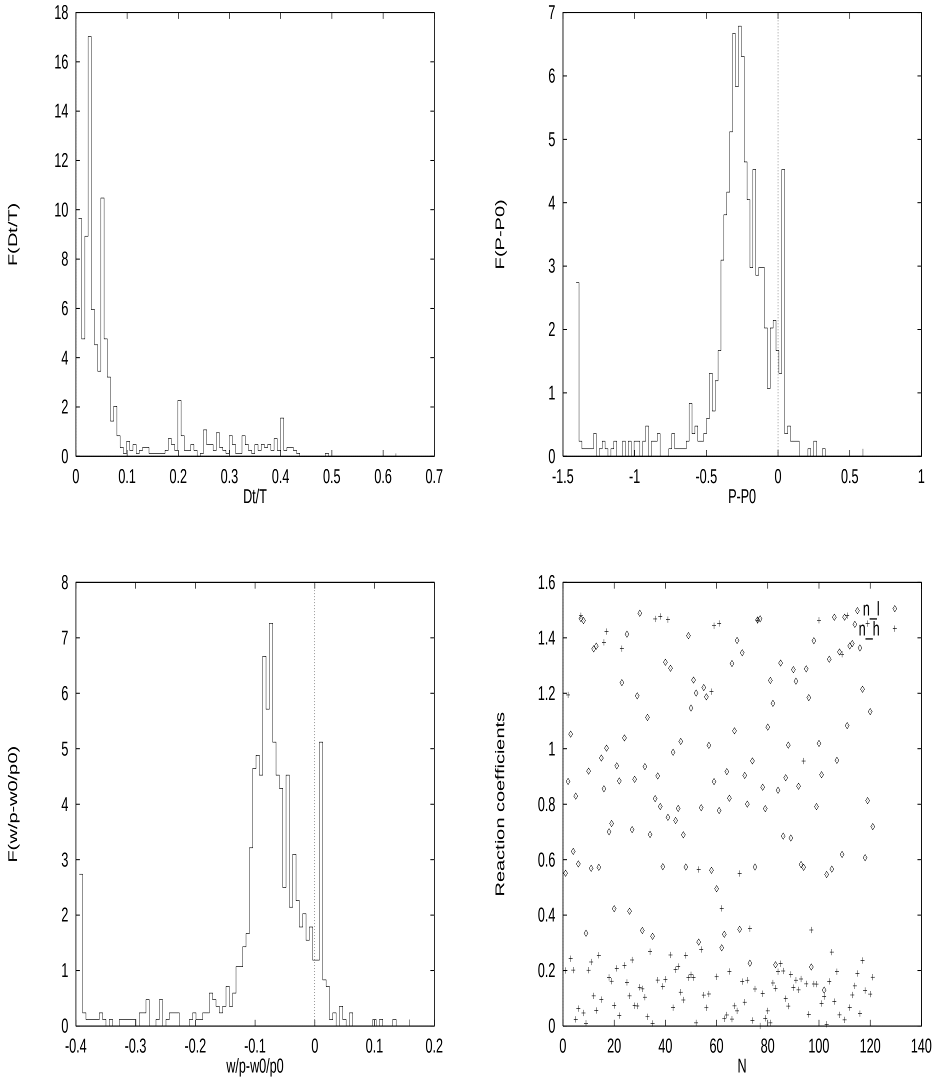


Figure 3.3: a) Histogram of the distribution of the length of the Ricardo machinery effect for a set of 900 simulations with reaction coefficients of prices and wages chosen at random in the interval $[0-1.5]$. T stands for the total duration of the simulations, here we have $T = 200$. b) Same as a) for the distribution of the real wages, more precisely for the difference between w/p at time T, i.e. at the end of the simulations, and $w0/p0$ at time 0, i.e. at the beginning of the simulations. c) Same as a) for the distribution of the productivity, more precisely for the difference between P at time T, i.e. at the end of the simulations, and $P0$ at time 0, i.e. at the beginning of the simulations. d) Coefficients of reaction of wages for the cases with negligible effect on employment, with ν_h and ν_l standing for the coefficients of reaction respectively to positive and negative labour excess demand.

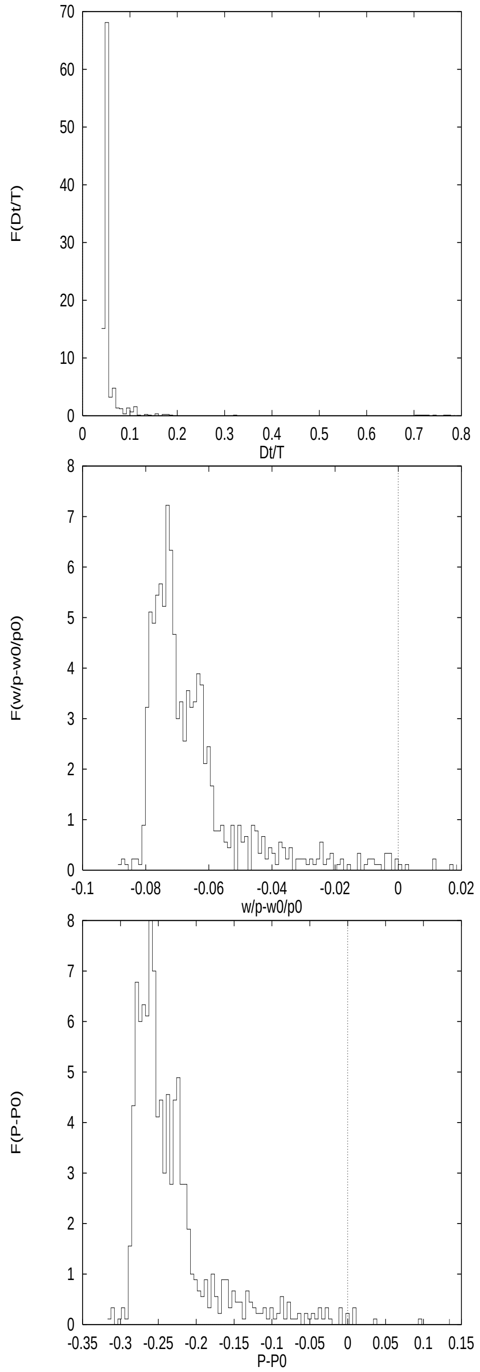


Figure 3.4: Same as Fig.3.3a,b,c for the case of rigidity of nominal wages, i.e. $\nu_l = 0$.

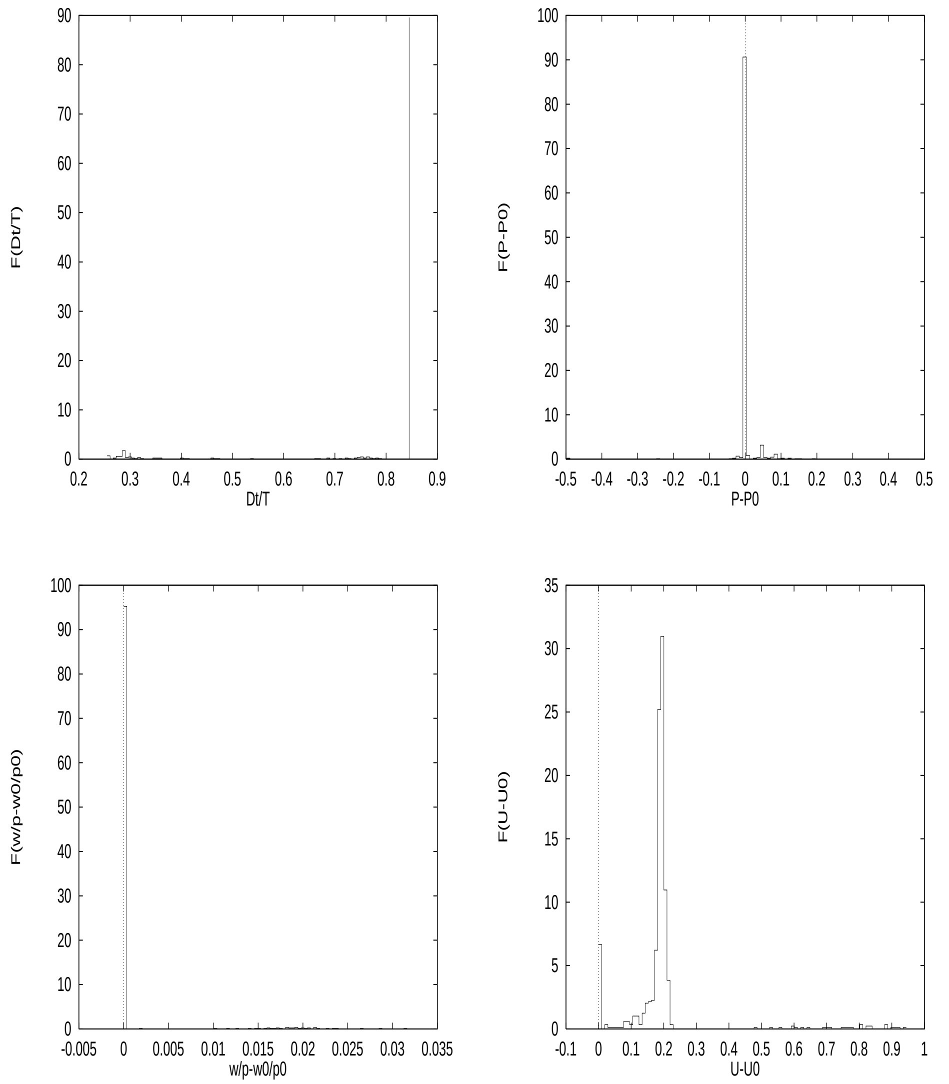


Figure 3.5: a) b) c)Same as Fig. 3.3a,b,c for the case of wages completely prices indexed. We remark that in a) there is a pick at $Dt/T = 0.85$ this is simply because the machinery effect starts at $t = 30$ and therefore when unemployment is not reabsorbed it lasts for $T - 30$ periods, i.e. $Dt/T = 0.85$. d) Histogram of the distribution of the final level of unemployment U.

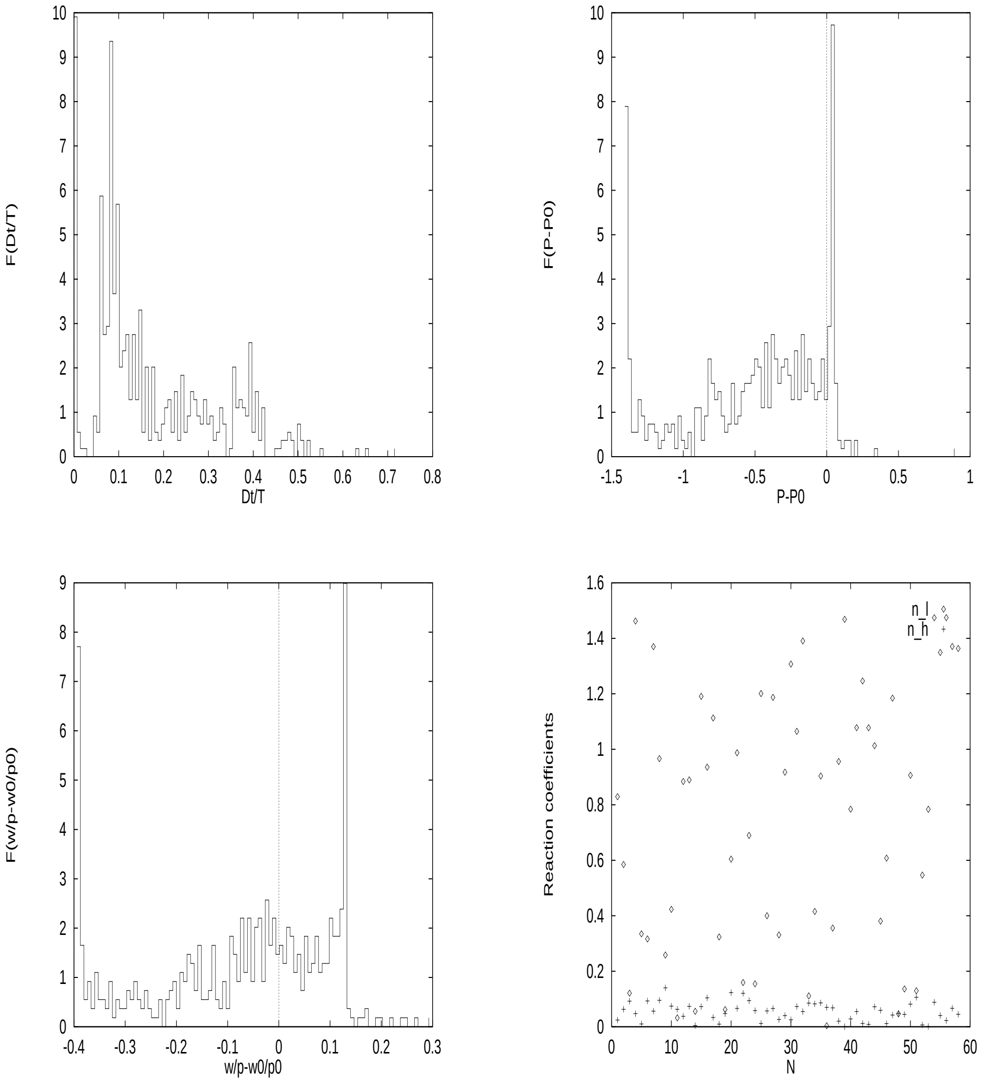
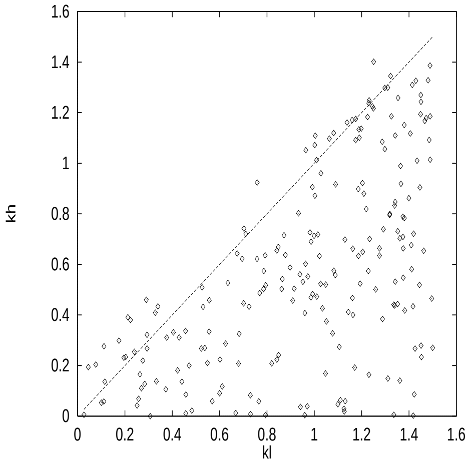


Figure 3.6: a) b) c)Same as Fig. 3.3a,b,c for a set of simulations with increasing saving. d) Coefficients of reaction of wages ν_h and ν_l for the cases with negligible effect on employment.



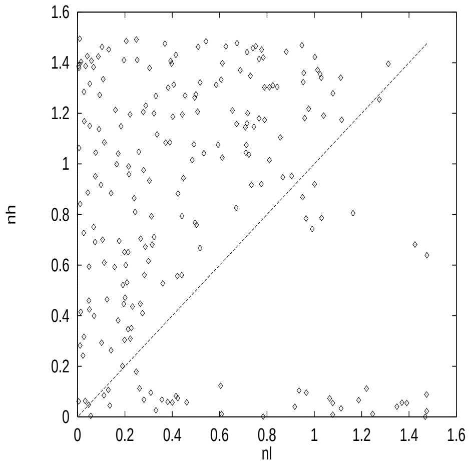


Figure 3.7: a) Values of the coefficients k_h versus k_l for the set of simulations of Fig.6 characterised by an increase of real wages. b) Values of the coefficients ν_h versus ν_l for the set of simulations of Fig.3.6 characterised by an increase of real wages.

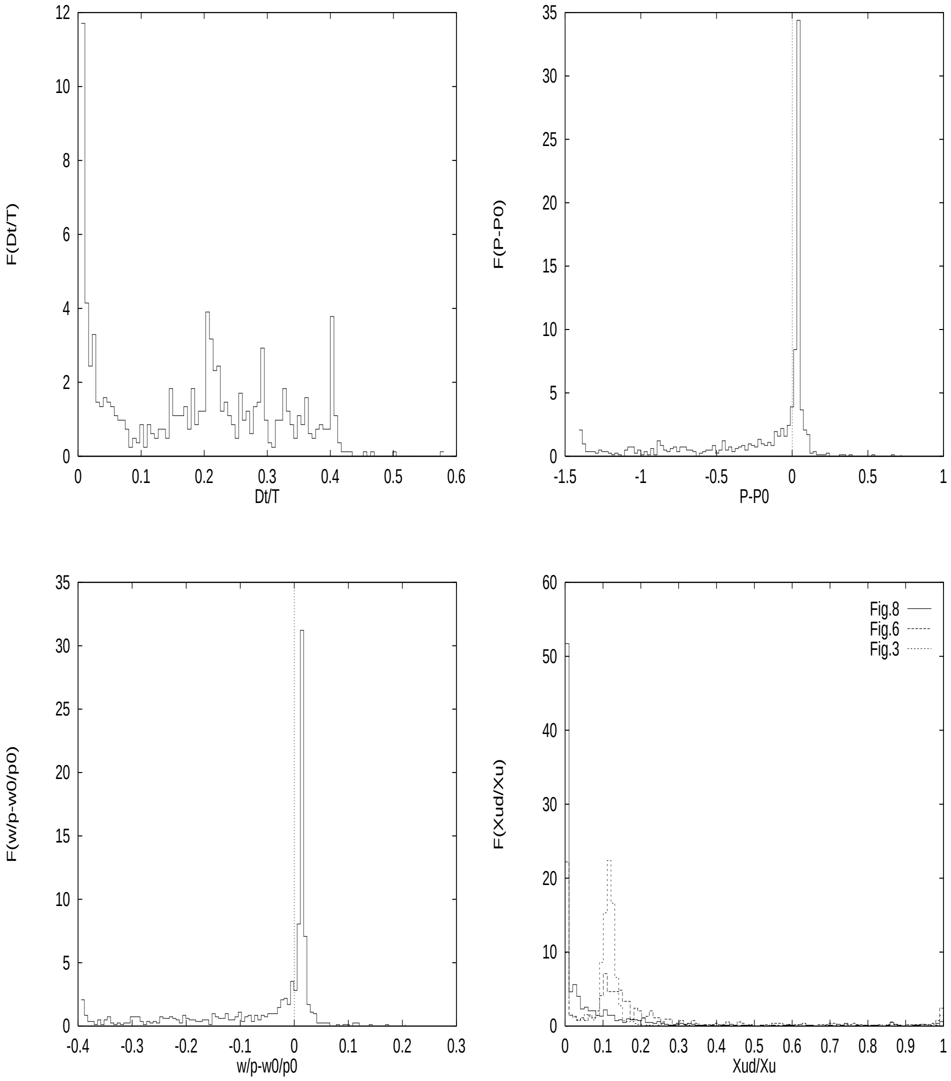


Figure 3.8: a) b) c)Same as Fig. 3.3a,b,c for a set of simulations with proportional saving and higher monetary supply growth rate. d) Histogram of the distribution of the ratio Xud/Xu, with Xud standing for the number of processes in the phase of utilization declassed at time T and Xu standing for the potential number of processes (in the phase of utilization) at time T. The 3 histograms correspond respectively, as indicated in the figure, to the set of simulations of Fig.3.3, Fig.3.6 and Fig.3.8.

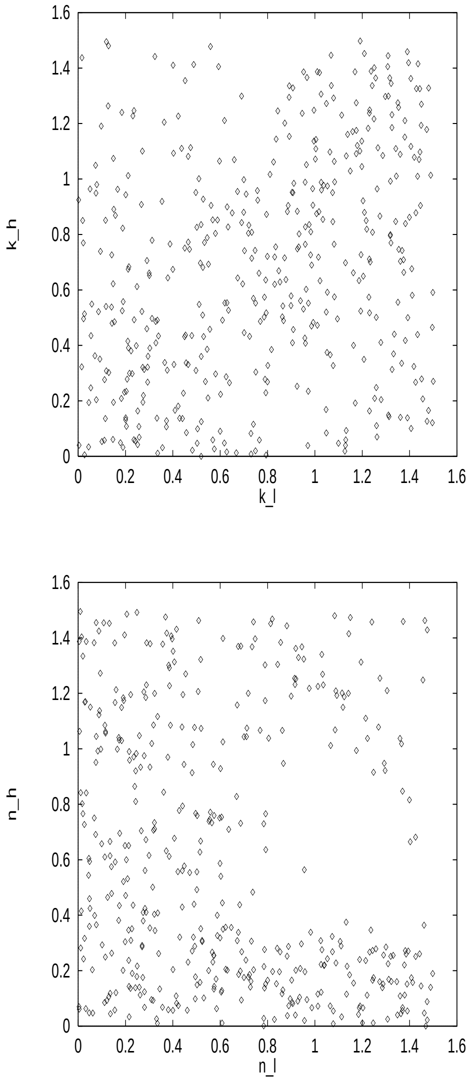


Figure 3.9: a) Values of the coefficients k_h versus k_l for the set of simulations of Fig.3.8 characterised by an increase of real wages. b) Values of the coefficients ν_h versus ν_l for the set of simulations of Fig.3.8 characterised by an increase of real wages.

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Chapitre 4

Caractérisation des sentiers d'équilibre et sentier de traverse

Ce papier a pur but l'analyse locale du modèle néo-autrichien. Une analyse approfondie du régime régulier ainsi que de l'analyse de deux économies définies par deux technologies differentes par une méthode de statique comparative est d'abord effectué. L'analyse des chemins hors de l'équilibre est ensuite realisée par la méthode habituelle qui consiste à analyser l'evolution temporelle de differentes variables économiques. Nous avons introduit une deuxième méthode dans laquelle les chemins de traverse sont optimaux par rapport à differents critères sociaux. L'évolution temporelle des variables économiques peut dans ce deuxième cas aider à la modélisation des differents variables clés. Le degré de rationalité accordé aux agents dans ce contexte nous a suggéré une modification du modèle qui sera intégrée dans les simulations du Chapitre 5.

REESTABLISHING EQUILIBRIUM PATHS IN A NEO-AUSTRIAN MODEL: ANALYSIS OF TRANSITORY REGIMES. [1]

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abstract

Using a neo-austrian representation of production processes we focus on the methods of analysis of the dynamical evolution of the system. We study the effects of a technological change starting from the classical approach of static comparison of steady states. We then study the shock directly running numerical simulations and analyzing the variation with time of the economic observables. Finally, in order to clarify and simplify the analysis, we introduce a different method of analysis which consists in considering a goal fixed by a social planner and in looking at the economical coherence of the time evolution of some economic key variables or control parameters. We claim that this method gives results easier to understand than the direct one and therefore can be used for the study of transitory paths as a guideline for a fruitful use of the direct analysis of numerical simulations.

Key words: neo-austrian representation, transitory regimes, dynamical evolution.

PACS numbers: 05.45

4.1 Introduction

The analysis of a global economic system is often made by means of comparison of steady states. This method allows to put into light the basic properties of the system as the study of fixed

[1]This paper has been delivered as an invited lecture at the 3rd International Summer School/Conference "Let's face Chaos through Nonlinear Dynamics" held at the university of Maribor, Maribor, Slovenia, on 24 June - 5 July 1996. This paper refers to a model which has been initiated by M.Amendola (University of Rome, La Sapienza) and J.L. Gaffard (University of Nice Sophia Antipolis) and extensively developed by M.Amendola, C.Froeschlé, J.L.Gaffard and E.Lega.

points for a general dynamical system.

Nevertheless, as a matter of fact economies are often out of equilibrium: for example they are bearing external shocks (like oil crises) or endogenous changes (like technological changes). In these cases economies are in a transitory regime, which may be characterized by some rate of unemployment (which is not a social optimum), and because of the complexity of the system it is very difficult to forecast the final state. In fact only a perfect knowledge of the system would allow agents to choose the optimal adjustment so that the economy may reach a new equilibrium path. Surely, the lack of coordination, due to the imperfect knowledge of the system, together with the fact that it takes time to react to an economic shock, make transitory regimes very complex.

Using a macroeconomic sequential model (for the analysis of the production process) we can analyze the effects of some typical real shocks. The common approach for this kind of study is to run numerical simulations and analyze the variation with time of the economic observables. This refers to the so called positive economics. Such an approach consists in asking what would be the working of an economy which is characterized by a given type of organization.

We introduce in this paper a different method of analysis: the dynamical evolution of the system is constrained by an objective corresponding to some social optimum and we study the viability of the system, i.e the coherence of the evolution with time of some key variables (or control parameters). This kind of approach belongs, in economics, to the so called welfare analysis. It consists in defining the optimum organization presented as the best way to satisfy the wants of economic agents or of a social planner

In Section 2 we present the model and the algorithm that we implemented for numerical simulations. In Section 3 we focus on the method of analysis and we introduce the new method. In Section 4 we analyze, as an example, the particular case of technological change.

4.2 A macro-economic model: the neo-austrian representation of the production process.

We consider a neo-austrian model (Hicks 1973, Amendola and Gaffard 1988, Froeschlé and Lega 1995, Amendola et al. 1996) where a homogeneous output is obtained from one primary input: labour. The aim is to study the evolution of a production process looking at the effects induced on the economy by shocks of different nature and magnitude, and searching the necessary conditions to recover stability.

We cannot give a quantitative forecast of the behaviour of macroeconomic observables as it is the case with some multi-sectorial linear models. Such models make possible predictions on a given interval of time (one year for example), provided that the coefficients computed from previous output series, analyzed by statistic institutes, do not change due to some shock in the economy. Using the neo-austrian model we would like to catch some basic phenomena of the economy in the same spirit, let us say, than the Hénon map (1976) concerning the study of strange attractors.

4.2.1 The production process

Before giving the fundamental equations governing this model we start by defining its basic "ingredients" for the discrete version that we have implemented for the numerical simulations presented in this paper:

- The time is discretized with a step Δt. In the following we will take $\Delta t = 1$ and the time i will indicate i units of time, i.e. $i\Delta t$.
- The construction of a production process is not instantaneous, but requires a given time n^c (c stands for construction).
- In order to build a single process one only primary input is taken into account, at each step Δt: the quantity a^c of labour.
- At time $n^c + 1$ the process starts to produce a quantity b of good, using a quantity a^u of labour, and the production lasts n^u (u stands for utilization) periods before the process becomes obsolete.

We call respectively $\vec{x}^c$ and $\vec{x}^u$ the number of processes in the construction and in the utilization phase and $\vec{x} = [\vec{x}^c, \vec{x}^u]$ the total number of processes:

$$\vec{x} = [x_0^c, x_1^c, ..., x_{n^c-1}^c, x_{n^c}^u, ..., x_{n^c+n^u-1}^u] \tag{4.1}$$

where the subscript stands for the age of the processes.

For instance, if we have $\vec{x}$ processes at time i, making the hypothesis that the economy growths at a constant rate g, the number of processes of age j at time i is equal to that of age $j+1$ increased by a factor g:

$$x_j(i) = x_{j+1}(i)(1+g) \tag{4.2}$$

Given the number of oldest processes $x_N = x_{n^c+n^u-1}$ (for simplicity we will write in the following N instead of $(n^c + n^u - 1)$) the previous recurrence relation allows to compute the whole set of components of $\vec{x}$:

$$x_j = x_N(1+g)^{N-j} \tag{4.3}$$

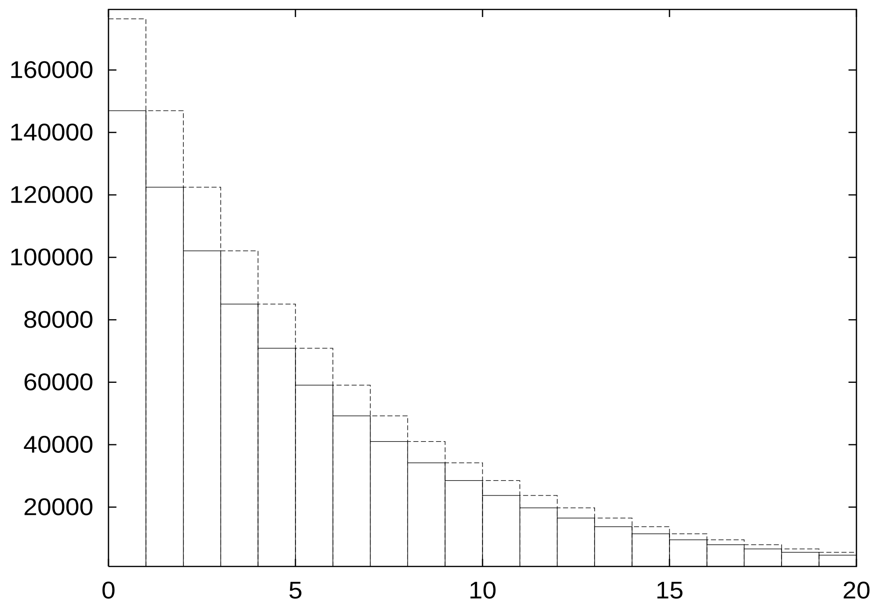

Figure 4.1: Profile of the structure of the production process, at a given time i, for a steady state with $n^c = 10$, $n^u = 10$ and a rate of growth $g = 0.2$ (solid line). Evolution, at time $i+1$, of the structure of the production process due to the only effect of aging (dotted line). The number of new processes of age zero is the result of a decision process and of some fundamental macro-economic equations (see text) which, in absence of shock, gives (as shown in the Figure): $x_0^c(i+1) = x_1^c(i+1)(1+g)$ (steady state).

Equation 4.2 defines the so-called steady state for which Fig.4.1 shows the profile of the structure of the set of x_j.

In the model the "engine" of the evolution is the natural ageing, i.e. the processes of age j at time $i+1$ are nothing but the processes of age $j-1$ at the previous period i:

$$x_j(i+1) = x_{j-1}(i) \tag{4.4}$$

Therefore, at time $i+1$, taking into account that the processes of age $n^c + n^u$ disappear, in order to have the whole set of x_j, we have to determine the number of new processes (x_0^c) of age zero (Fig.4.1). The number of x_0^c will be determined as the result of a decision process and of some fundamental equations. Let us remark that in the absence of shock the new processes are given by Eq.(4.2): $x_0^c(i+1) = x_1^c(i+1)(1+g)$.

4.2.2 Basic equations

Two classes of economic agents are considered: firms and households. All exchanges between them are intermediated by money: the resources required to carry out production and to sustain consumption are therefore financial resources, not physical output. In order to carry out production firms invest an amount of money W, the total wages of workers, given at time i by:

$$W(i) = m(i-1) + f(i) - c(i) + h^f_{nd}(i-1) \tag{4.5}$$

and households formulate their demand (y) for buying goods produced by firms:

$$y(i) = W(i) + c(i) + h^h_{nd}(i-1) \tag{4.6}$$

Following Eq.4.5 the wage fund W is given by the money proceeds (m) from sales of previous period, the financial resources given by banks (f), the eventual non desired balances of previous period (h^f_{nd}) and the taxes (c), or take out, taken from the state. The households (Eq.4.6) have at time i: the wage W, the state contribution c (schools, hospitals etc ...) and the eventual non desired balances h^h_{nd} of previous period.

The wage fund may be constrained by the human resources: $W > l_s w = \bar{W}$ where l_s is the labour supply, i.e. the total number of workers, and $w(i)$ the wage of one worker. In this case the wage fund becomes $\bar{W}$ and the difference $W - \bar{W}$ gives the non-desired balances of firms: h^f_{nd}.

4.2.3 The decision process

As we said in Section 2.1, the evolution of the production process depends on aging and on the decisions taken by firms. The knowledge of the demand at time $i-1$ allows firms to anticipate the demand at time i and therefore to try to adapt the supply of goods that they will produce at time i.

The demand (d, d^* for the expected demand) is considered in volume, i.e. the quantity y (Eq.4.6) is divided by the price p of product unit. The price is determined at the beginning of the period as explained in Section 2.4. Following this strategy of adaptative anticipation, the expected demand is:

$$d^*(i) = \frac{y(i-1)}{p(i)}[1 + g(i-1)] \tag{4.7}$$

where $g(i)$ is the rate of growth of economy at time i:

$$g(i) = \frac{m(i) - m(i-1)}{m(i-1)} \tag{4.8}$$

The difference between the expected demand and the stocks of goods (o), eventually unsold in the previous period, gives the expected production $q^*(i) = d^*(i) - o(i-1)$.

If the real capacity of production $q = b\sum x^u$ is greater than the expected production, agents estimate that a given quantity of goods will remain unsold and therefore some processes in excess are scrapped beginning from the older ones, otherwise the whole structure is maintained. The supply of goods at time i will be therefore: $s(i) = \min(q^*(i), q(i))$.

The knowledge of the total number of processes in the utilization phase and of per capita wage ($w(i)$), computed at the beginning of the period (Section 2.4), allows to compute the cost of utilization:

$$W^u(i) = w(i)a^u \sum_{j=n^c}^{N} x_j^u(i) \tag{4.9}$$

and therefore to obtain the total investment for construction: $W^c = W - W^u$. If W^c is not enough to pay all the existing processes in the phase of construction then some of them are scrapped beginning from the younger ones and $x_0 = 0$. Otherwise, once payed all the the existing processes, it is possible to pay, with the remaining money W_0^c, a number x_0 of new processes: $x_0 = W_0^c / wa^c$.

At the end of the period the production is sold. The money proceeds from sales are given by the minimum between households demand (Eq.4.6) and firms supply: $m(i) = \min(p(i)d(i), p(i)s(i))$. If the demand results to be greater than the supply of firms households remain with non desired balances: $h_{nd}^h(i) = p(i)(d(i) - s(i))$, otherwise firms have unsold stocks: $o(i) = s(i) - d(i)$.

4.2.4 Key variables

Once defined the basic equations and the temporal evolution of the production process it still remain to give the laws governing the variables which enter respectively in the decision process: p and w (Eq.4.7 and 4.9) and in the computation of the wage fund (Eq.4.5) and demand (Eq.4.6): c and f. We will call them key variables. From the point of view of dynamical systems they play the role of control parameters .

Prices and wages are in general computed according to the market laws of supply and demand respectively of good and of labour:

$$p(i) = p(i-1)[1 + k\frac{(d(i-1) - s(i-1))}{s(i-1)}] \tag{4.10}$$

$$w(i) = w(i-1)[1 + \nu\frac{(ld(i-1) - ls(i-1))}{ls(i-1)}] \tag{4.11}$$

where the coefficients k and ν indicates the speed of reaction of the market to a disequilibrium between supply and demand. The external financial resources and the take out can increase at the same rate g_0 of the initial steady state or at the rate g of the economy:

$$f(i) = f(i-1)(1 + \tilde{g}) \tag{4.12}$$

$$c(i) = c(i-1)(1 + \tilde{g}) \tag{4.13}$$

with $\tilde{g} = g_0$ or $\tilde{g} = g$. Of course other laws of evolution are possible, according to the kind of economy that we are interested to describe.

4.2.5 The algorithm

In order to summarize the model and to be more clear about its dynamic we show in Fig.4.2 the main steps of the algorithm that we have implemented for the numerical simulations that we will present in the following sections.

4.3 Methods of analysis

Given the complexity of the model and the number of parameters we consider very important to pay attention to the methods of analysis.

4.3.1 Classical approach

The starting point, as well as for any dynamical system, is the study of fixed points, or more generally of steady states and the study of two different economies can be made by comparison of their corresponding steady states. A steady state, as we said above, is characterized by an economy growing at a constant rate g where no disequilibrium occurs in the market of labour and of good: wages and prices are constant and no undesired balances are created. The basic equation for wage fund (Eq.4.5) becomes:

$$W(i) = m(i) - \alpha m(i) \tag{4.14}$$

where $m(i) = m(i-1) + f(i)$, i.e. the external financial resources correspond exactly to the growth of incomes and $\alpha m(i)$ corresponds to the take out $c(i)$, i.e. the state takes from firms a constant part α of the incomes. For instance f will be zero for a steady state with $g = 0$. Let us remark that the equilibrium of the market of goods implies that all the production is sold, i.e. that the demand of households corresponds exactly to the supply of firms (in volume), and in monetary terms to their incomes m. Eq.4.6 becomes:

$$m(i) = W(i) + c(i) \tag{4.15}$$

with $c = \alpha m$ Eq.4.15 is just the same than Eq.4.14: the steady state is completely described by only one equation. If now we develop Eq.4.14 we get:

$$a^c w \sum_{j=0}^{n^c-1} x_j^c + a^u w \sum_{j=n_c}^{N} x_j^u = (1-\alpha) pb \sum_{j=n_c}^{N} x_j^u \tag{4.16}$$

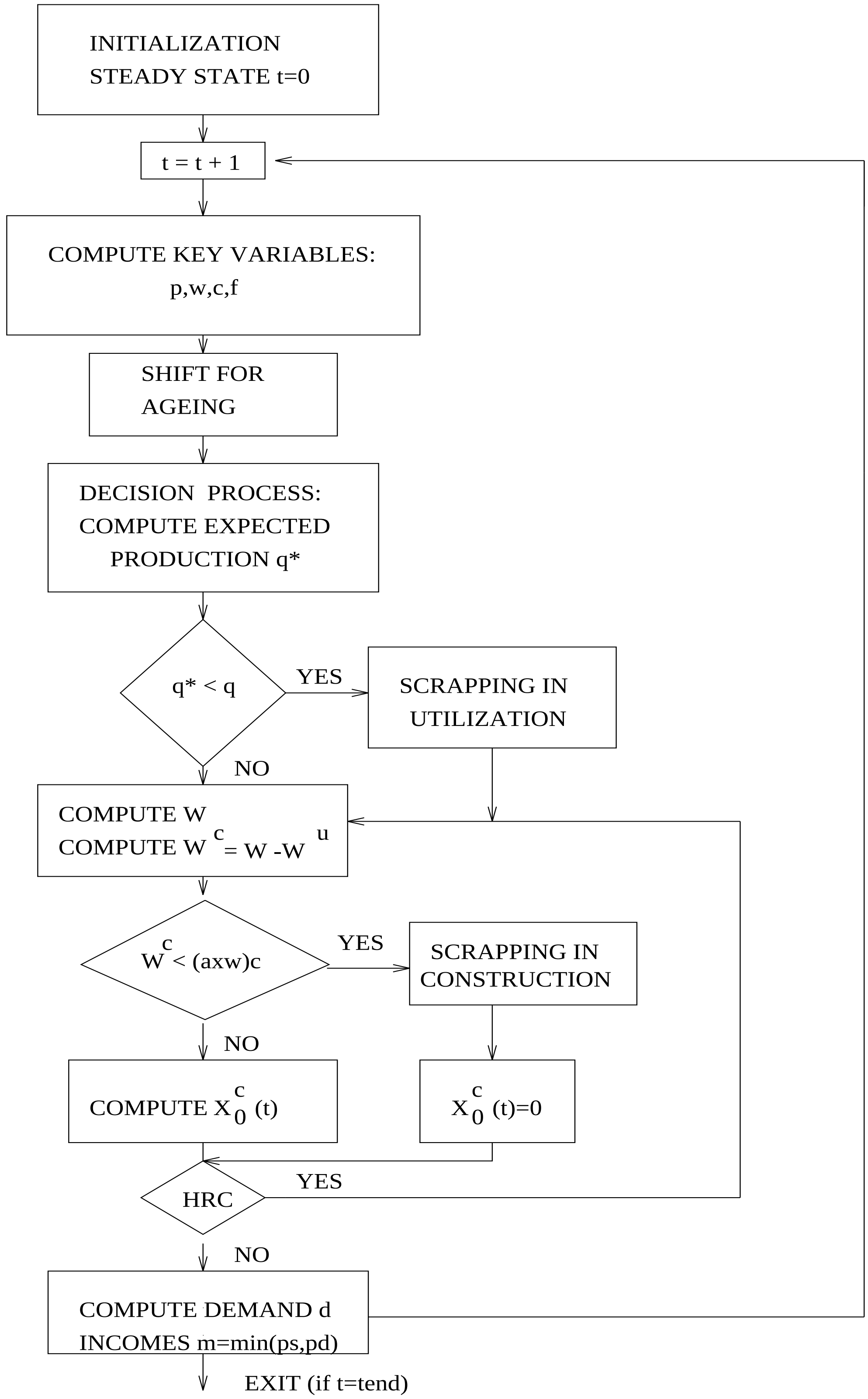

Figure 4.2: Algorithm of the neo-austrian model.

which, given the number of oldest processes x_N, and taking into account Eq.4.3 can be rewritten as:

$$a^c w x_N \sum_{j=0}^{n^c-1}(1+g)^{N-j} + a^u w x_N \sum_{j=n_c}^{N}(1+g)^{N-j} = (1-\alpha) b p x_N \sum_{j=n_c}^{N}(1+g)^{N-j} \tag{4.17}$$

The computation of the total number of processes in Eq.4.17 gives:

$$\frac{w}{p}\{a^c(1+g)^{n^u}[\frac{1-(1+g)^{n^c}}{1-(1+g)^{n^u}}] + a^u\} = (1-\alpha)b \tag{4.18}$$

Let us remark that the external financial resources do not appear in the equilibrium condition of Eq.(4.18): in a steady state the role of money is neutral. In the particular case in which the length of construction and utilization are the same ($n^c = n^u = n$) we get:

$$\frac{w}{p}[a^c(1+g)^n + a^u] = (1-\alpha)b \tag{4.19}$$

The sum inside the square bracket (A hereafter) represents the fundamental quantity of labour necessary to produce a unit b of good and to maintain on the steady state the whole production process. Taking into account that for the steady state, with the income $m = bp$ of one process in utilization (more precisely with $m - c$), firms pay wages for this process and for $(1+g)^n$ processes in construction, Eq.4.19 can be directly obtained from Eq.4.14. Moreover, we can infer that for the general case of Eq.4.18 the term which multiplies a^c gives the number of processes in construction corresponding to one process in utilization.

Let us remark that if the utilization phase is shorter than the construction phase ($n^c < n^u$) then A will be greater than in the case $n^u = n^c$ and if we want to have the same volume of production in the two cases then x_N has to be greater for the case with shorter utilization. In the following we will take $n^c = n^u = n$.

In order to study two economies characterized by a different technological level we can compare their corresponding steady states. We define the technological level by the quantity of labour necessary for produce a unit b of good and by the proportion of labour necessary for construction versus utilization. Hence, we will say that the technology $[\hat{a}^c, \hat{a}^u]$ is more efficient than $[\bar{a}^c, \bar{a}^u]$, if it needs less work, i.e.:

$$\hat{A} \equiv \hat{a}^c(1+g)^n + \hat{a}^u < \bar{A} \equiv \bar{a}^c(1+g)^n + \bar{a}^u \tag{4.20}$$

although more in construction: $\hat{a}^c > \bar{a}^c$ (more research for example) and less in utilization: $\hat{a}^u < \bar{a}^u$. From the comparison of Eq.4.19 for the two economies it follows that, if the unit of good produced is the same as well as the price and the rate of take out, then the wage $\hat{w}$ will be greater than $\bar{w}$. If the labour supply is the same for both economies then x_N (and therefore the global production: $bx_N \sum_{j=n}^{N}(1+g)^{N-j})$ will be greater for the country equipped with the more efficient technology:

$$x_N \bar{A} = h x_N \hat{A} \tag{4.21}$$

with $h > 1$.

This is a very simple but useful exercise in view of a dynamical analysis of a situation of disequilibrium.

4.3.2 Direct method

At this stage numerical simulations become a useful tool. A first method, called by economists positive approach, is to run simulations for a given state of the economy and to look at the variation with time of the economic observables.

In order to study the effect on economy of a technological shock let us start with a given state of the society, of the financial policy etc .., and we give the rules for the evolution of the key variables. We build the steady state and we inject, at a given time i, new values for the quantities of labour $[\hat{a}^c, \hat{a}^u]$.

Let us remark that the change is not instantaneous: starting at a given time i the new technology on the new processes $x_0^c(i)$ it will take $N = 2n$ steps of time to change the technology on the whole production process. This means that for a certain time, at least n periods, the fundamental quantity of labour will be greater than the initial one, therefore the technology will be less efficient. The greatest difference is after n periods: $\hat{a}^c(1+g)^n + a^u > a^c(1+g)^n + a^u$. It is easy to see that the transition can be very complex.

Let's take for instance the simple case of $g = 0$ with $n_c = n_u = n = 10$ and $[a^c, a^u] = [8, 8]$ for the initial technology and $[\hat{a}^c, \hat{a}^u] = [10, 5]$ for the new-one. Prices and wages are constant: $k = \nu = 0$ (Fig.4.3c,d), and and external financial resources (Fig.4.3a) and take out (Fig.4.3b) growth respectively at the rate of growth of the initial steady state and at the rate of growth of economy. The shock occurs at time $i = 20$. We analyze step by step the transition regime and the final equilibrium path.

For all the simulations discussed in this paper we show the temporal evolution of the following set of economic observables:

- a) the ratio of financial resources over incomes : $\beta = f/m$ and the ratio of financial resources over all the money injected the economic system: f/F, where $F(i) = m(0) + \sum_{j=1}^{i} f(j)$.
- b) the rate of take out: $\alpha = c/m$

- c) the price: p
- d) the per capita wage: w
- e) the real per capita wage: w/p; the productivity: $b\sum x^u/\sum ax$ and the per capita product: $b\sum x^u/l_s$
- f) the rate of unemployment
- g) the distortion of the productive capacity, i.e. the total number of processes in the construction phase over the total number of processes in the utilization phase: $\sum x^c/\sum x^u$
- h) the non desired balances of households and firms, more precisely the ratio: h_{nd}/W
- i) the number of new processes: x_0^c
- l) the scrapping of the production process.

It appears in Fig.4.3i that at period $i = 21$ the rate of start $x_0(21)$ suddenly drops. This is due to the increase of the quantity of labour necessary for construction. The same occurs for the following 10 periods. At time $i = 31$ less processes arrive in utilization, so the global production drops (Fig.4.3e dotted line) and as a consequence incomes decrease. It follows a decrease of the wage fund at period 32 and a phase of non zero unemployment

starts (Fig.4.3f). At the same time the quantity of labour in utilization begins to decrease and the transfer of workers from the utilization to the construction phase allows to start a greater number of new processes. At period $i = 41$ the new technology is active in the whole production process. The advantage of a lower quantity of fundamental labour $\hat{A}$ and the increase of the global production makes possible the decrease to zero of the rate of unemployment.

At period $i \simeq 80$ the system has reached a new equilibrium path characterized by a rate of start (Fig4.3i) which, although oscillating, has a mean value greater than that of the initial steady state. Fig.4.3l shows a non zero rate of scrapping in the utilization phase, indicating that the new equilibrium reached has an utilization phase shorter than the construction phase.

As we have shown comparing steady states (Section3.1) the technical change allows to increase the global production of goods but goods have still to be sold in order to have a real advantage in changing the technology. Therefore, the demand of households has to increase proportionally to the increase of production. In the simple case considered the demand of households remains constant. In fact, following Eq.4.5 and Eq.4.6, we have: $y(i) = m(i-1) + h_{nd}^h(i-1)$. When the supply of firms is less than the demand of household the last equation can be rewritten as: $y(i) = p(i)s(i-1) + p(i)[d(i-1) - s(i-1)] = y(i-1)$, otherwise $y(i) = m(i-1) = y(i-1)$.

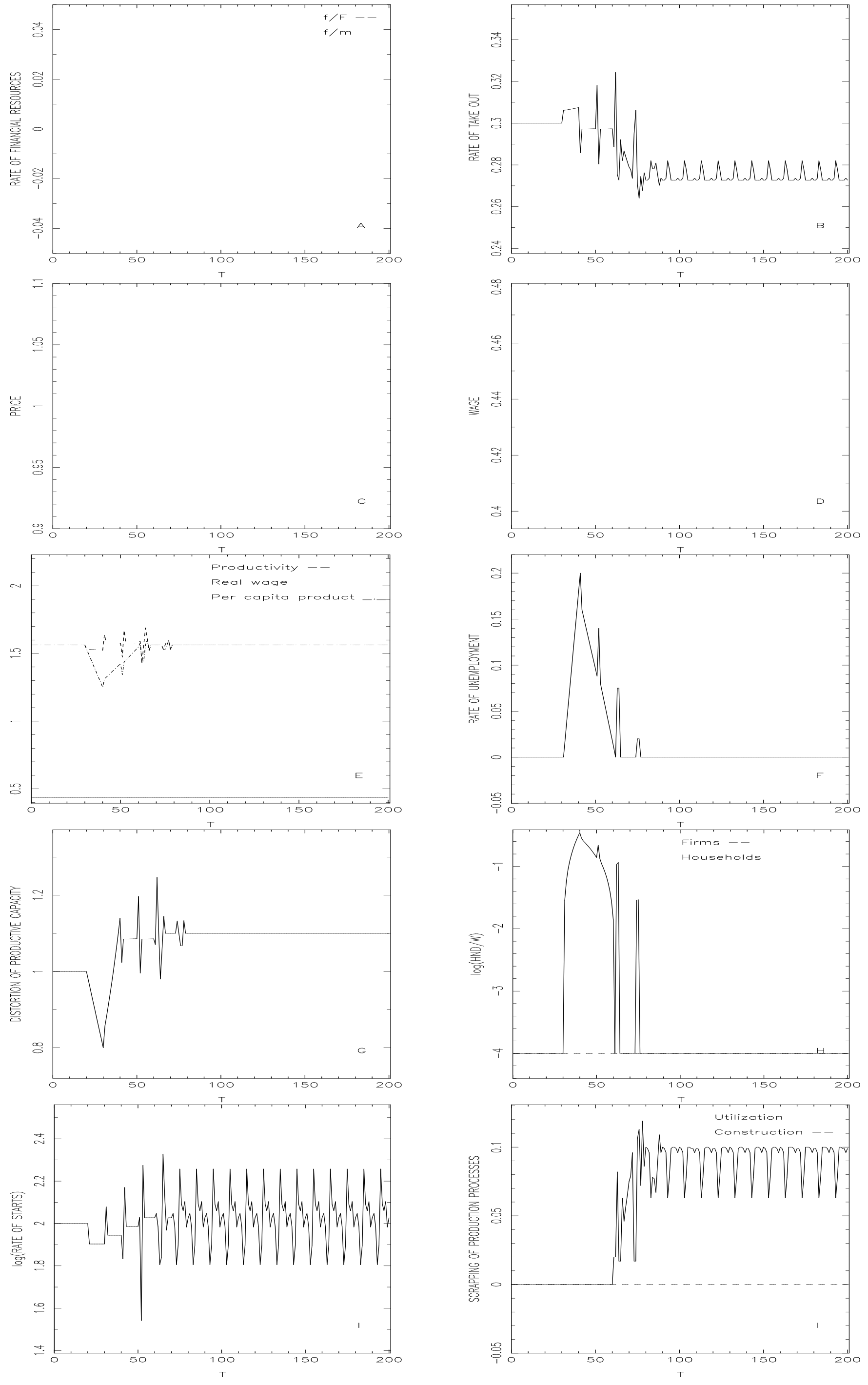

Figure 4.3: Analysis of a macro-economical shock: technological change. The initial steady state is characterized by: $n^c = 10$, $n^u = 10$ and a zero rate of growth . The shock occurs at time $i = 20$. The labour coefficients are changed from $[a^c, a^u] = [8, 8]$ to $[\hat{a}^c, \hat{a}^u] = [10, 5]$. Prices and wages are fixed and external financial resources and take out growth respectively at the rate of growth of the initial steady state and at the rate of growth of economy (the method of analysis corresponds to the so called positive approach).

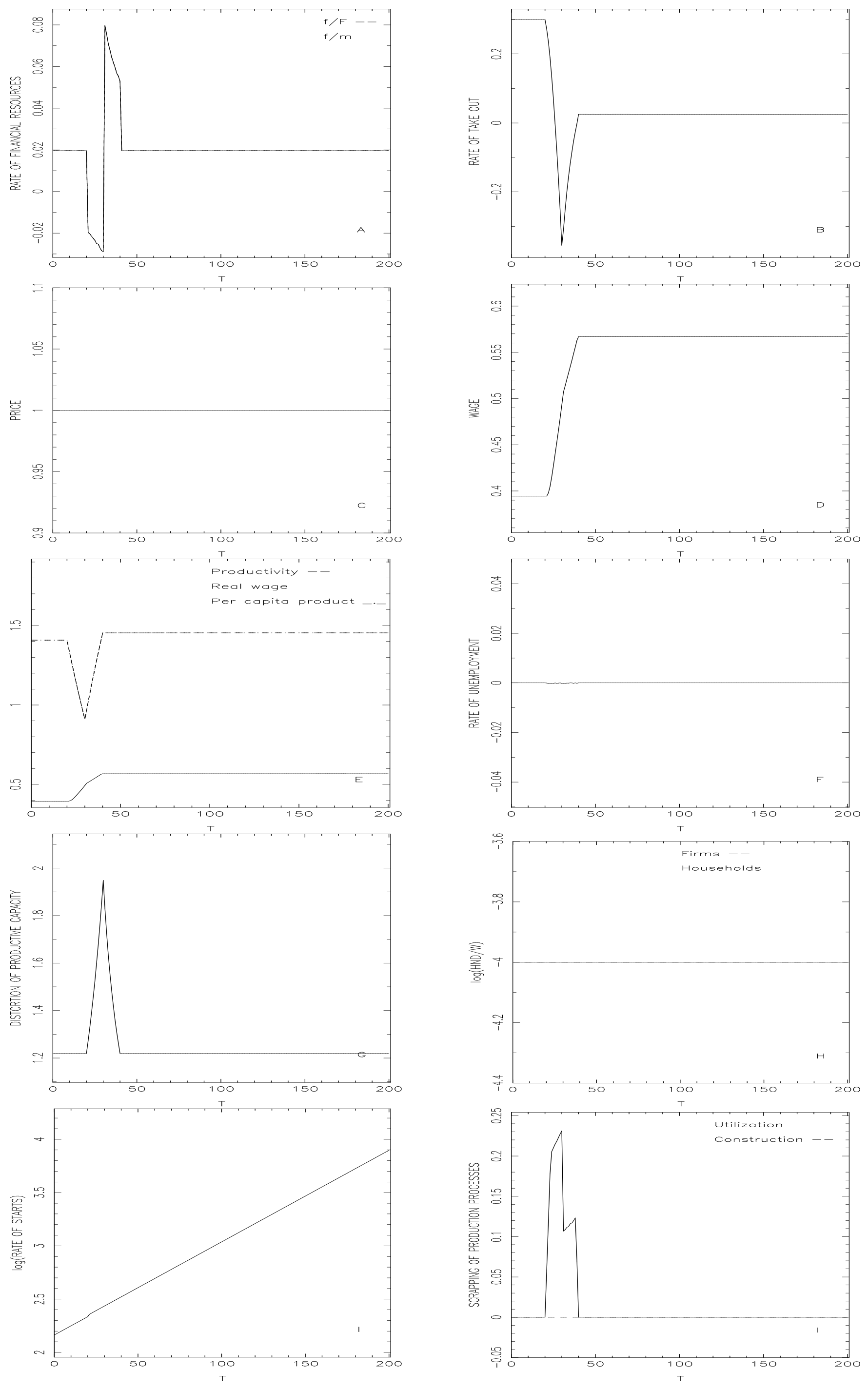

Figure 4.4: Same technological shock as Fig. 4.3. The initial steady state is characterized by: $n^c = 10$, $n^u = 10$ and a rate of growth $g = 0.02$. The control parameters are the external financial resources and the take out. Prices and wages follow the market laws and the coefficient of reaction to disequilibrium are $k = \nu = 0.5$ (the method of analysis corresponds to the so called normative approach). The objective is to reach as soon as possible the new equilibrium path at the price of scrapping processes in the utilization phase.

Taking into account that in the decision process agents anticipate the production on the basis of the demand observed in the previous period (Section 2.2), it is easy to understand the constant rate of scrapping of the final state. In other words, the volume of production in the initial and final state is the same (as shown in Fig.4.3e, dotted line):

$$nx_N = \hat{n}^u \hat{x}_N \tag{4.22}$$

where $\hat{n}^u$ is the new length of the utilization phase for the final state. The equivalence of the labour supply for the initial and final state gives:

$$nx_N[a^c + a^u] = n\hat{x}_N(\hat{a}^c + \hat{a}^u \frac{\hat{n}^u}{n}) \tag{4.23}$$

The solution of the last two equations gives: $\hat{x}_N = 1.1x_N$ and $\hat{n}^u = 0.9n$ in agreement with the numerical simulation.

The equilibrium obtained is one between all the possibility that we could have analyzed when comparing steady states, but we cannot consider it as an optimal final state: factories are destroyed before their natural death.

Of course this simulation with prices and wages constant is just a didactic case in which the system cannot reach a final optimum equilibrium, because there is not enough money to face the technological change. However, we have obtained qualitatively the same result for more complicated scenarii with non-zero rate of growth and different values of the parameters k and ν as well as for different laws of evolutions of f and c (Amendola et al.1996) [2].

4.3.3 A new method of analysis

Instead of giving the rules of evolution of all the key variables and analyze the transitory path to an unknown final state we propose here to fix the final state and to analyze transitory regimes between known steady states. For this kind of approach, called normative by economists, we consider a goal fixed by a social planner and a strategy to reach it. The goal consists in ensuring full employment and a greater level of production. Different strategies can be adopted: for example ensuring the full employment all over the transitory regime up to final state, or building processes in order to reach as quick as possible (turnpike strategy, Koopmans 1964) the new equilibrium path.

The number of degrees of freedom of the model is reduced given the constraints which fix the value of some of the key variables. The aim of the simulations in this case is to study the relevance, from an economic viewpoint, of the temporal evolution of such key variables.

In practice we decide first the structure of the production process at a given time i and then we compute, at time i, the value of some of the key variables which is necessary for having the desired structure of $\vec{x}$. Let us remark that this means that agents have a certain knowledge of the model, i.e. they know the final state and this allows them to make what economist call rational expectations.

Let us go back to the basic equations (Eq.4.5, Eq.4.6):

$$W(i) - m(i-1) - f(i) + c(i) - h^f_{nd}(i-1) = 0 \tag{4.24}$$

$$m(i) - W(i) - c(i) - h^h_{nd}(i-1) = 0 \tag{4.25}$$

where the demand $y(i)$ has been replaced by the incomes $m(i)$ because we make the hypothesis that the structure of production process is the optimal one, i.e. markets are cleared, there is no disequilibrium between supply and demand. Moreover we have:

$$m(i) = p(i)b \sum_{j=n^c+1}^{N} x^u_j(i) \tag{4.26}$$

and:

$$W(i) = w(i)a \sum_{j=0}^{N} x_j(i) \tag{4.27}$$

The structure of the production process is known once fixed the objective, therefore four unknown quantities remain: $p(i)$, $w(i)$, $f(i)$, $c(i)$; and because we have 2 equations we give the law of evolution of 2 variables and we observe and analyze, during the transitory state to the new path, the variation with time of the other two variables.

For example we can consider that prices and wages follow the market laws (Eq. (4.10) and Eq. (4.11)), then the take out and the external financial resources will be given by:

$$c(i) = m(i) - W(i) - h^h_{nd}(i-1) \tag{4.28}$$

$$f(i) = W(i) - m(i-1) + c(i) - h^f_{nd}(i-1) \tag{4.29}$$

and the analysis will focus on the relevance and on the complexity of the variation of these variables. In the following section we will give examples based exactly on the above case, but of course all other possible choices of variables can be analyzed: given the evolution law of f and c we can study the variation of p and w, given f and p we can study the variation of c and w and so on ..

4.4 Application to a real shock: the case of technological change.

We apply this method of analysis to the same technological shock that we have studied with the direct method of Fig.4.4.

We consider that the initial steady state growths at a rate $g = 0.02$ and for prices and wages we fix the coefficient of reaction to the disequilibrium between supply and demand to the value: $k = \nu = 0.5$. The number of periods for construction and utilization is $n^c = n^u = 10$ and the shock occurs at time $i = 20$. Prices and wages are governed by market laws while c and f are given by Eq. (4.28) and Eq. (4.29).

In the first simulation the strategy is to reach as soon as possible the new equilibrium path, i.e. we start at period 21 to construct the new processes at a rate corresponding to the steady state of the new technology. In order to make available the labour required for construction we have to scrap processes in the utilization phase (Fig.4.4l). The destruction of processes reaches its maximum value at period $i = 31$ when all the construction phase works with the new technology which requires more labour than the initial one ($\hat{a}^c > a^c$) while the utilization phase still works with the initial technology. At period 32 the scrapping decreases thanks to the decrease of the quantity of labour in the utilization phase ($\hat{a}^u < a^u$) which allows the transfer of workers from utilization to construction. No more scrapping is necessary when the new technology works on the whole structure of production (at period 41). The destruction of processes, in this case, reflects the fact that the labour demand is greater than the labour supply and this causes an increase of wage (Fig.4.4d) in the same interval: $21 \leq i \leq 41$.

In Fig.4.4g we can see, also as a consequence of the scrapping in utilization, an increase of the distortion of the production process in the ten periods which follow the shock. Again the maximum is reached at period 31, when there is still no advantage on the technological change.

The decrease of the volume of production ($s = b \sum x^u$) during the interval $21 \leq i \leq 31$ (Fig.4.4e dotted line) is followed by a sharp drop of incomes ($m = \min(sp, dp)$) up to time $i = 31$, and by a corresponding drop of the rate of take out α (Eq. (4.28) and Fig.4.4b).

The decrease of the rate of take out is in fact due to the coupled effect of decrease of m and increase of wage. In Fig. 4.4b, α, whose initial value is 0.3, not only becomes negative but it reaches a final equilibrium value of about 0.01, i.e. $c = 1\% m$, which is really not realistic. It is not the same for the external financial resources (Fig. 4.4a), whose rate $\beta = f/m$ varies in the interval $-0.02 < \beta < 0.08$ starting from an initial value of about 0.02, i.e. $f \simeq 2\% m$. Let us remark that a negative value for β means that firms give back money to the banks. In any case the system reaches a final state corresponding to the goal fixed by the social planner, i.e. to a higher purchasing power of households and no unemployment (Fig. 4.4d,f) but the transition and the final value for the take out make this scenario not realistic.

We can release the strong requirement of constructing immediately at the rate of the new technology forbidding the scrapping in utilization (Fig.4.5l). Fig.4.5i shows that it takes a longer

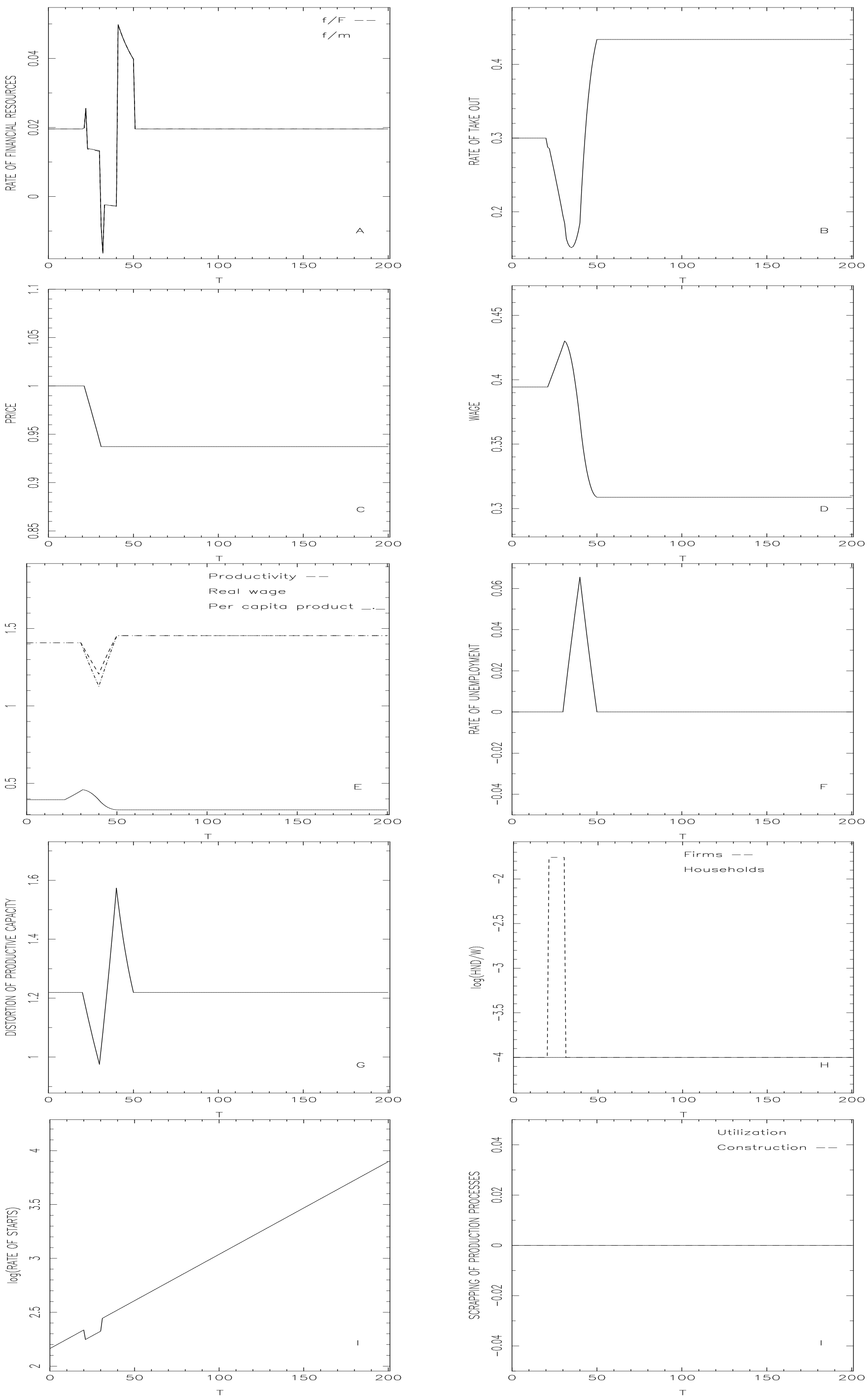

Figure 4.5: Same as Fig. 4.4 but the objective is to reach the new equilibrium path without scrapping in utilization.

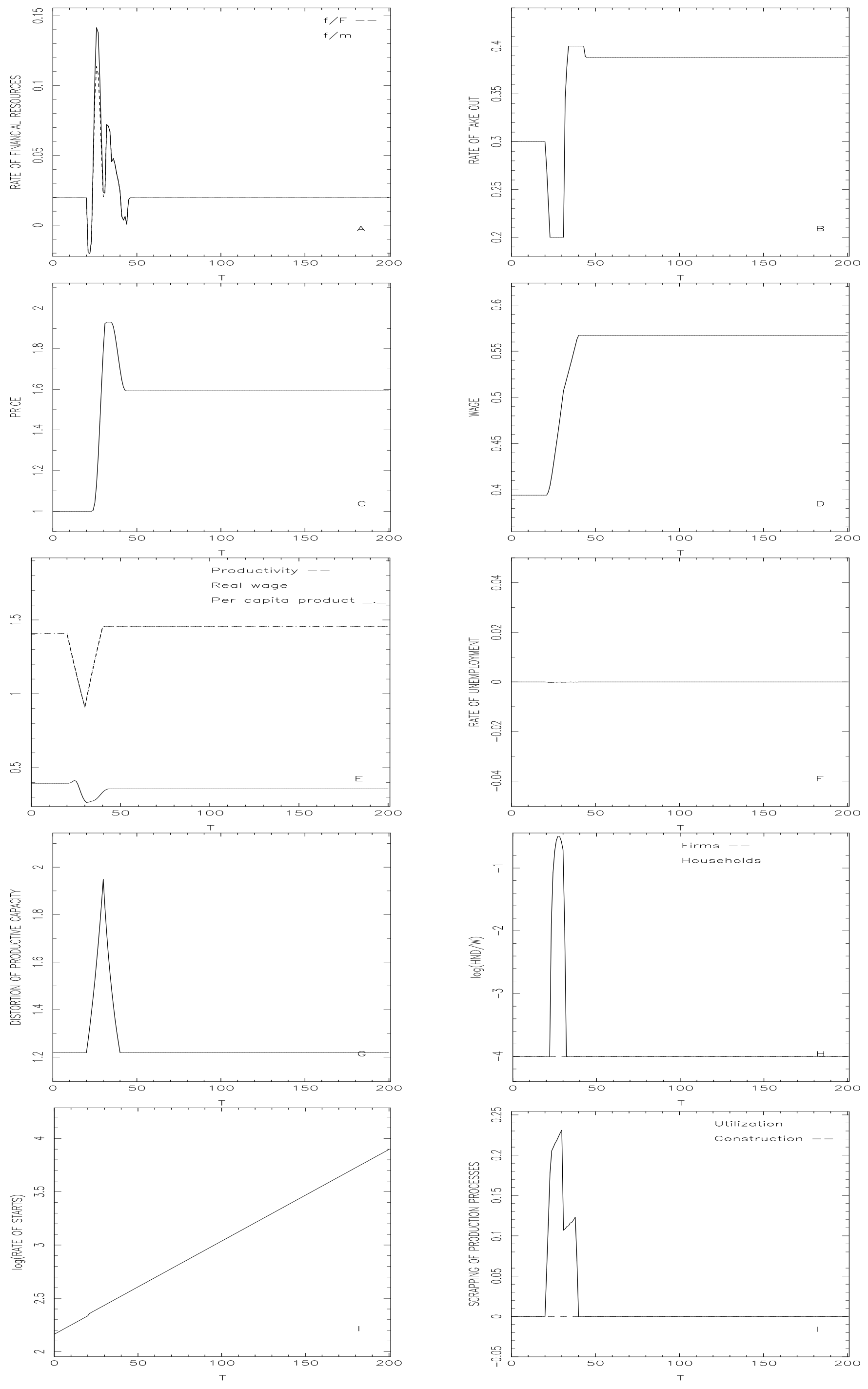

Figure 4.6: Same as Fig. 4.4 but the variation of the rate of take out α is constrained in the interval: $0.2 \leq \alpha \leq 0.4$.

time for the new processes (x_0^c) to reach the new path. The objective of the social planner is reached through a realistic phase of transition (Fig.4.5a,b) although during the transition we observe a non-zero rate of unemployment (Fig.4.5f).

Let us now suppose that policy and social forces impose some limitations to the change of c and f from one period to the other. In other worlds bounds prevent the desired variation of the key variables.

We start constraining the variation of α in the interval $0.2 \leq \alpha \leq 0.4$ (Fig.4.6). The value obtained for c following Eq.4.28 is replaced by the lower limit $c_l = 20\%m > c$ from $i = 21$ to $i = 31$ (Fig.4.6b). Therefore the demand of households is greater than what obtained by Eq.(4.25):

$$y(i) = W(i) + c_l(i) + h_{nd}^h(i-1) \geq W(i) + c(i) + h_{nd}^h(i-1) \tag{4.30}$$

i.e. a disequilibrium is created between supply of firms (the right term in Eq.4.30) and demand. Demand is in fact greater than supply, therefore the price increases (Fig.4.6c) and households have non desired balances (Fig.4.6h). The opposite occurs when c is replaced by its upper bound: demand is lower than supply, the price decreases up to a final value which is higher than the initial one. In order to obtain the desired structure of production process in this case we need a higher flexibility of the external financial resources (Fig.4.6a) because a discontinuous take out enter in Eq.(4.29). For the same reason the value of β is no longer equal to ratio f/F, i.e. there is a disequilibrium between the value of incomes and the creation of money F. The final regime, with the intervention of the state on the possible range of variation of the take out, becomes now realistic.

When the social planner try to channel the process of change of the take out, shrinking the interval of variation of α, the transition is quite the same, the variation of f is almost unchanged an the system reaches the desired final state characterized by a different level of wage and take out. From the economic viewpoint the new repartition of take out and wage in the composition of the demand of households may not be accepted by workers.

A different situation occurs when we constrain the variation of f. In the simulation presented in Fig. 4.7 we have constrained the variation of the rate of financial resources β such that: $\beta \leq 0.06$, leaving unchanged the limits on the rate of take out ($0.2 \leq \alpha \leq 0.4$). When comparing to Fig. 4.6 we can see that the duration of the transition is unchanged while the variation with time of β and α (Fig.4.7a,b) is more noisy and during the transition a non zero level of unemployment (Fig.4.7f) appears, due to the lack of money for investment caused by the upper bound on β. It is easy to understand that a higher reduction of the external financial resources can have drastic consequences even on the viability of the final regime.

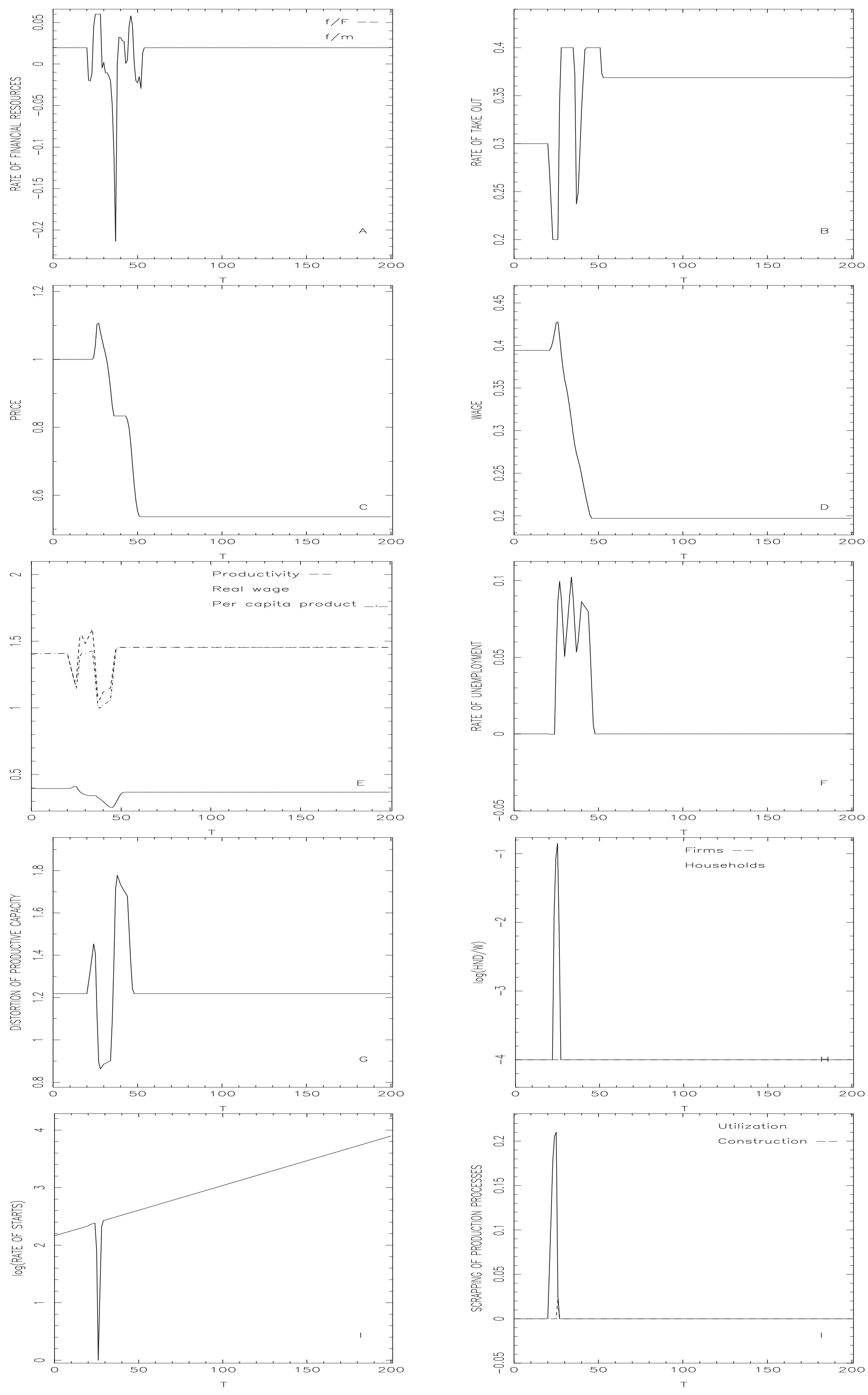

Figure 4.7: Same as Fig. 4.4 but the variation of the rate of take out α is constrained in the interval: $0.2 \leq \alpha \leq 0.4$ and the variation of the rate of external financial resources β can not be greater than: 0.06.

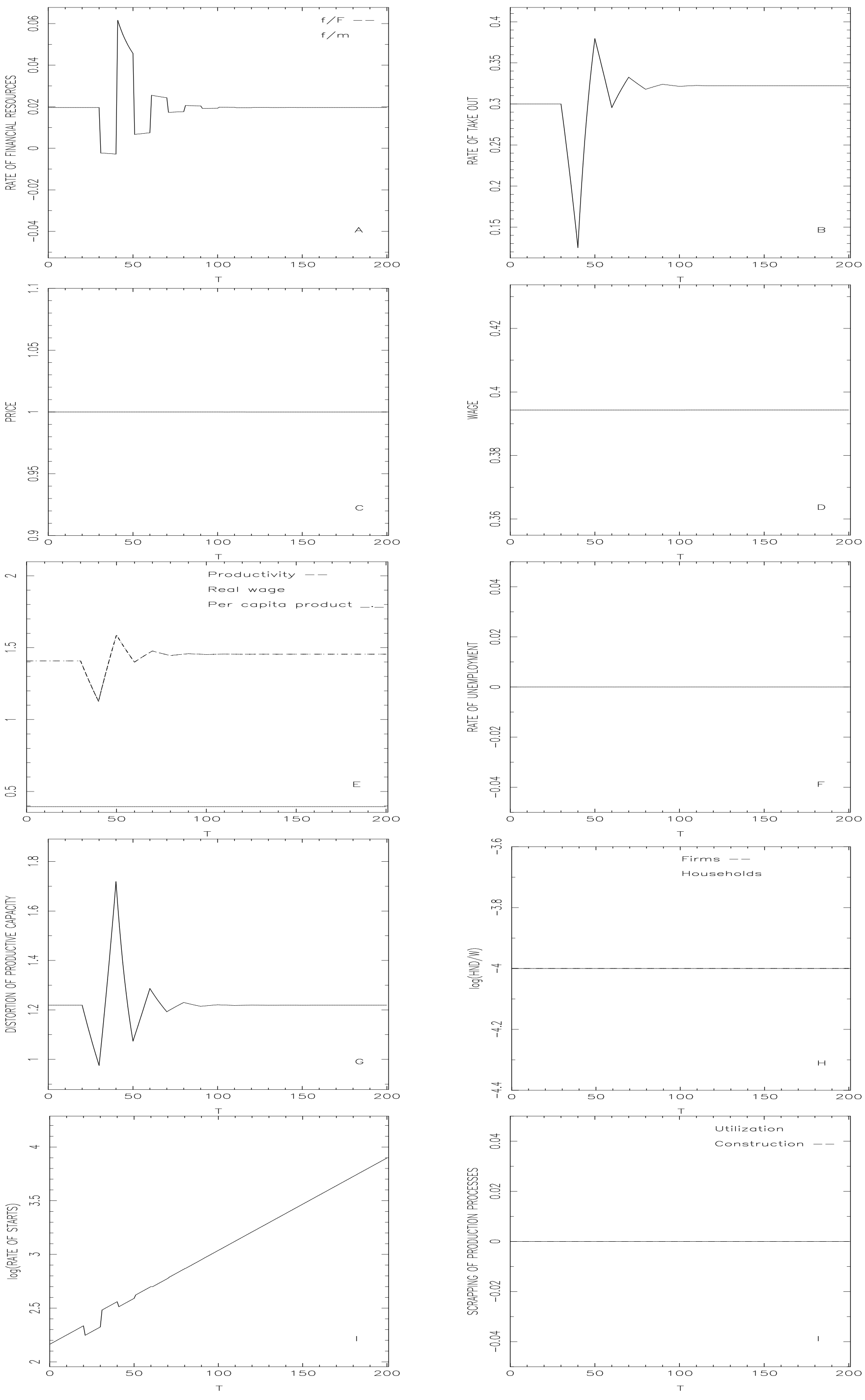

Figure 4.8: Same as Fig. 4.4 but the objective is to reach the new equilibrium path with no unemployment during the transition and of course in the final state.

We have repeated the same set of experiences for another objective: we ensure the full employment all over the transitory regime up to the final state. The transition requires now a longer time (Fig.4.8) because the structure of productive capacity is adapted to the labour supply and not immediately to the final regime as it was in Fig.4.4, but the changes of f and c are now more realistic. In fact, although α drops to a value of 0.13 (Fig.4.8b) it does not becomes negative and the final regime has a rate of take out ($\alpha = 0.32$) quite similar to the initial one ($\alpha = 0.3$).

If we limit the variation of α to the interval $0.2 \leq \alpha \leq 0.4$ the transition is almost unchanged (Fig.4.9) but a higher flexibility of the external financial resources is required (Fig.4.9a).

An upper bound of 0.05 on the rate of external financial resources (Fig.4.10a) is at the origin of a transitory phase of unemployment (Fig.4.10f) which can have drastic consequences on the viability of the system if the constraint becomes too strong, i.e. for lower values of β.

4.5 Conclusion

Although a model is always constructed in the aim of simplifying a given reality, a macroeconomic model results to be a complex system (because of the very complex reality that it tries to describe) which is not very easy to handle. Therefore the use of numerical simulations becomes very useful in order to analyze and try to understand some general behaviours of macro-economy. But, because of the complexity of the systems and of the number of parameters even the analysis of a simple shock, like a technological change can become puzzling.

Therefore we suggest a careful preliminary study of the steady states, i.e. the comparison of different equilibrium paths in order to catch some basic properties of the system that we want to analyze. We have shown here an example for a technological change.

We have then proposed a method of analysis which is at midway between the comparison of steady states and the direct analysis of numerical simulations. It implies the definition of a social optimum and a certain knowledge of the model by agents, which are supposed to know the final state, i.e. to be able to make rational expectations. We have analyzed again a technological change showing the effects on economy of a policy of regulation of the variation of the external financial resources and of the take out. We have shown that the transition regime is easier to handle and to understand than with the direct method of analysis and that it is possible to have a deep insight of the properties of the model.

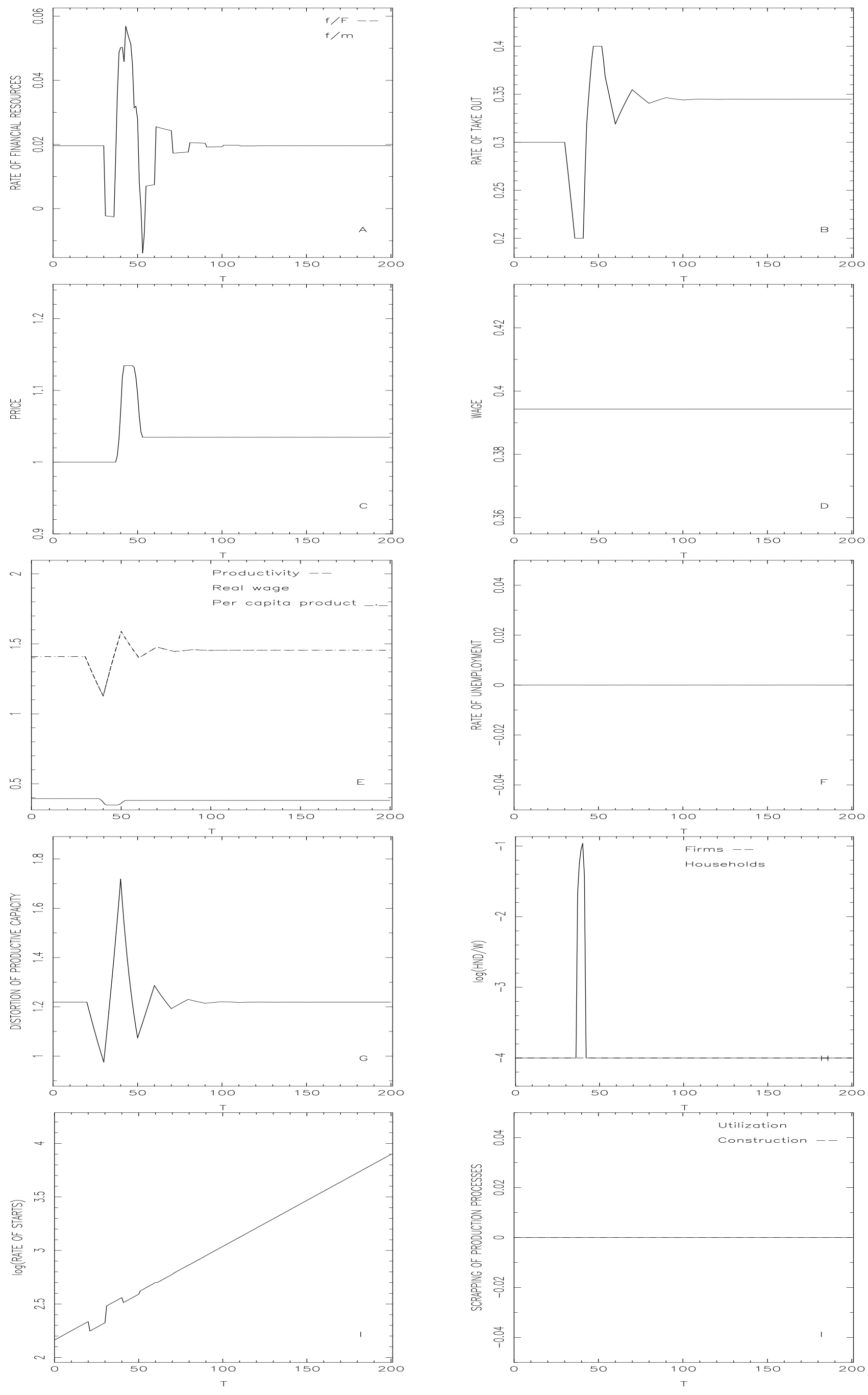

Figure 4.9: Same as Fig. 4.8 but the variation of the rate of take out α is constrained in the interval: $0.2 \leq \alpha \leq 0.4$.

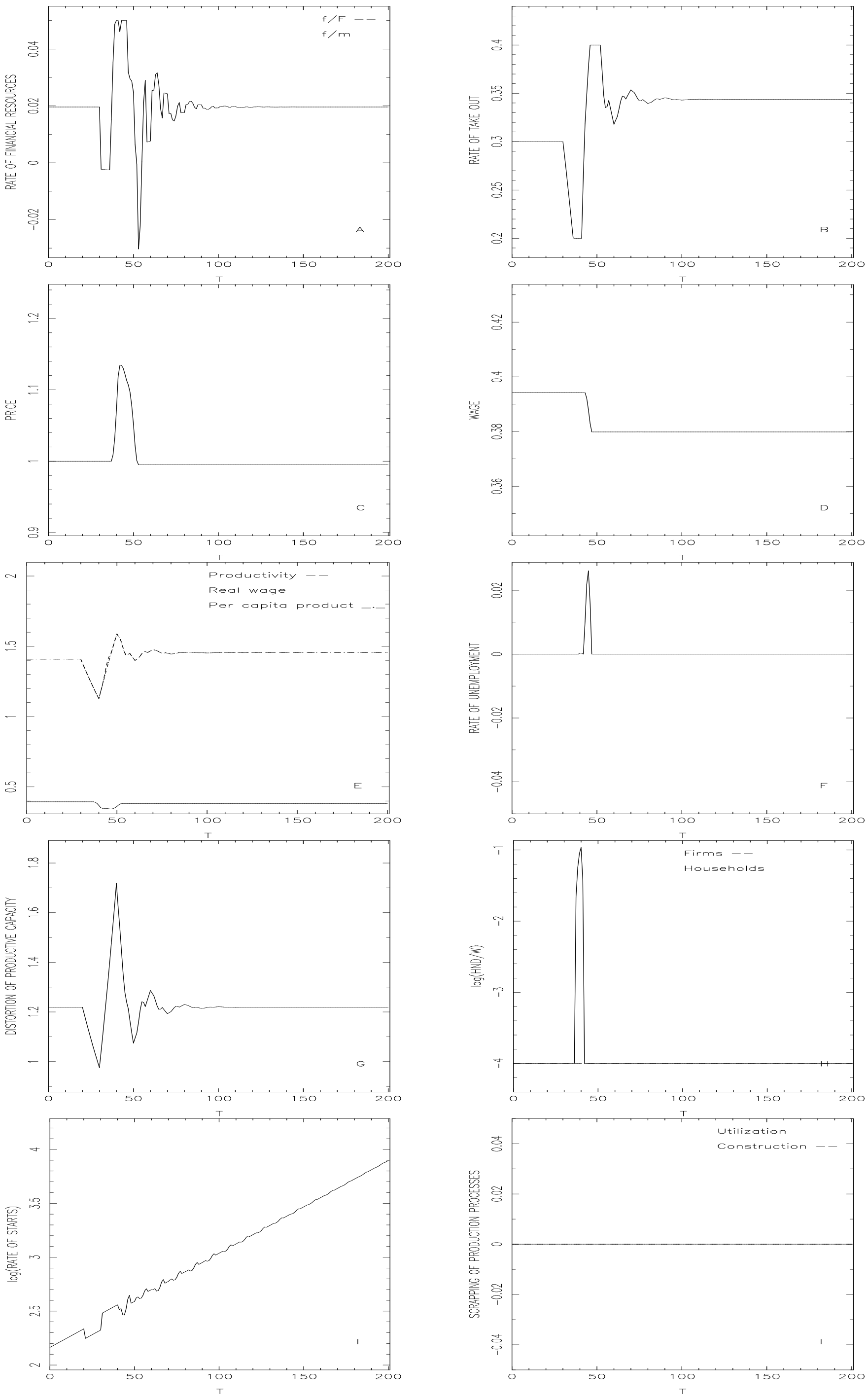

Figure 4.10: Same as Fig. 4.8 but the variation of the rate of take out α is constrained in the interval: $0.2 \leq \alpha \leq 0.4$ and the variation of the rate of external financial resources β can not be greater than: 0.06.

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Chapitre 5

Innovation, paradoxe de la productivité et chômage

Cet article a pour but de donner une explication possible du paradoxe de la productivité c'est à dire de la chute de gains de productivité et de la hausse du taux de chomage en dépit d'avancées technologiques substantielles. Nous montrons que ce qui arrive à la productivité et à l'emploi est le produit d'une séquence de déséquilibres qui traduisent les défaults de coordination des agents économiques. Pour ce faire différents niveaux de connaissance du modèle sont accordés aux agents. Un niveau plus elevé de la productivité et des salaires réels est obtenu, moyennant une inflation modérée, grace à l'augmentation de l'offre de monnaie.

INNOVATION, PARADOXE DE LA PRODUCTIVITE ET CHÔMAGE:
LES ENSEIGNEMENTS DE L'APPROCHE NEO-AUTRICHIENNE [1]

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5.1 Introduction

La chute des gains de productivité dans la plupart des éco nomies occidentales et singulièrement en Europe continentale, depuis le début des années 70, en dépit d'avancées technologiques substantielles, constitue encore un puzzle non résolu. Différentes analyses ont, certes, été proposées: les unes se référent à l'épuisement des potentialités de rattrapage des économies européennes vis à vis de l'économie américaine, d'autres à un déplacement de la composition sectorielle de l'économeie vers des activités à faibles gains de productivité, d'autres encore à l'insuffisance des investissements en R&D, d'autres, enfin, au délai requis pour maîtriser complétement les nouvelles technologies. Ces analyses rendent certainement compte d'une partie de la réalité. Elles ont en commun de mettre l'accent sur la spécificité des technologies considérées ou sur la vitesse intrinsèque de pénétration et d'adoption de ces technologies. Elles ont le défaut de proposer des explications qui ne relèvent pas du champ de l'économie, et d'attribuer aux nouvelles technologies ce qui relève des conditions de la gestion économique des processus de changement. Elles se heurtent au fait que des pays, qui font face aux mêmes bouleversements technologiques, connaissent des évolutions très contrastées. Pour avoir une claire compréhension des phénomènes en question en restant sur le terrain de l'analyse économique, plutôt que de mettre l'accent sur la technologie elle même, il faut souligner l'importance, dans un processus d'évolution hors de l'équilibre initié par le changement technologique, des problèmes de complémentarité intertemporelle de la production et de coordination intertemporelle de l'activité économique.

Dans cette perspective, le paradoxe de la productivité traduit le divorce entre la productivité d'une technologie donnée - telle qu'elle peut seulement être mesurée dans un état d'équilibre de l'économie dont la capacité productive est pleinement adaptée à cette technologie - et la productivité effective de l'économie déterminée par ce qui se produit durant l'évolution hors de l'équilibre stimulée par le changement de technologie. La dynamique de la productivité et

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de l'emploi est l'expression d'un processus de transformation de la structure productive qui met en jeu le fonctionnement de l'ensemble de l'économie au cours du temps. Son analyse implique de considérer les phases successives du processus de production qui consiste dans la construction puis dans l'utilisation d'une nouvelle capacité de production. Elle implique aussi de considérer les difficultés de coordination associés à ce processus.Le propos de cet article est de formuler une explication essentiellement économique de ce qu'il est convenu d'appeler le paradoxe de la productivité et de la hausse du taux de chômage qui l'accompagne, précisément en mettant l'accent sur les problèmes de coordination associés à l'existence de phénomènes de complémentarité intertemporelle de la production. Dans cette perspective, un cadre analytique spécifique sera élaboré (section 5.2) et un modèle sera construit (section 5.3). Une première série de simulations effectuées avec ce modèle permettra d'identifier les sources du paradoxe de la productivité et de l'effet-machine de Ricardo (section 5.4). D'autres simulations permettront de retracer différents scénarios d'évolution associés à des mécanismes distincts de coordination, et par là même d'éclairer certains aspects des évolutions observées en Europe et ailleurs, notamment dans les années 90 (section 5.5). Ce faisant, il sera possible d'énoncer les conditions dans lesquelles, suite à un changement de technologie, la productivité de l'économie rejoint celle de la nouvelle technologie et le plein emploi est rétabli, autrement dit les conditions dans lesquelles un régime quasi-régulier est finalement atteint.

5.2 Le cadre analytique

Les théories modernes de la croissance ou des cycles sont conues suivant une problématique de l'équilibre. Elles suggèrent que les évolutions de l'emploi et de la productivité procèdent simplement des données fondamentales (préférences, technologie, institutions) et de leur évolution déterminée ou stochastique. Ainsi l'emploi et le chômage sont-ils le reflet des conditions qui gouvernent le marché du travail et, plus particulièrement, des arbitrages entre revenu et loisir des offreurs de travail (Kydland and Prescott 1982), de la stratégie des firmes pour connaître le niveau réel de productivité des travailleurs, du processus d'appariement entre offre et demande de travail (Aghion and Howitt 1994), en bref et généralement, de données relatives aux préférences, à la technologie des échanges, aux institutions, et à des structures d'information. Ainsi l'évolution de la productivité peut elle être décomposée entre sa tendance strictement exogène et attribuable à des changements technologiques (y compris des chocs stochastiques) et des variations qui peuvent être qualifiées de conjoncturelles et qui relèvent de la variabilité relative du produit et de l'emploi, elles mêmes expliquées par des rigidités liées aux institutions ou à la nature des comportements stratégiques. Suivant cette perspective, le ralentissement des gains de productivité et la montée du chômage en Europe, et singulièrement, en Europe continentale seraient le résultat de chocs technologiques spécifiques modifiant la nature même du progrès technique, dans un environnement institutionnel inchangé. Seul des changements institutionnels seraient, alors, susceptibles de rétablir le plein emploi. La proposition est ainsi faite de la mise en oeuvre d'un nouveau "contrat social" consistant dans "the possibility of firms and their workers making credible commitments to a high investment/ wage moderation equilibrium" (Crafts and Toniolo 1996 p. 31). Ce type d'analyse est inhérent à une approche de type croissance équilibrée. L'analyse de la croissance hors de l'équilibre (Amendola and Gaffard 1988, 1998) s'inscrit dans une autre perspective. L'emploi et la productivité y apparaissent, nécessairement, comme des variables essentiellement endogènes. Leur évolution traduit des défauts de coordination de l'activité économique, qui, certes, peuvent tenir à des mécanismes institutionnels, mais qui sont, surtout, la conséquence des effets de rétroaction et d'interdépendance entre les variables économiques dans le contexte de régimes de croissance irréguliers. Le premier offets en question, pour pouvoir, ensuite, évaluer les règles et les mécanismes institutionnels qui font que le jeu de ces effets ne met pas en cause la viabilité de l'économie. A cet égard, il sera possible de montrer que certaines des règles et des institutions qui, suivant l'analyse en termes d'équilibre, sont censées piéger l'économie dans un état sous optimal, suivant l'analyse séquentielle, en garantissent la viabilité.L'analyse de la croissance hors de l'équilibre, qui cherche à mettre en évidence les effets de la rupture des relations de complémentarité et de coordination intertemporelles, requiert une représentation de la production qui, précisément en privilégie la dimension temporelle. Une représentation néo-autrichienne du processus de production (Hicks 1970, 1973), qui le décrit comme un processus verticalement intégré où la ressource humaine est la seule ressource explicitement considérée, dans la mesure où le capital est spécifique du processus lui même, est bien adaptée à ce propos. Ce processus prend place le long d'une séquence de périodes qui est constituée d'une phase de construction suivie d'une phase d'utilisation de la capacité productive. Dans ce contexte, une structure équilibrée de la capacité productive est représentée une population cohérente de processus de différents âges, c'est à dire une population de processus reliés entre eux par une relation stable de proportionnalité qui garantit le maintien d'un régime régulier. Cette structure par âges de la capacité productive revêt, non seulement, une dimension horizontale (le nombre et la structure parâges des processus de production à chaque moment du temps), mais aussi une dimension verticale (la configuration temporelle). Elle fait que l'investissement et la consommation, l'offre et la demande de bien final sont cohérents entre eux, à chaque moment du temps et au cours du temps.

Un choc de quelque nature que ce soit, et en particulier un choc technologique, se traduit, nécessairement, par une distorsion de la structure de la capacité productive au regard de sa configuration d'équilibre. Cette rupture de la complémentarité intertemporelle de la production signifie aussi l'apparition de problèmes de coordination temporelle de l'activité. Quand la construction et l'utilisation de la capacité productive ne sont plus en cohérence l'une avec l'autre, alors il en est de même pour l'investissement et la consommation et pour l'offre et la demande de bien final. Les réactions à ces déséquilibres et les ajustements requis de la capacité productive, à moins, comme nous le verrons, que des hypothèses très restrictives soient faites, enclenche un processus d'évolution hors de l'équilibre qui est à l'arrière plan du paradoxe de la productivité et de l'effet-machine de Ricardo. L'articulation séquentielle du processus de production et la définition de la structure de la capacité productive caractéristiques, de la représentation néo-autrichienne, rendent possible de faire apparaM-ntre au niveau analytique les problèmes de complémentarité intertemporelle qui surgissent comme résultat d'une rupture de régime régulier. Pour que les problèmes de coordination apparaissent également au niveau analytique, il faut compter avec l'articulation séquentielle, non seulement du processus de production, mais aussi du processus de décision. Ceci est obtenu en retenant la méthode des prix fixes qui consiste à supposer (en conformité d'ailleurs avec l'observation du fonctionnement des marchés des biens industriels) que les décisions de prix et de production sont prises en temps discret, de telle sorte que prix et flux de produit changent seulement lors du passage d'une période à la suivante en réaction aux signaux du marché. C'est le délai de réponse qui fait que les déséquilibres viennent à la surface plutôt que d'être instantanément résorbés, comme ce serait le cas si l'on supposait une réponse instantanée.L'économie est, naturellement, une économie monétaire, c'est à dire une économie dans laquelle la monnaie n'est pas neutre, ni dans le court terme, ni dans le long terme quand celui-ci ne se confond pas avec un régime régulier. La rupture de complémentarité intertemporelle de la production, qui est au coeur des processus de changement hors de l'équilibre, implique, en effet, que inputs et output, coûts et recettes sont désormais dissociés dans le temps. Pour que l'économie reste viable, un pont doit être lancé dans le temps qui relie inputs et output, coûts et recettes. Ce pont est, en quelque sorte, fourni par la monnaie, non pas une monnaie simple contrepartie de l'activité productive, parce que, hors de l'équilbre, il y a des moments où cette contrepartie n'existe pas, mais une monnaie qui a son existence propre, monnaie centrale ou monnaie de crédit qui constitue une variable de contrôle dans le modèle à construire.

5.3 Le modèle néo-autrichien

Un modèle séquentiel avec les caractéristiques qui viennent d'êt re mentionnées est présenté dans Amendola et Gaffard (1998). La structure générale est, seule, reprise ci-après.

Le système est décrit par des variables d'état et des variables de contrôle.

Les variables d'état sont les suivantes:

- $x(t)$, le vecteur des processus de production,

- $m(t)$, le montant des ventes de produit final,
- $h^f(t)$, les encaisses monétaires oisives des firmes,
- $h^n(t)$, les encaisses monétaires oisives des ménages,
- $o(t)$, le stock de produit final,
- $\omega(t)$, le fonds des salaires,
- $y(t)$, la valeur monétaire de la demande finale.

Les variables de contrôle sont les suivantes:

- $u(t)$ le vecteur dont les éléments représentent les décisions d'investissement qui procèdent des taux de natalité, des taux de déclassements des processus de production et du taux d'utilisation de la capacité productive,
- $p(t)$, le prix du produit final,
- $w_h(t)$, le taux de salaire par qualifications,
- $h_d^f(t)$, les encaisses désirées par les firmes (qui peuvent aussi faire figure de désendettement),
- $f(t)$, le flux des ressources financières externes (monnaie centrale ou monnaie de crédit),
- $c(t)$, le montant du prélèvement (" take out"), c'est à dire la part des ressources financières qui ne sont pas dédiées à la mise en oeuvre des processus de production et qui constitue les dépenses improductives au sens des classiques,
- $\eta(t)$, la fraction des stocks de produit final remise sur le marché.

Elles sont déterminées, dans les simulations, de manière exogène (variables de contrôle en boucle ouverte) ou suivant des mécanismes de rétroaction (variables de contrôle en boucle fermée).

5.3.1 Le processus de production

La production est réalisée au moyen de processus de type néo-autrichien. Un processus élémentaire est défini par le vecteur d'inputs de travail:

$$a_j = [a_{jk}]; \quad k = 1, ..., N \tag{5.1}$$

dont les éléments représentent les quantités de travail requises dans les périodes successives de la phase de c (de 1 à n^c) et suivant celle-ci, de la phase d'utilisation u (de $n^c + 1$ à N) de la capacité productive de la marchandise j, de telle sorte que:

$$a_j = [a_j^c, a_j^u]; \tag{5.2}$$

avec $a_{jk} = a^c$, $\forall k = 1, ...n^c$ et $a_{jk} = a^u$, $\forall k = n^c + 1, ..., N$ et par le vecteur d'output:

$$b_j = [b_{jk}]; \;\; avec \quad b_{jk} = b \quad \forall k = n^c + 1, ..., N \tag{5.3}$$

A chaque moment donné t la capacité productive de la marchandise j est représentée par le vecteur:

$$x_j(t) = [x_j^c(t), x_j^u(t)]; \tag{5.4}$$

dont chaque element est un nombre de processus de production élementaires d'unâge particulier, encore en phase de construction ou déjà en phase d'utilisation. D'autre part, comme conséquence du vieillissement et du déclassement, le nombre effectif de processus de production de type j de differents âges mis en oeuvre au même moment t est tel que:

$$x_{jk}(t) = \tilde{x}_{jk}(t) - u_{jk}(t) \tag{5.5}$$

$$\tilde{x}_{jk}(t) = x_{jk-1}(t-1); \quad \tilde{x}_{j1}(t) = u_{j1}(t) \tag{5.6}$$

où $u_{j1}(t)$ est le taux de natalité des nouveaux processus de type j.

Le déclassement des processus de production se produit quand les contraintes de ressources financières sont si fortes qu'elles ne permettent pas que puissent être mis en oeuvre tous les processus de production hérités du passé. Une alternative au déclassement est une utilisation partielle des processus en phase d'utilisation, ce qui, cependant, implique un coût.

5.3.2 Les contraintes

Dans chaque période, le niveau d'activité de l'économie est contraint par les ressources financières disponibles ou, alternativement, par les ressources humaines disponibles. Alors, le fonds des salaires est tel que:

$$\omega(t) = \min[\tilde{\omega}(t), \hat{\omega}(t)] \tag{5.7}$$

où $\tilde{\omega}(t)$ est le fonds de salaires contraint par les ressources financières:

$$\tilde{\omega}(t) = m(t-1) + h^f(t-1) + f(t) + c(t) - h_d^f(t) \tag{5.8}$$

et $\hat{\omega}(t)$ est le fonds des salaires contraint par les ressources humaines d isponibles.

$$\hat{\omega}(t) = w(t)\hat{L}^D(t) \tag{5.9}$$

Dans le contexte de séquence, les prix sont fixes dans chaque période donnée et peuvent seulement changer, s'il y a lieu, d'une période à la suivante. En conséquence:

$$m(t) = \sum_j \min[p_j(t)d_j(t), p_j(t)s_j(t)] \tag{5.10}$$

Des variations de stocks de bien final se substituent aux variations de prix qui ne peuvent pas prendre place à l'intérieur de la période élémentaire. Un excédent de demande se traduit par une accumulation d'encaisses non désirées par les ménages:

$$h^f_{nd}(t) = \sum_j p_j(t) \max[0, d_j(t) - s_j(t)] \tag{5.11}$$

Un excédent d'offre se traduit par une accumulation de stocks non désirés par les firmes:

$$o_j(t) = \max[0, s_j(t) - d_j(t)] \tag{5.12}$$

Le montant total de l'offre de travail à chaque période t, quand le travail est hétérogène, est donnée par le vecteur suivant:

$$L^S_s(t) = [l^S_1(t), l^S_2(t), ..., l^S_h(t)] \tag{5.13}$$

La contrainte ainsi constituée évolue au cours du te mps sous l'effet de la croissance démographique, des variations d e salaires et d'un mécanisme d'apprentissage par la pratique. La demande de biens finals est formée d'une demande de co nsommation à partir du fonds des salaires et d'une demande de con sommation à partir des profits ou d'une demande de consommation s ocialisée, auxquelles s'ajoute, le cas échéant un report de d emande matérialisé par l'existence d'encaisses non désir ées. Elle s'écrit:

$$y(t) = \omega(t) + c(t) + h^h(t-1) \tag{5.14}$$

5.3.3 Les décisions de production, de consommation et d'investi ssement

L'offre de biens est telle que:

$$s_j(t) = [y^*_j(t)/p_j(t)] \tag{5.15}$$

où $y^*_j(t)$ est la demande anticipée par les firmes. Celle-ci obéit à un principe d'anticipations adaptatives ou d'anticipations qualifiées de rationnelles. La production de bien final j est donnée par:

$$q_j(t) = s_j(t) - \eta o_j(t-1) \tag{5.16}$$

avec:

$$q_j(t) = \tau_j(t) \sum_{k=n^c+1}^{N} b_{jk} x_{jk}(t) \tag{5.17}$$

où τ_j est le taux d'utilisation de la capacité productive La demande de biens est telle que:

$$d_j(t) = [y_j(t)/p_j(t)] \tag{5.18}$$

où y_j est une fraction de la demande globale en valeur. L'investissement (i.e. le taux de natalité) est déterminé de différentes faqns alternatives. Il peut être déterminé par le montant des liquidités disponibles, ce qui n'a véritablement de sens que lorsque l'économie reste au voisinage de l'équilibre. Il peut être déterminé en relation directe avec le taux de croissance courant de l'économie, auquel cas des ajustements drastiques sont possibles qui engendrent une instabilité typiquement harrodienne. Il peut, enfin, être déterminé par référence à un taux de croissance " naturel", auquel cas il est tel que la structure temporelle de la capacité productive reste équilibrée, à moins que ne se manifestent des contraintes de ressources. Les déterminations de l'investissement autres que celle qui procède des liquidités disponibles conduisent à la formulation d'une demande globale de financement externe qui doit être confrontée à une offre institutionnellement fixée.

$$f(t) = min[f^d(t), f^s(t)] \tag{5.19}$$

5.3.4 Les processus de contrôle

Les prix sont fixes à l'intérieur de chaque période élémentaire et changent d'une période à la suivante en réaction aux déséquilibres entre la demande et l'offre observés sur le marché:

$$gp_j(t) = k_j \phi_j(t-1) \tag{5.20}$$

où ϕ_j est le taux de demande excédentaire de bien j. Les salaires sont fixés par les firmes en début de période et changent seulement d'une période à la suivante, soit en réaction aux déséquilibres perųs sur le marché, soit en application de règles conventionnelles tel les que l'indexation sur les prix ou sur la productivité:

$$gw_h(t) = \nu_h \psi_h(t-1) \tag{5.21}$$

où ψ_h est le taux de demande excédentaire de travail de qualification h. L'offre de monnaie obéit à différentes règles alt ernatives: elle peut être telle que son taux de croissance est éga l au taux de croissance du régime régulier ou, encore, au taux de croissance courant de l'économie hors de l'équilibre; elle peut aussi être fixée de telle manière à réprimer toute hausse du niveau général des prix; elle peut, enfin, être fixée de telle manière à combler l'écart temporaire entre les coûts et les recettes qui fait suite à un choc technologique. Le prélèvement obéit à différentes règles alternatives: il peut être tel que son taux de croissance est égal au taux de croissance du régime régulier ou, encore, au taux de croissance courant de l'économie hors de l'équilibre; il peut être aussi fixé en relation avec les besoins de financement des dépenses productives qui surgissent à la suite d'un choc technologique.Ce modèle décrit une économie dont la source fondamentale de croissance est une augmentation de la population. Des gains de productivité réguliers et exogènes ne sont pas ici envisagés. Des chocs technologiques sont, en revanche, susceptibles d'accroître définitivement le produit par tête à chaque période et de provoquer une augmentation temporaire du taux de croissance.

5.4 De la pleine coordination à la coordination imparfaite aux sources du paradoxe de la productivité et de l'effet-machine de Ricardo

L'économie étudiée est une économie qui subit des chocs technologiques. Ces chocs ont la propriété de permettre à l'économie de bénéficier, à l'équilibre, d'un accroissement de la productivité du travail pour un même taux de croissance. En d'autres termes, les technologies nouvelles sont, systématiquement, des technologies supérieures. Elles ont, toutefois, comme autre caractéristique d'avoir un coût réel de construction de la capacité de production correspondante systématiquement plus élevé. Chaque choc rend nécessaire des ajustements de la capacité productive qui demandent du temps . Il se traduit par l'apparition de déséquilibres du fait, d'une part, du poids de contraintes héritées du passé qui ne sont pas immédiatement relaxées, d'autre part, de défauts de coordination, notamment de coordination intertemporelle. Imaginons, dans un premier temps, qu'il n'y a pas de défaut de coordination. Les agents, en l'occurrence ici les entreprises, connaissent le modèle de fonctionnement de l'économie et font des anticipations que l'on pourrait qualifier de rationnelles. Cela signifie qu'ils savent que des distorsions de la structure temporelle de la capacité de production sont préjudiciables au bon fonctionnement de l'économie. Ils vont, donc, déterminer leur investissements et, en particulier, le taux de natalité des processus de production en se référant à un taux de croissance " naturel" de l'économie, de telle manière à maintenir la cohérence temporelle de la production. Ils vont, en outre, anticiper, à chaque période, une demande finale égale à la capacité de production en cours d'utilisation héritée de la période précédente et ne pas procéder à des déclassements de processus de production. Les chocs induits de demande sont systématiquement jugés transitoires et les déséquilibres de marché qui les signalent sont considérés comme si ils étaient le fruit d'erreurs strictement aléatoires n'impliquant pas de révision des plans. Dans ce contexte, un choc technologique, du type décrit plus haut a pour effet une augmentation temporaire du taux de chômage (l'effet-machine de Ricardo-Hicks) et, conjointement, une chute, également temporaire, de la productivité (le paradoxe de la productivité). Cependant, l'économie converge, rapidement, vers un nouvel équilibre complet ("full equilibrium"), sans chômage, ni déclassement de processus de production, caractérisé par un niveau plus élevé de la productivité et des salaires réels (Fig. 5.1).

Les effets mentionnés résultent de ce que lest pas élastique de telle sorte que le choix des firmes de maintenir l'équilibre temporel de la capacité, en dépit de l'augmentation de la quantité de travail requise par unité de capacité, se heurte, momentanément, à la contrainte de disponibilité de main d'oeuvre. La pleine coordination intertemporelle de l'activité assure, néanmoins que ces effets sont résorbés. Ce résultat est, toutefois, subordonné à l'absence de contraintes de financement: l'offre de monnaie est endogène, elle est déterminée par les besoins de financement des firmes, ou si l'on préfère, les autorités monétaires font aussi des anticipations rationnelles et fixent le montant des crédits en conséquence. Il faut, par ailleurs, noter que les effets sur l'emploi et la productivité ne sont pas, intrinsèquement, liés à la nature de la nouvelle technologie, mais à un défaut temporaire de complémentarité dû à une insuffisance de l'offre de travail. Il aurait suffi que celle-ci soit élastique, qu'il y ait une augmentat ion de la durée du travail dans le cadre d'un arbitrage revenu-loisir, pour que ces effets disparaissent.Le cas qui vient d'être considéré est le seul cas dans lequel l'analyse effectuée conduit aux mêmes résultats que l'analyse standard, et conclut à la convergence de l'économie vers un nouvel état d'équilibre complétement déterminé par l'un des fondamentaux, en l'occurrence les propriétés de la nouvelle technologie. La raison de cette similarité de résultat est que l'hypothése d'anticipations rationnelles conduit à annuler la séquentialité du processus de décision et par suite la structure séquentielle du modèle lui même.

Or, il n'y a aucune raison d'imaginer que les anticipations sont rationnelles et que la coordination est complète, pas plus qu'il n'y a de raison d'imaginer que les contraintes de ressources sont relaxées, instantanément, par le jeu de mécanismes incitatifs appropriés. Des déséquilibres peuvent être mal interprétés et donner lieu à des réactions qui traduisent des défauts de coordination et qui, en raison de l'existence de complémentarités intertemporelles, engendrent des régimes irréguliers ou, plus simplement, des équilibres sous optimaux. R. Lucas (1975) formulait une analyse de ce type quand il décrivait des fluctuations engendrées par la "fossilisation" des erreurs commises du fait de l'interprétation des chocs nominaux comme étant des chocs réels. Ainsi, il se peut que les firmes formulent des anticipations de demande adaptatives. Elles retiennent, alors, les déséquilibres de marché comme des signaux de la demande future et sont, le cas échéant, amenées à procéder à des déclassements de processus de production en cours d'utilisation. En revanche, elles maintiennent un comportement d'investissement guidé par des anticipations rationnelles. Le même type de choc technologique a pour conséquence que l'économie converge vers état, certes caractérisé par un taux de croissance constant, mais aussi, par rapport à l'ancien sentier de régime régulier, par un taux de chômage positif constant, par un niveau de productivité inférieur, par des salaires réels plus faibles, par une part des salaires dans le revenu constante, par un niveau de prix plus élevé et par une diminution de la durée de

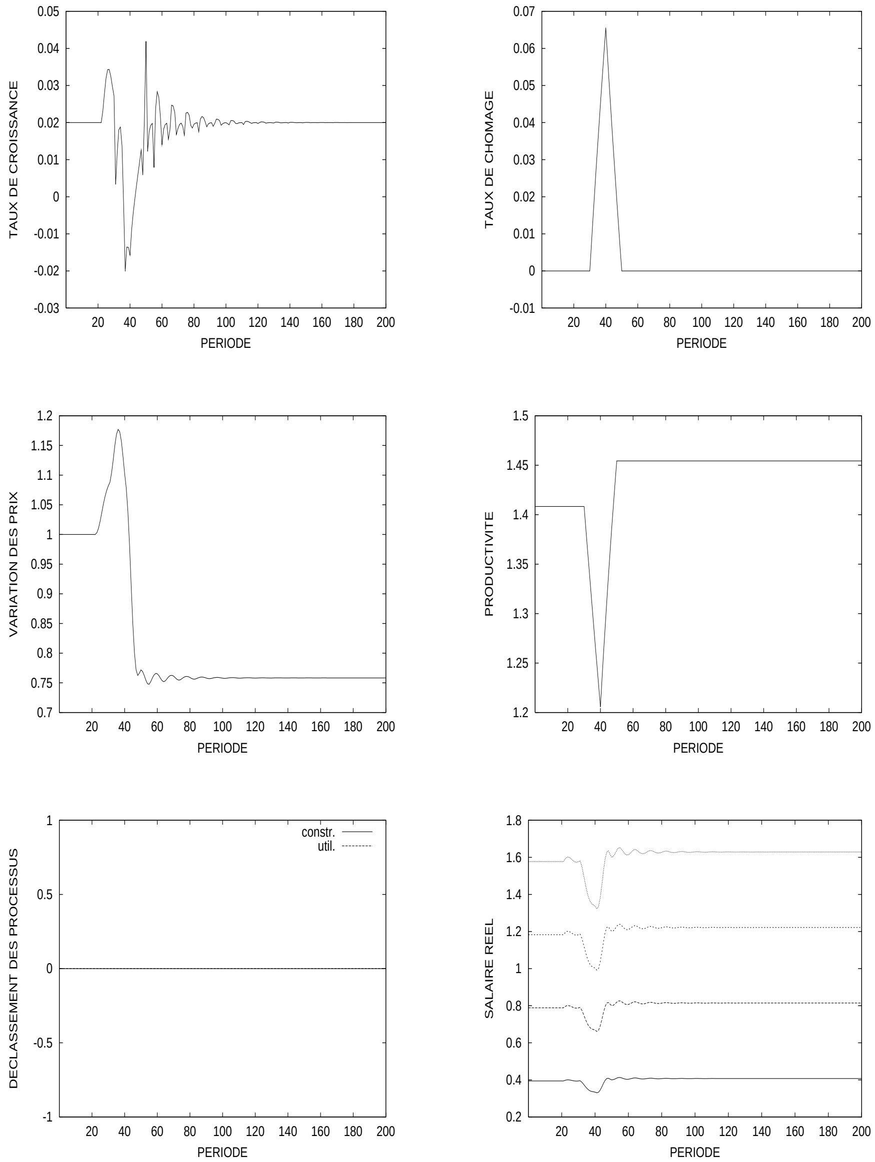
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Figure 5.1:

vie des processus de production (qui prend la forme de déclassements prématurés) (Fig. 5.2).

Cet équilibre illustre assez bien ce que peut être le paradoxe de la productivité, c'est à dire, la chute de la productivité malgré l'introduction d'une technologie plus productive. Il se produit un divorce entre la productivité de la technique, mesurable, en l'occurrence, quand cette technique équipe une économie en régime régulier, et la productivité de l'économie hors du régime régulier. Ce divorce n'a, évidemment, rien à voir avec les techniques concernées. Il s'explique par les défauts de coordination de l'activité dans un contexte où il existe des complémentarités intertemporelles de la production. Ce même équilibre révèle, aussi, que le chômage persistant, loin d'être le résultat de dysfonctionnements du seul marché du travail puisque les salaires nominaux et réels ont diminué, ou une caractéristique structurelle liée à la nature de la nouvelle technologie, est un phénomène qui renvoie aux conditions générales de coordination hors de l'équilibre. Il est à noter, d'ailleurs, que l'équilibre atteint est un équilibre keynésien standard de sous emploi dont la stabilité est dépendante de l'existence d'un plancher fixé pour les variations de salaires nominaux.

5.5 Les dynamiques “ évolutionnaires” alternatives

Les simulations précédentes constituent des cas d'école. Pour apprécier plus clairement les enjeux de la coordination intertemporelle, il faut abandonner toutes sortes d'hypothèses d'anticipations rationnelles et considérer un processus de décision pleinement séquentiel. Les défauts de coordination possibles concernent aussi bien les décisions d'investissement que les décisions de production. Ils proviennent de ce que les agents adoptent des comportements adaptatifs impliquant d'ajuster leurs décisions d'une période à l'autre sur la base de l'information de marché. Ces adaptations, qui portent sur les prix, la production et l'investissement, sont plus ou moins fortes et c'est l'un des objets de l'analyse numérique que déterminer les types d'évolution qu'elles engendrent.Reprenons le cas d'un choc technologique biaisé, c'est à dire impliquant un alourdissement des coM-ts de construction. Supposons que les prix et la production courante résultent de mécanismes essentiellement adaptatifs. Supposons, en outre, que le taux de natalité des processus de production est déterminé par les ressources disponibles sous la réserve importante que les ressources financières libérées par le déclassement prématuré de processus de production en cours d'utilisation, au lieu d'être investies, sont conservées en encaisses (ce qui équivaut, du point de vue des conséquences macroéconomiques à un désendettement des firmes). La simulation réalisée fait apparaître des fluctuations du taux de croissance, une chute de la productivité et un chômage stationnaire (Fig.5.3).

L'introduction, dans ce contexte, d'une politique monétaire restrictive, dédiée à maintenir stable le niveau général des prix, se solde par un ralentissement durable de la croissance, une

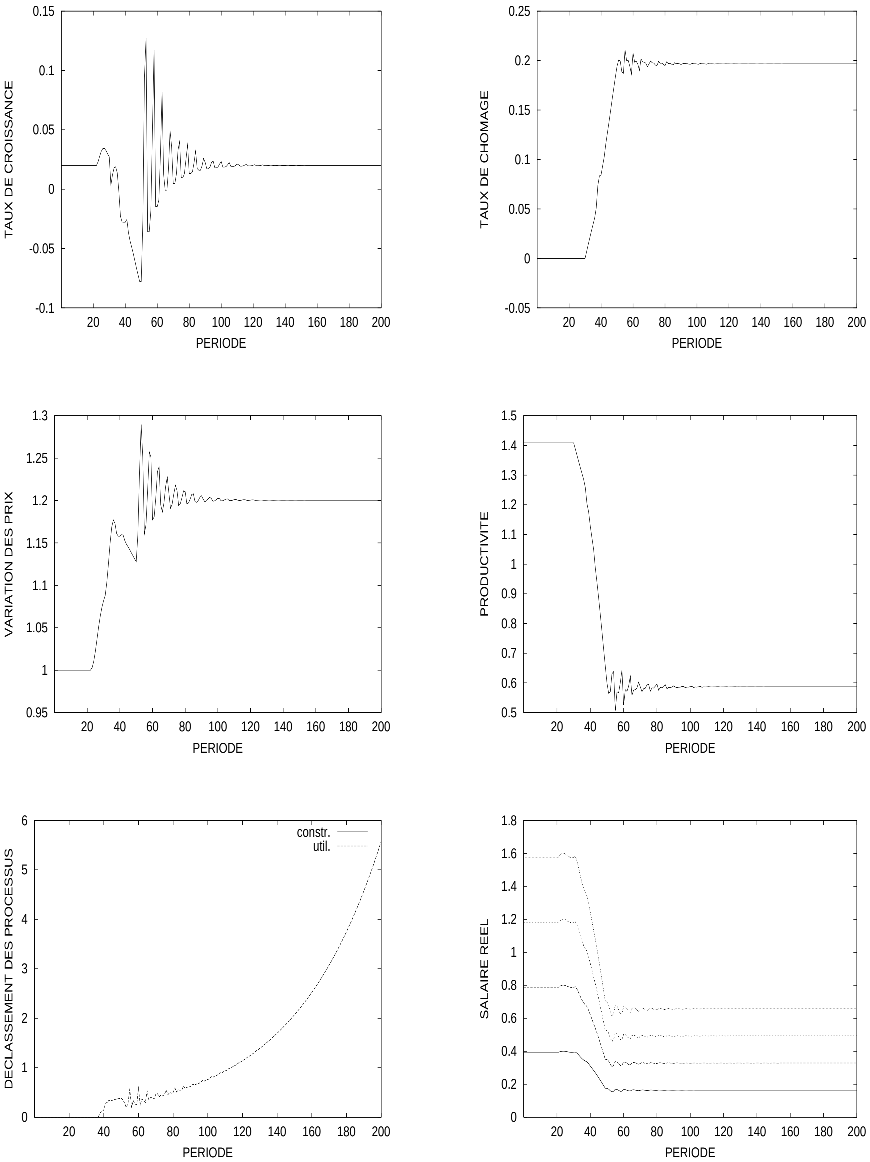
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Figure 5.2:

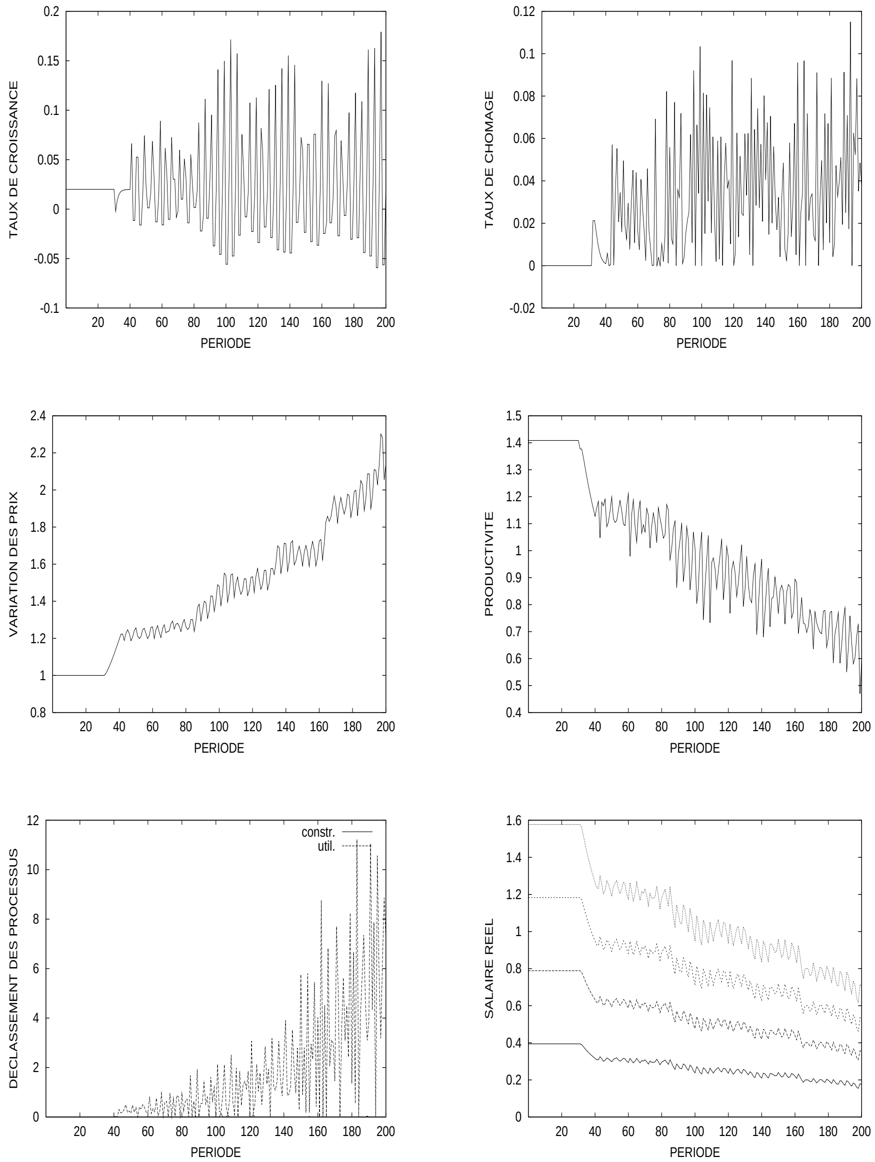
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Figure 5.3:

chute de la productivité, et une augmentation asymptotique du taux de chômage (Fig.5.4).

Ce scénario rend assez bien compte de ce qui a pu se produire en Europe continentale dans les années 90.Une baisse du taux de prélèvement pour faire face à l'accroissement des coûts de construction (i.e. une augmentation du taux d'épargne des firmes) et une expansion modérée de l'offre de monnaie se traduisent, au contraire, par la convergence vers un nouveau régime régulier caractérisé par un niveau plus élevé de la productivité et des salaires réels, moyennant une inflation modérée (Fig.5.5). Ce scénario rend assez bien compte de ce qui a pu se produire aux Etats Unis dans les années 90.

5.6 Conclusion

L'analyse conduite en faisant usage de simulations numériques révèle que le paradoxe de la productivité, tout comme le chômage qui lui est associé, est un phénomène qui dépend de problèmes de coordination de l'économie hors de l'équilibre plutôt que de facteurs structurels tels que la nature des technologies. Ce qui arrive à la technologie, aussi bien que ce qui arrive à l'emploi, comme résultat d'un choc technologique, est le produit d'une séquence de déséquilibres qui mettent en jeu les contraintes de ressources humaines et financières et qui traduisent un manque de coordination entre les mécanismes de formation des prix et des salaires, le mécanisme de formation de l'épargne et la politique monétaire.

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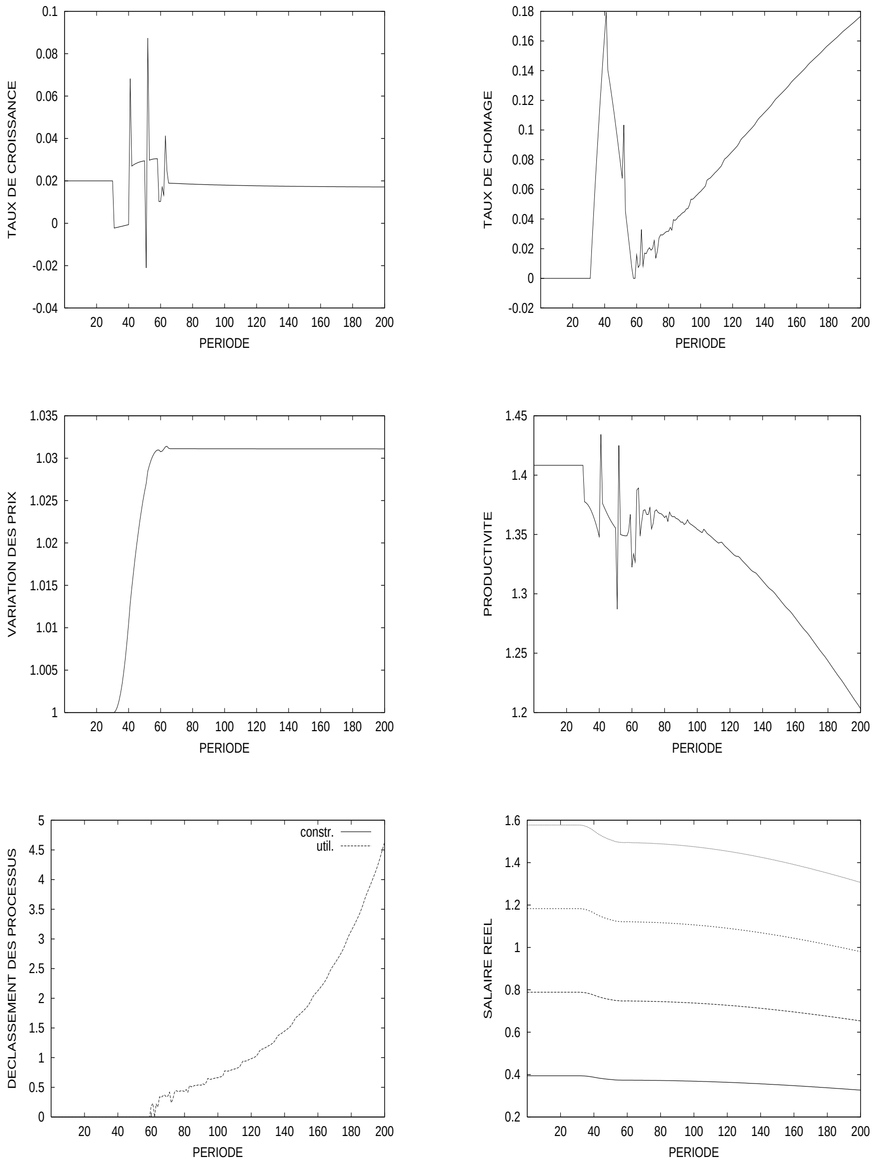
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Figure 5.4:

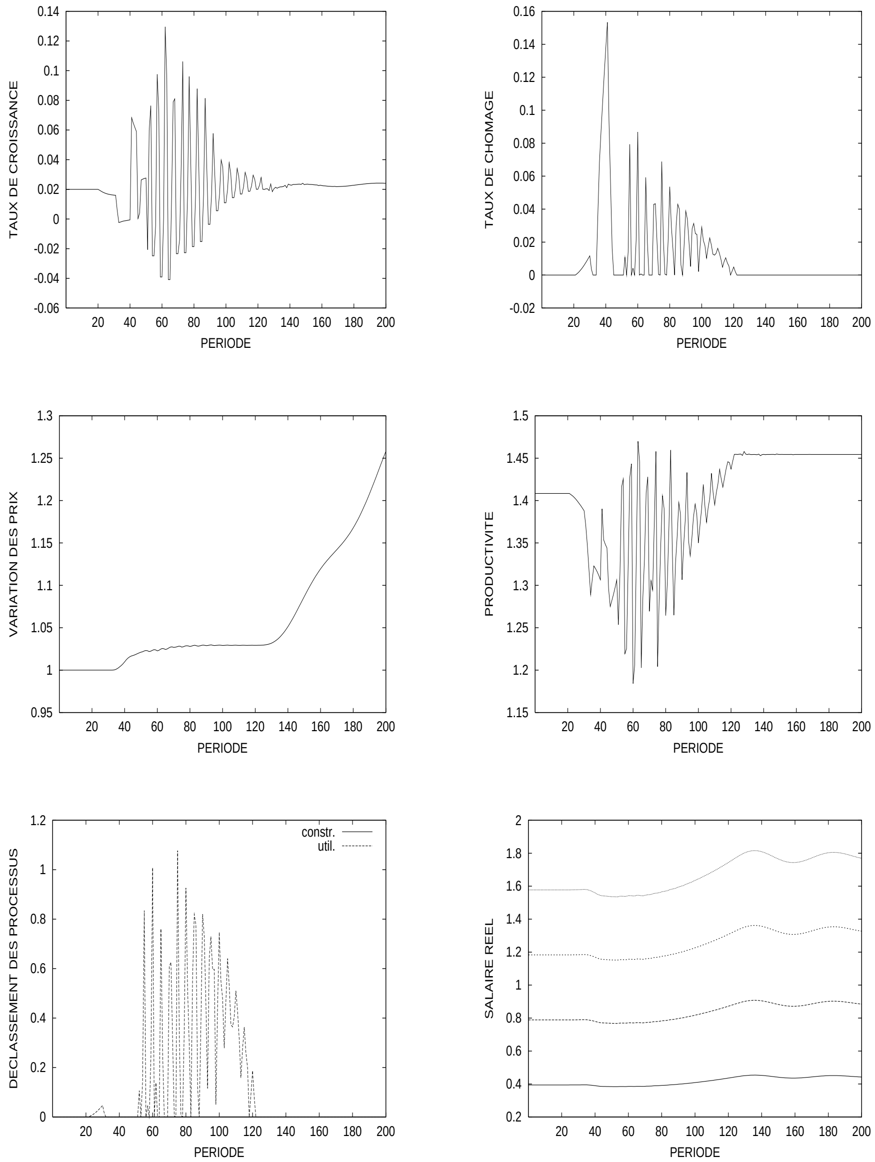
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Figure 5.5:

Chapitre 6

Un essai d'analyse globale des orbites confinées.

La problematique de l'ordre et du chaos est au coeur de ce chapitre. Etant donnée une série temporelle chaotique, il est déterminant pour le dynamicien comme pour l'économiste d'établir si elle est associée à un phénomène purement stochastique ou déterministe. Dans le cas déterministe la question est de pouvoir établir, en général par la mesure des exposants de Lyapunov, l'intensité du chaos. Or le calcul des exposant des Lyapunov, bien maitrisé si l'on connait le modèle n'est pas trivial quand on connait seulement des séries temporelles. Plusieurs méthodes ont été introduite en litterature. Dans ce papier nous avons utilisé celles qui concernent l'analyse des séries courtes (maximum 1000 données) sur un modèle connu. Afin de résoudre les problèmes existants pour l'analyse des séries régulieres et du chaos faible nous avons introduit une nouvelle méthode. Ces méthodes pourraient se révèler intéressantes pour l'analyse des séries macro-économiques (où on n'a pas plus que trente ou quarante ans de données mensuelles).

ON THE COMPUTATION OF LYAPUNOV EXPONENTS FOR DISCRETE TIME SERIES. APPLICATIONS TO TWO DIMENSIONAL SYMPLECTIC AND DISSIPATIVE MAPPINGS.

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abstract

Many techniques have been developed for the measure of the largest Lyapunov exponent of experimental short data series. The main idea, underlying the most common algorithms, is to mimic the method of computation proposed by Benettin and Galgani (1979). The aim of the present paper is to provide an explanation of the reliability of some algorithms developed for short time series. To this end, we consider two dimensional mappings as model problems and we compare the results obtained using the Benettin and Galgani method to those obtained using some algorithms for the computation of the largest Lyapunov exponent when dealing with short data series. In particular we focus our attention on conservative systems, which are not widely investigated in the literature. We show that using standard techniques the results obtained for discrete series related to area–preserving mappings are often unreliable, while dissipative systems are easier to analyze. In order to overcome the problem arising with conservative systems, we develop an alternative method, which takes advantage of the existing techniques. In particular, all algorithms provide a good approximation of the largest Lyapunov exponent in the strong chaotic symplectic case and in the dissipative one. However, the application of standard algorithms provides results which are not in agreement with the theoretical expectation

for weak chaotic motions, and sometimes also for regular orbits. On the contrary, the method that we propose in this paper seems to work properly for the weak chaotic case. Because of the speedness of computation, we suggest to use all algorithms to cross–checking the results.

6.1 Introduction

One of the most used, and widely accepted, methods to determine the degree of chaoticity of a dynamical system relies on the computation of the *Lyapunov characteristic indicators* (hereafter LCIs) whose limits, as time goes to infinity, are the Lyapunov Characteristic Exponents (hereafter LCEs). The LCIs provide a measure of the exponential divergence of nearby trajectories. A positive LCI indicates that the system has an expanding direction. A regular dynamics is characterized by a zero Lyapunov Indicator. In 1968 Oseledec provided the general theory of LCEs as well as the idea to compute them (Oseledec, 1968). The computation of the largest LCI as a test of stochasticity was already introduced in the seventies by Froeschlé (1970) and Froeschlé and Scheidecker (1973a, 1973b). A simple procedure to compute the whole set of the LCIs was given by Benettin et al. (1980) and by Contopoulos et al. (1978) (for a review see Froeschlé 1984, Hénon 1980). Their procedure relies on the knowledge of the system of differential equations governing the dynamics.

However one is often involved with discrete time series which come out from experimental situations for which the explicit dynamical system is not known. In this case the problem of recognizing determinism and of measuring the chaoticity becomes a quite difficult task. Due to the great interest related to the study of experimental series in many scientific domains, a lot of methods for recognizing determinism (Packard et al. 1980, Sugihara and May 1990, Gao and Zheng 1993, Wayland et al. 1993) and for computing Lyapunov indicators for discrete time series (Wolf et al.1985, Sano and Sawada 1985, Eckmann et al. 1986, Sato et al. 1987, Bryant et al. 1990, Wales 1991, Zheng et al. 1991, Gao and Zheng 1993, Rosenstein et al. 1993, Kantz 1994, Celletti et al. 1999a, 1999b) have been developed.

Some methods allow to compute the whole spectrum of Lyapunov exponents (Sano and Sawada 1985, Eckmann et al. 1986, Bryant et al. 1990, Zheng et al. 1991) from tangent maps obtained reconstructing the dynamics on a suitable m-dimensional space. For example, with the method proposed in (Wolf at al., 1985) one can recover two or three positive exponents by constructing a fiducial trajectory along the line of the method of Benettin et al. (1980). However, all these methods require large data sets (64000 points in a 3-dimensional space in (Eckmann et al., 1986) 5000 points in a 3-dimensional space in (Zheng et al. 1991). For shorter data sets (up to 2000 points in a 3-dimensional space) one can just determine the presence of chaos by computing only the largest Lyapunov indicator (Sato et al. 1987, Wales 1991, Rosenstein et al.

1993, Gao and Zheng 1993, Celletti et al. 1999a, 1999b).

The common idea of all but the Wales's (1991) method, which is based on a prediction approach, is to mimic the method of computation proposed by Benettin and Galgani (1979) (hereafter called BG method). The aim of this paper is to provide an explanation of the reliability of the algorithms based on the BG method. To this end we first recall the numerical technique developed by these authors, which make use of the explicit equations describing the dynamical system. The BG method depends on two parameters (the norm and the direction of the tangential vectors); we provide evidence of the robustness with respect to the above parameters. We then recall the most common methods used for short time series, underlining their similarities and differences and we apply them to analyze the behaviour of paradigm systems, like the conservative 2–dimensional standard map and the dissipative 2–dimensional Hénon mapping. We show that the above algorithms provide a good approximation of the largest LCI, especially for the conservative example in a strong chaotic regime and for the dissipative case. However some dynamics is not covered by these methods, in particular when dealing with regular curves or weak chaotic motion. To this end, we introduce a new technique and show that it provides good results for any kind of dynamics, including weak chaotic motion.

This paper is organized as follows. In Section 2 we review the standard technique for the computation of the largest LCI; in Section 3 we recall some algorithms developed for the computation of the largest Lyapunov exponent for short data series. In Section 4 we apply such algorithms to some model problems, like the standard map and the Hénon's map. An alternative method, which takes advantage from the algorithms recalled in Section 3, is proposed at the end of Section 4. In Section 5 the influence of noise is studied for all the algorithms analysed in Section 4. Conclusions and future developments of this work are provided in Section 6.

6.2 Classical numerical computation of the Largest Lyapunov Characteristic Indicator

We recall the algorithm of computation of the largest LCI in the case of a mapping (see Froeschlé 1984 for a review). We consider, as a model problem for the symplectic case, the 2-dimensional standard map (Froeschlé 1970, Chirikov 1979), which is described by the equations:

$$\left\{ \begin{array}{lcll} x_{i+1} & = & x_i + \epsilon \sin(x_i + y_i) & \mod (2\pi) \\ y_{i+1} & = & x_i + y_i & \mod (2\pi). \end{array} \right. \tag{6.1}$$

Figure 6.1 shows some typical orbits of this mapping: a circulation torus (Fig. 6.1a) a libration island (Fig. 6.1b), a weak chaotic region surrounding the hyperbolic point at the origin of the phase space (Fig. 6.1c) and a chaotic orbit obtained for a greater value of the perturbing parameter: $\epsilon = 1.5$ (Fig. 6.1d). For $\epsilon = 10$ the whole phase space is filled by points which seem to be randomly distributed (Fig. 6.1e). As a dissipative example, we consider the Hénon's mapping (Hénon, 1976), whose equations are:

$$\begin{cases} x_{i+1} &= 1 - ax_i^2 + y_i \\ y_{i+1} &= bx_i \end{cases} \tag{6.2}$$

Figure 6.1f shows the phase space portrait of this map for $a = 1.4$ and $b = 0.3$.

6.2.1 Using the divergence of two nearby orbits.

As it is well known (Benettin et al. 1980) the whole spectrum of the Lyapunov exponents is computed following the long term evolution of a set of tangential vectors. A renormalization procedure is applied at given intervals of time to avoid overflow of the lengths of the vectors in the case of a chaotic orbit. The tangential vector may be approximated by small distance vectors and the renormalization procedure is obtained as follows (Benettin and Galgani 1979, Froeschlé 1984). Consider two orbits starting at points P_0 and $P_0^{'}$ separated by a distance d. After a time $h\tau$ ($\tau = 1$ is the period of the mapping and h is an arbitrary integer) P_0 is transformed in P_1 and $P_0^{'}$ in $P_1^{'}$; we denote by d_1 the distance between P_1 and P_1'. By a homothesis of the center P_1 and of ratio d/d_1 we get two new starting points P_1 and $P_1^{''}$ separated by a distance d and we iterate the process, obtaining a sequence of points separated by distances d_1, d_2, d_3,...

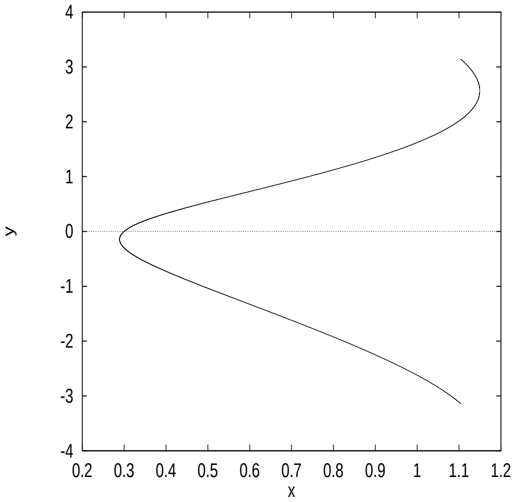
It is well known that the quantity:

$$\gamma(P_0, n, d, h) = \frac{1}{n} \sum_{i=1}^{[n/h]} \ln(\frac{d_i}{d}) \tag{6.3}$$

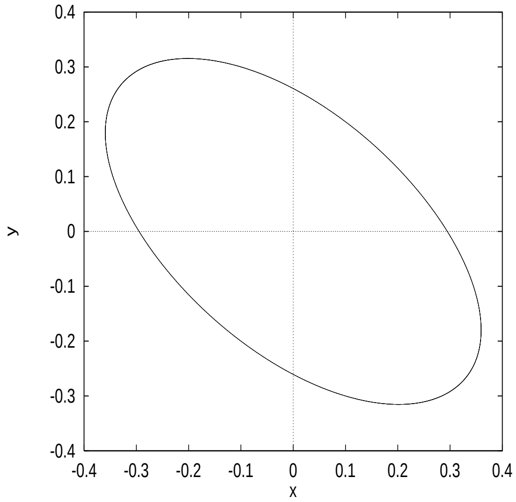
tends to a limit, which is the largest LCE as the number n tends to infinity and as the distance d tends to zero. The method developed in (Wolf et al., 1985) mimics this algorithm for the case of a series of data. For any point on the trajectory, they find its nearest neighbour and follow the evolution of both points, which are replaced as their separation goes beyond a certain threshold. The replacing elements must define a small angular separation from the evolved pair and a distance smaller then the threshold, i.e. they are required to approximate the tangent vector in norm and direction. Starting from the definition of $\gamma(P_0, n, d, h)$ (Eq.6.3) we can quantify the dependence of the computed largest Lyapunov Indicator using as "key" parameters: the norm and the direction of the approximated tangent vector.

Robustness with respect to the key parameters

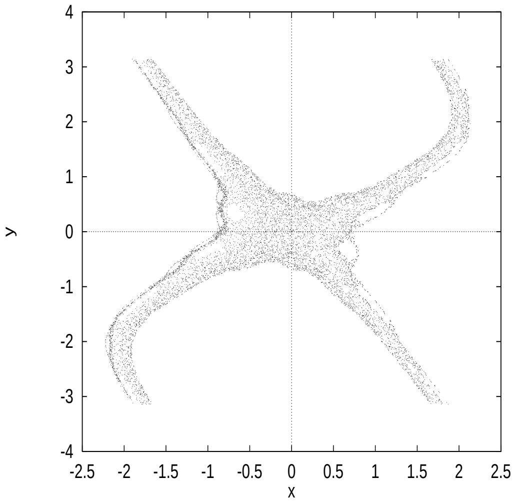
Let us slightly modify the homothesis procedure as follows: instead of considering the pair P_1 and $P_1^{''}$ as starting points, we consider the pair P_1 and $P_1^{'''}$ obtained rotating the vector formed



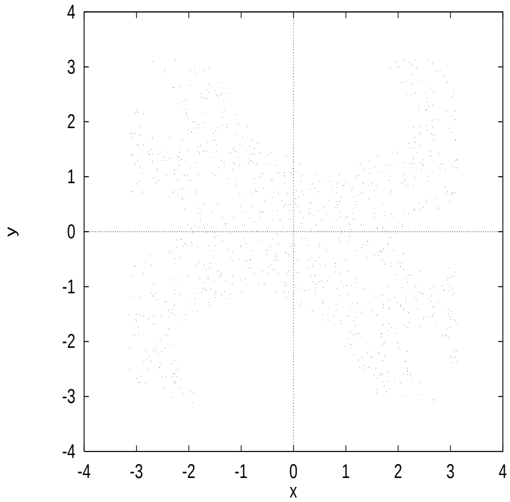
(a) Circulation torus: $\epsilon = 0.3$, $x_0 = 0.3$, $y_0 = 0$



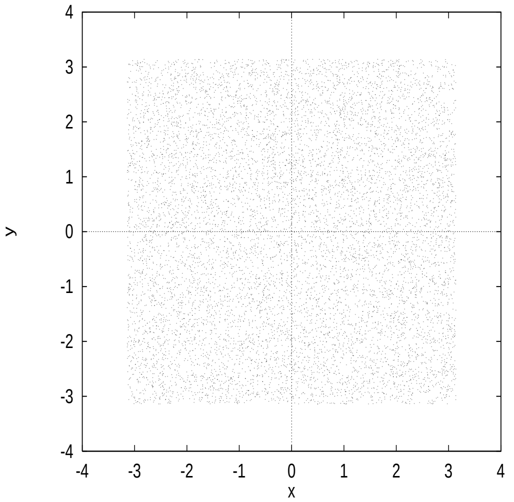
(b) Libration island: $\epsilon = -1.3$, $x_0 = 0.1$, $y_0 = 0.2$



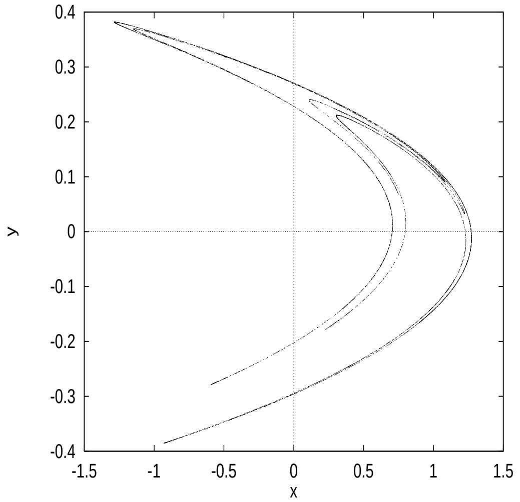
(c) Weak chaos: $\epsilon = 0.9$, $x_0 = 10^{-5}$, $y_0 = 0$



(d) Chaos: $\epsilon = 1.5$, $x_0 = 2$, $y_0 = 0$



(e) Strong chaos: $\epsilon = 10$, $x_0 = 0.1$, $y_0 = 0.1$



(f) Hénon map: $a = 1.4$, $b = 0.3$, $x_0 = 0$, $y_0 = 0$

Figure 6.1: a,b,c,d,e: orbits of the standard map obtained with 10 000 iterations, f) A typical orbit of the Hénon mapping obtained with 10 000 iterations.

by P_1 and P_1'' by an angle β_1 with β_1 randomly chosen in a suitable interval $[-\alpha, \alpha]$, for a real α. Let $d_{1,\alpha}(\beta_1)$ be the distance between P_1 and P_1'; we define

$$\tilde{\gamma}(P_0, n, d, 1, \alpha) = \frac{1}{n} \sum_{i=1}^{n} \ln\left(\frac{d_{i,\alpha}(\beta_i)}{d}\right) , \tag{6.4}$$

where we have fixed the period of normalization as $h = 1$. In the following discussion we will simply write $\tilde{\gamma}(d, \alpha)$, omitting the other arguments. For the set of orbits of Fig. 6.1 we have computed $\tilde{\gamma}(d, \alpha)$ as a function of d, for two values of α, namely $\alpha = 0$ (which corresponds to the quantity defined in Eq.6.3) and $\alpha = \pi/2$. The whole set of experiments is performed with $n = 1000$ iterations. Fig. 6.2a shows that for a circulation torus the value of $\tilde{\gamma}(d, 0)$ is constant up to $d = 10^{-4}$ and then grows monotonically. At a distance $d = 0.1$ the LCI is twenty times larger then the value obtained for $d < 10^{-4}$. This phenomenon might be explained as follows: for large values of d, the tangent is badly approximated, in particular close to the origin due to the curling of the curve; as a consequence, the effect of the differential rotation of the points P_0 and P_0' is larger than it would be along the tangent. Since the differential rotation increases the distance between orbits, we obtain a value of $\tilde{\gamma}(d, 0)$ which grows with d. This effect becomes negligible for smooth invariant curves as it is for instance in Fig. 6.2b for the libration island of Fig. 6.1b. For weak chaos, Fig. 6.2c, we observe a slight dependence on the distance as $d > 0.01$; since differential rotation is still present we have the same effect as for the circulation torus. For $\epsilon = 1.5$ Fig. 6.2d, as well as for the strong chaotic orbit, Fig. 6.2e, and for the strange attractor, Fig. 6.2f, $\tilde{\gamma}(d, 0)$ does not really depend on the value of d. For very large d, $\tilde{\gamma}(d, 0)$ decreases since the system is bounded in phase space and the separation of two orbits cannot exceed the extension of the phase space itself.

Figure 6.2 provides also the values of $\tilde{\gamma}(d, \pi/2)$, where $\alpha = \frac{\pi}{2}$ is the second (key) parameter on which our method depends. We can see that the effect of this second "key" parameter changes with respect to the dynamical character of the orbits. For the two invariant curves (Figs. 6.2a, 6.2b) $\tilde{\gamma}(d, \pi/2)$ is respectively 30 and 50 times larger than $\tilde{\gamma}(d, 0)$ even for very small values of d (say $d = 10^{-8}$). In fact, the differential rotation is larger on the approximated tangent vector than it would be on the true tangent vector. As we have described above, a more important contribution of the differential rotation means larger values of the distances d_i and therefore of their mean value, i.e. $\tilde{\gamma}(d, \pi/2) > \tilde{\gamma}(d, 0)$. This effect, although less important, is still present for the orbit of Fig. 6.2c. For the strong chaotic orbit of Fig. 6.1e, $\tilde{\gamma}(d, \pi/2)$ becomes lower than $\tilde{\gamma}(d, 0)$: $\tilde{\gamma}(d, \pi/2) = 0.85\tilde{\gamma}(d, 0)$. In this case the terms $\ln\frac{d_i}{d}$ are almost always positive, and the divergence effect decreases if the direction is not preserved. In fact, for strong chaos the direction of the tangent vector coincides with that of maximum divergence (Froeschlé et al. 1998). It is worth noticing that for the orbit of Fig. 6.1d we obtain $\tilde{\gamma}(d, \pi/2) \simeq \tilde{\gamma}(d, 0)$; it seems that there is a balance between the two opposite contributions: the differential rotation (still present) which increases the terms $\frac{d_i}{d}$ and the effect of projection on the expanding direction which reduces the terms $\frac{d_i}{d}$. For the strange attractor, $\tilde{\gamma}(d, \pi/2)$ is about 0.3 $\tilde{\gamma}(d, 0)$. Chaos occurs along the attractor and its effect is highly reduced when the direction is not preserved.

It appears clearly that the norm and the rotation of the approximated tangent vector play different roles with respect to the dynamical character of the orbit. We will explain in the next section how this two "key" parameters affect the computation of the largest Lyapunov indicator when dealing with discrete series.

6.3 Methods of computation of the largest LCI for short time series

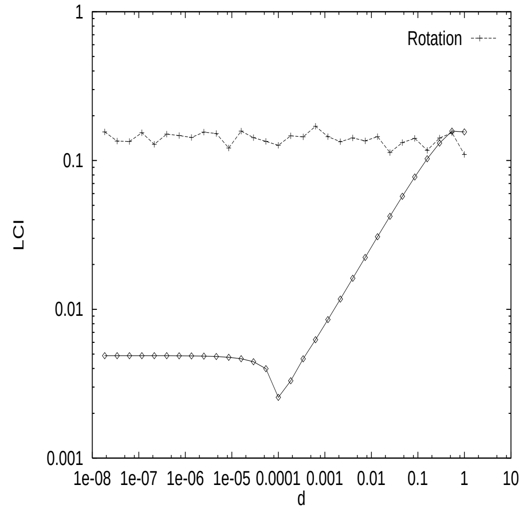
The basic idea of the methods developed in the literature (Sato et al. 1987, Rosenstein et al. 1993, Gao and Zheng 1993, Celletti et al. 1999a, 1999b) is to compute some quantities, from a single experimental time series, which mimic the terms $\frac{d_i}{d}$ of Eq.6.3 .

The standard techniques are based on three main steps. The first step is to reconstruct the dynamics. To this end, it is common to all techniques to use the method of delays (Packard et al. 1980, Takens 1981). Let $\{x_j\}$, $j = 1, ..., N$, be a discrete time series formed by N points $x_j \in \mathbf{R}$. We define a set of *delay* coordinates in an m–dimensional embedding space $\mathcal{E}_m$ as

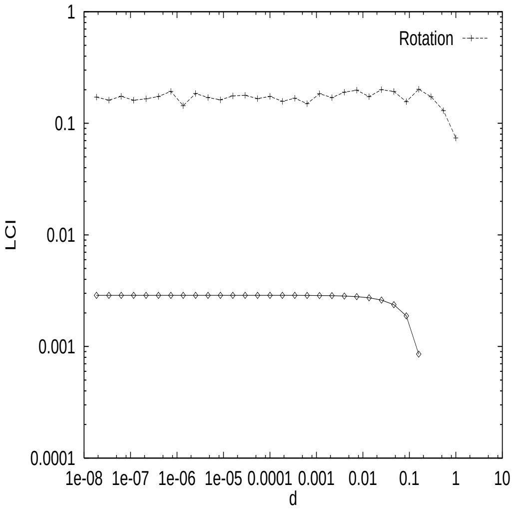
$$\begin{aligned} y_1 &\equiv (x_1, ..., x_m) \\ y_2 &\equiv (x_2, ..., x_{m+1}) \\ &\dots \\ y_M &\equiv (x_M, ..., x_N) \ , \end{aligned}$$

where $M \equiv N - m + 1$. Reconstruction of the dynamics means to determine the minimum acceptable embedding dimension m. Since we want to compute distances between neighbour points of the orbit (in order to get some approximation of the terms d_i), we have to be sure that the points are not "false nearest neighbours", i.e. they do not appear to be neighbours because the orbit is being viewed in a too small embedding space (Abarbanel, 1996). The problem of finding the optimal embedding has been largely discussed (see Gao and Zheng, 1993, and references therein). For instance, the methods of computation of the largest LCI allow at the same time to determine m. In the present paper we assume that the embedding dimension has been already determined and we set $m = 2$.

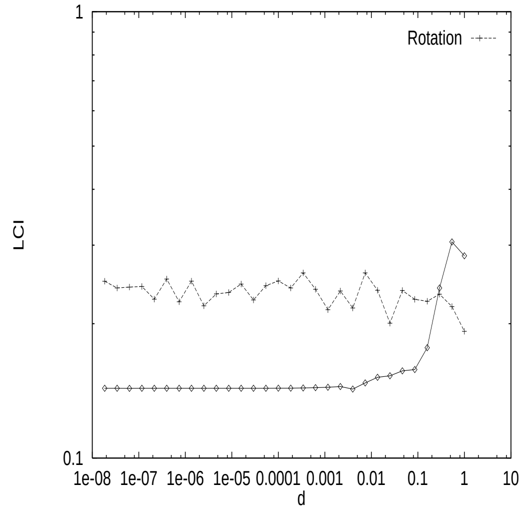
The second step is to find neighbouring points on the orbit. To this end, there are essentially two possibilities:



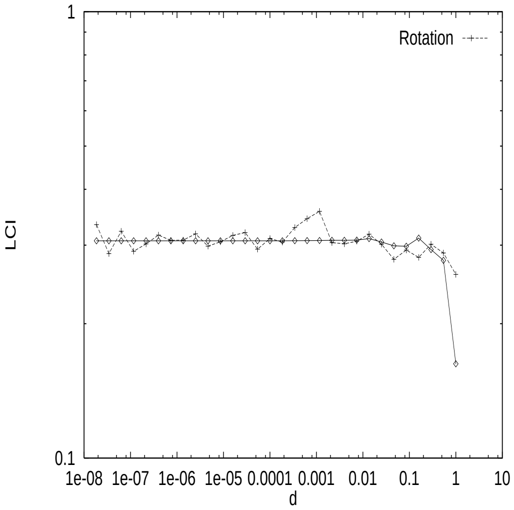
(a) Circulation torus: $\epsilon = 0.3$, $x_0 = 0.3$, $y_0 = 0$



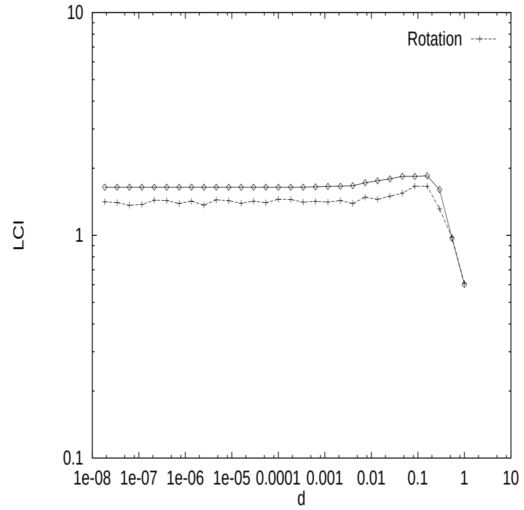
(b) Libration island: $\epsilon = -1.3$, $x_0 = 0.1$, $y_0 = 0.2$



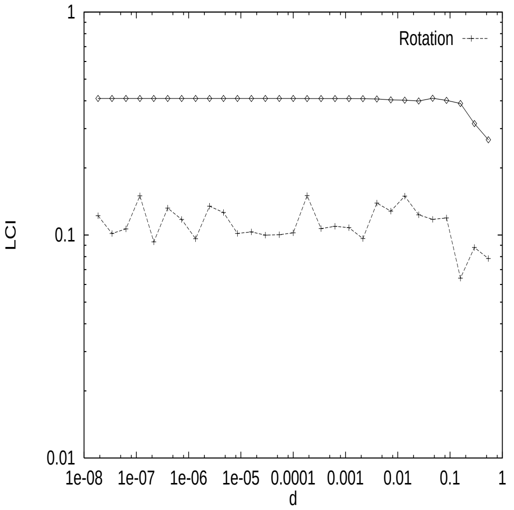
(c) Weak chaos: $\epsilon = 0.9$, $x_0 = 10^{-5}$, $y_0 = 0$



(d) Chaos: $\epsilon = 1.5$, $x_0 = 2$, $y_0 = 0$



(e) Strong chaos: $\epsilon = 10$, $x_0 = 0.1$, $y_0 = 0.1$



(f) Hénon map: $a = 1.4$, $b = 0.3$, $x_0 = 0$, $y_0 = 0$

Figure 6.2: The largest Lyapunov Indicator $\tilde{\gamma}(d, 0)$ (see Eq.4.4) as a function of the renormalization parameter d obtained for the orbits of Fig.4.1 using 1000 iterations; a,b,c,d,e: orbits of the standard map , f) Hénon mapping. The label 'Rotation' stands for the computation of $\tilde{\gamma}(d, \pi/2)$ (see Eq. 4.4).

1. Let $P_j \in \mathcal{E}_m$ with coordinates $P_j \equiv (x_j, ..., x_{j+m-1})$. For a given value $d_0 > 0$ we select all pairs (P_i, P_j) (let $n(d_0)$ be their number), with $i < j$, whose distance is less than or equal to the threshold d_0. This method (Gao and Zheng 1993, Celletti et al. 1999a, 1999b) focuses on the importance of having small distances in order to approximate tangent vectors.

2. For each point $P_j \in \mathcal{E}_m$ let locate its nearest neighbour P_i. More precisely, P_i is the point that minimizes the Euclidean distance to P_j. In this case (Sato et al. 1987, Rosenstein et al. 1993) it is privileged to visit the whole orbit, rather than to have small initial distances between points as in 1.

The last step is to estimate the largest LCI. We denote by $P_j^{(k)} \in \mathcal{E}_m$ the image of P_j after k iterations, i.e. $P_j^{(k)} \equiv (x_{j+k}, ..., x_{j+k+m-1})$. Denoting by $\|\cdot\|_m$ the Euclidean distance in $\mathcal{E}_m$, let $d_{i,j}^{(0)} \equiv \|P_i - P_j\|_m$, $d_{i,j}^{(k)} \equiv \|P_i^{(k)} - P_j^{(k)}\|_m$ and $\alpha_{ij}^{(k)} \equiv \ln \frac{d_{i,j}^{(k)}}{d_{i,j}^{(0)}}$. Following Gao an Zheng (1993) and Celletti et al. (1999a, 1999b), we call $\lambda_k(d_0)$ the average of $\alpha_{ij}^{(k)}$ over all pairs satisfying the condition $d_{i,j}^{(0)} < d_0$:

$$\lambda_k(d_0) = \frac{1}{n(d_0)} \sum_{\{i,j:d_{i,j}^{(0)} \leq d_0\}} \alpha_{ij}^{(k)} \ . \tag{6.5}$$

An estimate of the largest LCI is provided by $\lambda_1(d_0)$ or equivalently by $\lambda_k(d_0)/k$. The quantity $\lambda_k(d_0)$ depends only on the parameter d_0, i.e. the initial radius. A criterion to select an optimal value for d_0 was proposed in Celletti et al. (1999b). More precisely, d_0 was selected as the value of the initial distance such that the number of pairs within the hypersphere of radius d_0 is related to the total number N of observables by $n(d_0) \equiv \frac{N^2}{100}$. The reader can refer to Celletti et al. (1999b) for further details and for the use of the quantities $\lambda_k(d_0)$ (Eq. 7.1) to discern between deterministic and stochastic behaviour and, in the deterministic case, to determine the embedding dimension. In the following computations, we set $k = 2$ which seems to be a convenient choice of the delay for 2–dimensional mappings. Therefore, let $\Lambda(d_0) = \frac{\lambda_2(d_0)}{2}$.

Following the same procedure, we call $\hat{\lambda}_k$ the average of $\alpha_{ij}^{(k)}$ over the $(M-k)$ pairs of nearest neighbours having distances $d_{i,j}^{(0)}$:

$$\hat{\lambda}_k \ = \ \frac{1}{(M-k)} \sum_{\{i=1,...,M-k;\ j:d_{i,j}^{(0)}=\min_{\hat{j}} d_{i,\hat{j}}^{(0)}\}} \alpha_{ij}^{(k)} \ . \tag{6.6}$$

As before, an estimate of the largest Lyapunov exponent is provided by $\hat{\lambda}_1$ or equivalently by $\hat{\lambda}_k/k$. Let $\hat{\Lambda} = \frac{\hat{\lambda}_2}{2}$. Actually, the method of Sato et al. (1987) consists in extracting the largest LCI locating the plateau on the plot of $\hat{\lambda}_k/k$ as a function of k. Rosenstein et al. (1993) claim that "locating the plateau is sometimes problematic, and the resulting estimates of the largest LCI are unreliable". Therefore, they compute the largest Lyapunov exponent as the slope obtained by a least square fit to the "average" line $y_k = \frac{1}{(M-k)} \sum \ln d_{i,j}^{(k)}$ where the sum is over the pairs i, j such that $d_{i,j}^{(0)} = \min_{\hat{j}} d_{i,\hat{j}}^{(0)}$. As we will show in the next section, the computation of $\hat{\Lambda}$ provides already a good estimate of the largest LCI for almost all kind of orbits. In our opinion, the computation of $\hat{\Lambda}$ becomes very useful in case of analysis of experimental series, such that the automatic determination of the interval of time in which y_k is a linear function of k may be a non trivial task.

6.4 Applications to short time series.

Figure 6.3 shows the values of the largest Lyapunov indicator computed on the series $\{x_i,\ i = 1, 1000\}$ for the set of orbits of Fig.6.1. The curves labeled with A correspond to the computation of $\Lambda(d_0)$ (Eq.7.1) as a function of the threshold d_0, those labeled with B correspond to the computation of $\hat{\Lambda}$. Since $\hat{\Lambda}$ does not depend on any parameter, we have plotted the resulting value for d_0 equal to the mean value of the distances between nearest neighbours. The curves labeled with C correspond to the algorithm that we will introduce at the end of this section. For comparison, points labeled with D correspond to the values (already discussed in Fig.6.2) obtained with the BG method. Notice that in cases A and C, one cannot produce curves below a given d_0 due to the lack of statistics.

In order to control the effect of rotation with respect to the tangent direction, we show in Fig.6.4 the distribution of the angles formed by the tangent vectors $\vec{v}_i$ at points P_i and distance vectors $P_i\vec{P}_j$ for the pairs of points that satisfy the condition $d_{i,j}^{(0)} \leq d_0$. Tangent vectors $\vec{v}_i$ are obtained applying the tangent mapping. In order to reduce the transitory effect that occurs for a given initial vector, for example $\vec{v} = (1, 0)$, to become really tangent to the orbit, we have iterated the map 10 000 times and then we have computed the distribution of the angles on the successive $n = 1000$ iterations.

For the invariant torus we obtain a positive $\Lambda(d_0) \simeq 0.2$ (Fig. 6.3a), while the expected value is $5 \cdot 10^{-3}$ (we call "expected value" of the LCI the result provided by $\tilde{\gamma}(d, 0)$ as in Eq. 6.4 for small d). The problem is that the distribution of points along the torus is not homogeneous and the selection of pairs P_i, P_j is biased by the portion of the curve having high density. For the torus, the density of points is higher near the origin and it appears that the pairs which contract are less than those which expand. Therefore, there is no balance between negative and positive terms $d_{ij}^{(k)}/d_{ij}^{(0)}$ and their resulting mean value is positive.

The computation of $\hat{\Lambda}$ does not exhibit the same problem since each point of the orbit is considered once. Therefore, the result ($\hat{\Lambda} \simeq 4 \cdot 10^{-4}$) is in agreement with the dynamical character of the curve, and it is even lower than the expected value since the points involved in the computation belong to the orbit and therefore the differential rotation does not give any contribution as it is for the BG method. Fig.6.4a shows that, for the case of the torus, distance vectors $\vec{P_iP_j}$ such that $|\vec{P_iP_j}| \leq d_0$ have the same direction as the tangent vectors obtained applying the tangent mapping. It is not surprising, since the points of the series are on the orbit and the orbit is just a distorted line, i.e. is one-dimensional. Therefore the rotation with respect to the "true" tangent vectors does not play any role.

In the case of the island (Fig. 6.3b), for small values of d_0 ($10^{-4} < d_0 < 0.005$) the pairs of points with distances less than d_0 cover the curve quite uniformly, while for $d_0 \geq 0.005$ more pairs are selected in the expanding zones than in the contracting ones. We remark that the evolution of $\Lambda(d_0)$ as a function of d_0 depends on the covering of the curve which is related to the degree of rationality of its rotation number. Nevertheless $\Lambda(d_0)$, though becoming larger than the expected value for $d_0 \geq 0.005$ (indeed it is $\Lambda(d_0) \simeq 0.01$), still provides a good indication of regular motion. The curve is less distorted than the previously analyzed torus and therefore the non-homogeneity in the distribution of points has a minor effect on the computation of $\Lambda(d_0)$. Concerning $\hat{\Lambda}$ the result is in agreement with the dynamics of the orbit as it was for the case of the torus. Figure 6.4b shows that, as for the torus, the distance vectors are oriented as the true tangent vectors.

The cases of weak chaos ($\epsilon = 0.9$ and $\epsilon = 1.5$) are more difficult to analyze. The values $\Lambda(d_0)$ are too large in both cases (Figs.6.3c, 6.3d); the distribution of points is still non homogeneous and also in these cases the distance vectors are not in the same direction than the real tangent vectors (Fig.6.4c, 6.4d) with a resulting overestimate of the largest LCI in both cases, i.e. when $\epsilon = 0.9$ and $\epsilon = 1.5$ (as explained in Section 2, Fig.6.2c).

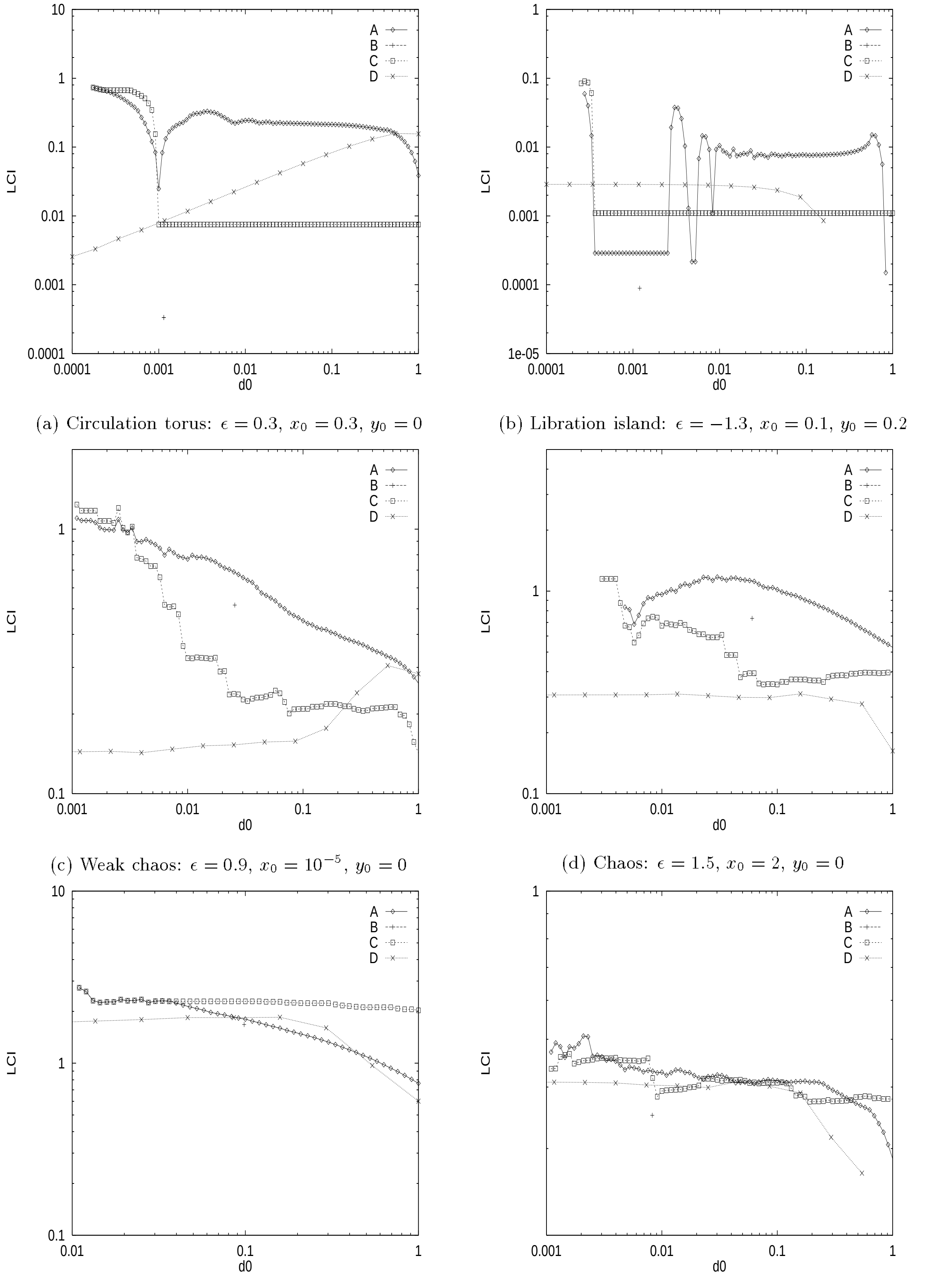
Concerning $\hat{\Lambda}$ we obtain a value larger than the expected one. As it is for $\Lambda(d_0)$, we can consider that the error in the estimate comes from the angular separation existing between distance and tangent vectors. In order to justify this statement, let us remark that using delay vectors is equivalent to change the system of coordinates. It is well known that the Lyapunov exponents are invariant under a change of coordinates, but the LCI's convergence to the Lyapunov exponents depends on the system of coordinates which is adopted for the computation. Therefore, in order to compare the values of $\hat{\Lambda}$ to those obtained with the BG method (Figs. 6.2c, 6.2d) we have repeated the experiment replacing the delay vectors (x_i, x_{i+1}) by (x_i, y_i) , i.e. considering both coordinates of the standard mapping. In this case we obtain a value close to $\tilde{\gamma}(d, \pi/2)$ confirming that $\hat{\Lambda}$ is greater than $\tilde{\gamma}(d, 0)$, due to the effect introduced by the angle between distance vectors and tangent vectors. For $\epsilon = 1.5$, we know from Fig. 6.2d that the rotation plays no effect and we have verified that $\hat{\Lambda}$ becomes close to $\tilde{\gamma}(d,0)$ when considering both coordinates of the standard mapping. Table 1 shows for each of the previous six orbits the values of $\hat{\Lambda}$ obtained using delays vectors (x_i, x_{i+1}) and using the (x_i, y_i) vectors.

Orbit	$\tilde{\gamma}(10^{-6},0)$	$\tilde{\gamma}(10^{-6},\pi/2)$	$\hat{\Lambda}$ on (x_i, x_{i+1})	$\hat{\Lambda}$ on (x_i, y_i)
Torus	$5 \cdot 10^{-3}$	0.150	$4 \cdot 10^{-4}$	$7 \cdot 10^{-3}$
Island	$3 \cdot 10^{-3}$	0.144	$9 \cdot 10^{-5}$	$8 \cdot 10^{-4}$
Weak chaos ($\epsilon = 0.9$)	0.143	0.250	0.516	0.332
Weak chaos ($\epsilon = 1.5$)	0.306	0.307	0.822	0.410
Strong chaos	1.645	1.422	1.675	1.398
Hénon	0.410	0.120	0.350	0.347

Table 1: Comparison between the expected largest LCI $\tilde{\gamma}(10^{-6},0)$ and the indicator $\hat{\Lambda}$ computed both on delays vectors (x_i, x_{i+1}) and on (x_i, y_i). In order to remark the influence of the differential rotation with respect to the true tangent vectors we have also written in Column 2 the values of $\tilde{\gamma}(10^{-6},\pi/2)$.

In the strong chaotic case of Fig.6.1e ($\epsilon = 10$), the dynamical character of the orbit is well detected using $\Lambda(d_0)$ and $\hat{\Lambda}$, but for large values of d_0, $\Lambda(d_0)$ decreases (as for $\tilde{\gamma}(d,0)$), since the system is bounded in phase space and the separation of the pairs of points cannot exceed the extension of the phase space. In particular, this is true for d_0 selected in agreement with the criterion provided in (Celletti et al., 1999b). A better agreement between $\Lambda(d_0)$ and the expected LCI is obtained taking 1000 pairs, i.e. when the majority of points P_j such that $d_{i,j}^{(0)} \leq d_0$ are in fact the nearest neighbours to the points P_i. Therefore, it may be likely to compute $\hat{\Lambda}$ which, moreover, does not depend on the choice of the parameter d_0. The orbit covers in this case the whole phase space and the distance vectors are not oriented anymore as the true tangent vectors (Fig. 6.4e). The value obtained for $\hat{\Lambda}$ is only 1% higher than $\tilde{\gamma}(d,\pi/2)$ for d corresponding to the mean value, $\bar{d}$, of the distances between nearest neighbours: $\hat{\Lambda} = 1.675$, $\tilde{\gamma}(\bar{d},\pi/2) = 1.682$. Let us remark that for $d = \bar{d}$, $\tilde{\gamma}(\bar{d},0) = 1.847$ due to a bad approximation of the tangent. It seems that the good agreement of $\hat{\Lambda}$ with the expected largest LCI ($\tilde{\gamma}(10^{-6},0) = 1.64$) is due to the balance between two opposite effects: an increase of the mean value (since the distances between nearest neighbours are such that the tangent vectors are badly approximated) and a decrease of the mean value (due to the rotation of distance vectors with respect to the true tangent vectors). For some pairs a saturation of the distances $d_{i,j}^{(k)}$ which cannot exceed the extension of the phase space contributes to the above result. This last effect is even more evident when considering the (x_i, y_i) vectors (Table 1).

For the dissipative case as shown in the literature (Rosenstein et al. 1993, Gao and Zheng



(a) Circulation torus: $\epsilon = 0.3$, $x_0 = 0.3$, $y_0 = 0$

(b) Libration island: $\epsilon = -1.3$, $x_0 = 0.1$, $y_0 = 0.2$

(c) Weak chaos: $\epsilon = 0.9$, $x_0 = 10^{-5}$, $y_0 = 0$

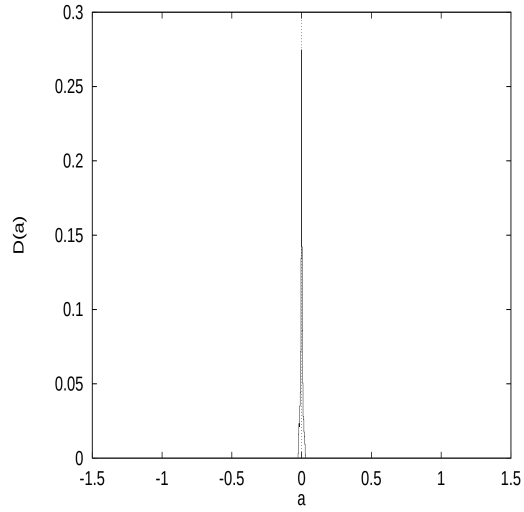
(d) Chaos: $\epsilon = 1.5$, $x_0 = 2$, $y_0 = 0$

(e) Strong chaos: $\epsilon = 10$, $x_0 = 0.1$, $y_0 = 0.1$

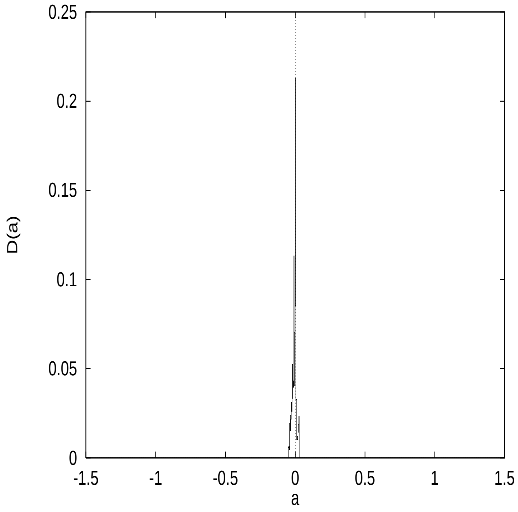
(f) Hénon map: $a = 1.4$, $b = 0.3$, $x_0 = 0$, $y_0 = 0$

Figure 6.3: The largest Lyapunov indicator obtained for the different algorithms explained in the text as a function of the parameter d_0 obtained for the orbits of Fig.6.1; a,b,c,d,e: orbits of the standard map , f) Hénon mapping.

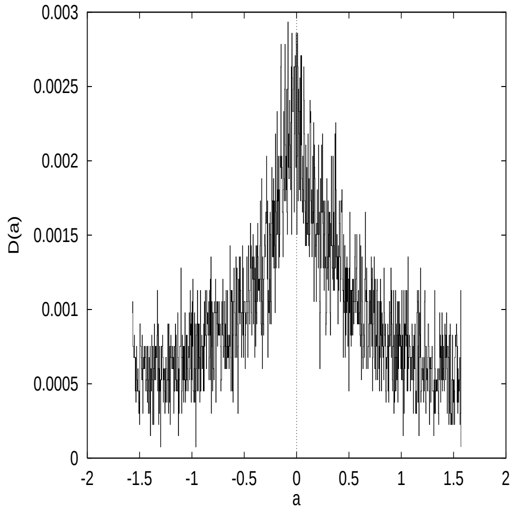
1993, Celletti et al. 1999a, 1999b), both $\Lambda(d_0)$ and $\hat{\Lambda}$ provide a good estimate of the largest LCI; concerning $\Lambda(d_0)$ the optimal choice of d_0 was suggested in (Celletti et al., 1999b). For this value d_0, we obtain $\Lambda = 0.413$, which is a very good estimate of the largest Lyapunov indicator ($\tilde{\gamma}(10^{-6}, 0) = 0.410$) and definitely better than $\hat{\Lambda} = 0.350$. In this case, there is no problem of orientation of distance vectors, since they form very small angles with the true tangent vectors (Fig. 6.4f). Concerning $\hat{\Lambda}$, the underestimate seems to be related to a problem of "saturation" of some of the terms $d_{i,j}^{(k)}$. In fact, choosing only the pairs having nearest neighbours at distance $d_{i,j}^{(0)}$ smaller than a threshold d_0, we recover the expected value. In this case the choice of small initial distances seems to be preferable than considering the whole orbit.



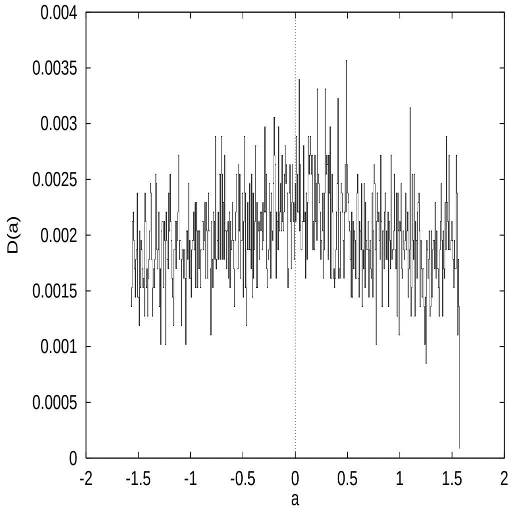
(a) Circulation torus: $\epsilon = 0.3$, $x_0 = 0.3$, $y_0 = 0$



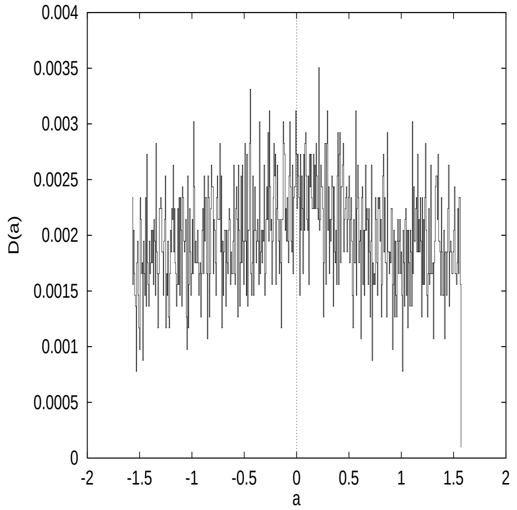
(b) Libration island: $\epsilon = -1.3$, $x_0 = 0.1$, $y_0 = 0.2$



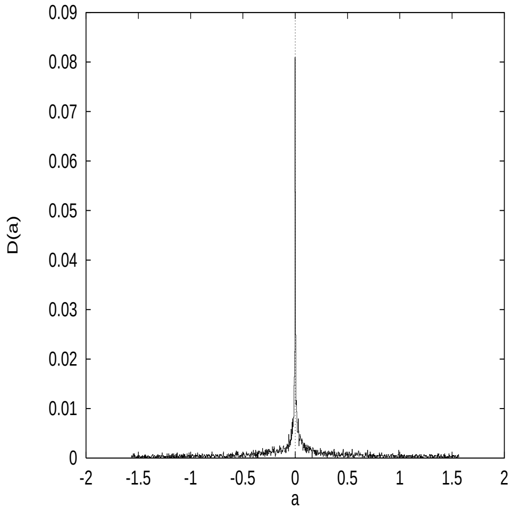
(c) Weak chaos: $\epsilon = 0.9$, $x_0 = 10^{-5}$, $y_0 = 0$



(d) Chaos: $\epsilon = 1.5$, $x_0 = 2$, $y_0 = 0$



(e) Strong chaos: $\epsilon = 10$, $x_0 = 0.1$, $y_0 = 0.1$



(f) Hénon map: $a = 1.4$, $b = 0.3$, $x_0 = 0$, $y_0 = 0$

Figure 6.4: Distribution of the angles formed by tangent vectors at points P_i and distance vectors $\vec{P_iP_j}$ for the orbits of Fig.6.1; a,b,c,d,e: orbits of the standard map , f) Hénon mapping.

6.4.1 A mixed method

From the previous discussion it is evident that the choice of neighbours is essential for the computation of the largest LCI and that both the criteria for $\Lambda(d_0)$ and for $\hat{\Lambda}$ are important, i.e. it is essential to have small distances between points, but also to consider uniformly the whole set of points. Therefore, we have implemented a sort of mixed method along the following lines: for each point $P_j \in \mathcal{E}_m$ let locate its nearest neighbour P_i. Order the distances $d_{i,j}(0)$ between nearest neighbours from the smallest to the largest and consider only the first h pairs of nearest neighbours. In what follows we have considered the $h = (M-k)/10$ first pairs, since it seems to be a good compromise between the fact of covering as much as possible the orbit and having small initial distance vectors.

We consider the images $P_i^{(k)}$ and $P_j^{(k)}$ after k iterations until the distance $d_{i,j}^{(k)}$ becomes greater than a given threshold d_0. Let $k(P_i, P_j)$ be the time necessary for $d_{i,j}^{(0)}$ to become greater than the threshold d_0. The idea is that the sequence $(P_i^{(k)}, P_j^{(k)})$ provides a good covering and that the upper threshold on the distance d_0 allows to control the approximated tangent.

Excluding the pairs whose initial distance is greater than d_0, we define as in Eqs.7.1 and 6.6

$$\alpha_{i,j}^{k(P_i,P_j)} \equiv \frac{\ln d_{i,j}^{k(P_i,P_j)}}{d_{i,j}^{(0)}}$$

and we call $\tilde{\lambda}(d_0)$ the average over the $h = (M-k)/10$ first pairs of nearest neighbours (renumbered from 1 to h) having distances $d_{i,j}^{k(P_i,P_j)} > d_0$ after $k(P_i, P_j)$ iterations:

$$\tilde{\lambda}(d_0) \quad = \quad \frac{1}{h} \sum_{\{i=1,\dots,h;\ j:d_{i,j}^{(0)}=\min_{\hat{j}} d_{i,\hat{j}}^{(0)}\}} \frac{\alpha_{ij}^{k(P_i,P_j)}}{k(P_i, P_j)} \tag{6.7}$$

The curves labeled with C in Fig.6.3 show the results obtained computing $\tilde{\lambda}(d_0)$. Let us remark that d_0 is now an upper threshold and not a lower threshold as it is for $\tilde{\gamma}$ and for $\Lambda(d_0)$. For the invariant curves of Fig.6.3a and Fig.6.3b, $\tilde{\lambda}(d_0)$ allows to identify the regular character of the orbits independently of the threshold.

In the weak chaotic samples ($\epsilon = 0.9$, Fig.6.3c, and $\epsilon = 1.5$, Fig.6.3d), $\tilde{\lambda}(d_0)$ provides a good approximation of the expected largest LCI as far as the threshold is sufficiently large in order to have a good covering of the orbit. In the strong chaotic case of Fig.6.3e, we remark that $k(i,j)$ is always equal to 1 or 2: the exponential divergence is so strong that the image distances reach immediately the threshold. Therefore it is equivalent to compute $\hat{\Lambda}$ for the first h pairs of nearest neighbours. The value so far obtained is larger than the expected one: $\tilde{\lambda}(d_0) = 2.264$ at $d_0 = \bar{d} + \sigma$, where σ is the standard deviation of the distribution of distances $d_{i,j}^{(0)}$ between nearest neighbours. This is mainly due to the fact that we are using the x-delay vectors. For comparison, we show in Table 2 the values of $\tilde{\lambda}(d_0)$ obtained using both delays vectors (x_i, x_{i+1}) and the (x_i, y_i) vectors.

Orbit	$\tilde{\gamma}(10^{-6}, 0)$	$\tilde{\gamma}(10^{-6}, \pi/2)$	$\tilde{\lambda}(\bar{d}+\sigma)$ on (x_i, x_{i+1})	$\tilde{\lambda}(\bar{d}+\sigma)$ on (x_i, y_i)
Torus	$5 \cdot 10^{-3}$	0.150	$7.50 \cdot 10^{-3}$	$6.40 \cdot 10^{-3}$
Island	$3 \cdot 10^{-3}$	0.144	$1.09 \cdot 10^{-3}$	$3.50 \cdot 10^{-3}$
Weak chaos ($\epsilon = 0.9$)	0.143	0.250	0.224	0.137
Weak chaos ($\epsilon = 1.5$)	0.306	0.307	0.350	0.270
Strong chaos	1.645	1.422	2.264	1.540
Hénon	0.410	0.120	0.416	0.437

Table 2: Comparison between the expected largest LCI $\tilde{\gamma}(10^{-6}, 0)$ and the indicator $\tilde{\lambda}(d_0)$ computed both on delays vectors (x_i, x_{i+1}) and on (x_i, y_i) for $d_0 = \bar{d} + \sigma$. In order to remark the influence of the differential rotation with respect to the true tangent vectors we have reported in Column 2 the values $\tilde{\gamma}(10^{-6}, \pi/2)$.

For the strong chaotic orbit, the value $\hat{\Lambda}$ obtained using the x-series provides a better approximation of the expected LCI; this is due to an underestimate of the image distances which are bounded by the extension of the phase space, occurring for pairs of nearest neighbours having quite large distances d_0. Taking only the first h pairs of nearest neighbours, we obtain for $\hat{\Lambda}$ the same value as $\tilde{\lambda}(d_0)$. Let us remark that the mixed method seems to work better for weak chaotic motion and for strong chaos than the preceeding methods, since it avoids the problem of "saturation" of distances in the case of a bounded phase space. Moreover, $\tilde{\lambda}(d_0)$ has the advantage of taking small initial distances and to provide a good covering of the available space.

In the case of the strange attractor (Fig.6.3f), we obtain a very good agreement between $\tilde{\lambda}(d_0)$ and $\tilde{\gamma}(d, 0)$.

6.5 Noise dependence

In view of the computation of the largest Lyapunov indicator for experimental data series it is important to study the influence of noise for the different methods of computation analysed. For each of the six orbits analysed in Section 4 we have added to the original series a noise by replacing each point $P_j \equiv (x_j, x_{j+1})$ with the point $P_j(r/L) =\equiv (x_j + \delta_j, x_{j+1}) + \delta_{j+1})$ with $(\delta_j, \delta_{j+1}$ randomly chosen inside a circle of radius r/L, L being the amplitude of the signal, i.e. the portion of the phase space visited by the x-coordinate of the orbit. We have computed the largest Lyapunov indicators for increasing values of r/L, using the alghoritms described in Section 4 and the method presented in Section 2. It appears in Fig. 6.5a shows the values of the largest LCI as a function of r/L

6.6 Conclusions

We have investigated some methods for computing the largest Lyapunov indicator for short data series. We have revisited the BG method (which is used when the explicit dynamical system is known) and we showed that the influence of the free parameters on the resulting Lyapunov indicator depends on the dynamical character of the orbit.

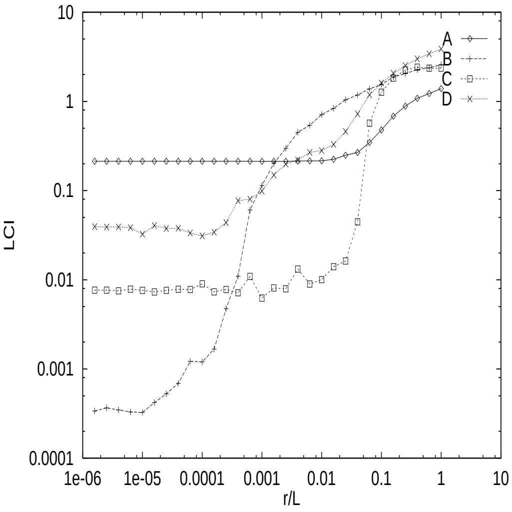
We have deeply studied the application of the methods of computation of the largest Lyapunov indicator to the case of the two dimensional standard map showing that it is not always trivial to obtain a good estimate, in particular in the cases of weak chaotic motion.

We have shown that for $\Lambda(d_0)$, it is necessary to have a homogeneous distribution of points, while $\hat{\Lambda}$ provides good results for all but the weak chaotic motion. For such orbits the distance vectors are not oriented as the true tangent vectors resulting in an overestimate of the Lyapunov indicator.

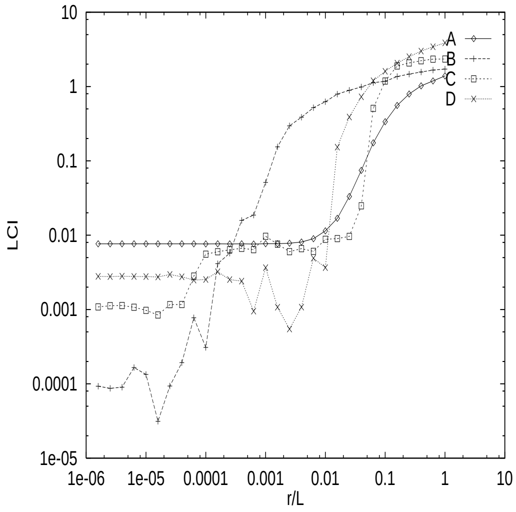
We have introduced a third method which takes advantage of having both small initial distance vectors and a uniformly covering of the orbit. This method provides a good estimate of the largest LCI for all kind of orbits. Although in the case of strong chaos, the quantity $\hat{\Lambda}$ seems to provide better results, we can explain this phenomenon as a spurious effect related to the saturation of the image distances for large initial distances.

As far as a dissipative system is concerned, i.e. the Hénon's map, the computation of the largest LCI starting from the experimental data series is an easier task, mainly because there are not complicated structures as the weak chaos may be and also because the scale invariance of an attractor allows to catch the dynamics of the orbit even with a short data series.

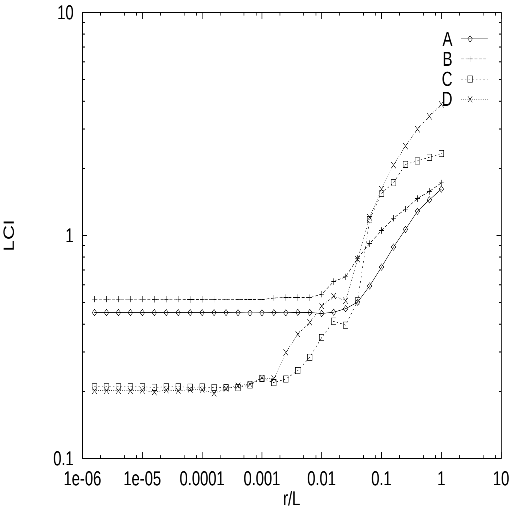
Since the implementation of the algorithms presented is very easy and since they do not require a lot of computation time, we suggest to use all of them to have a complete control of the results. We plan to perform a systematic study of more dimensional mappings in a future work.



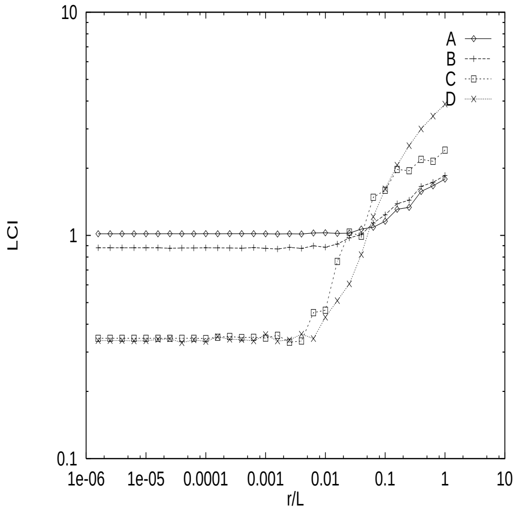
(a) Circulation torus: $\epsilon = 0.3$, $x_0 = 0.3$, $y_0 = 0$



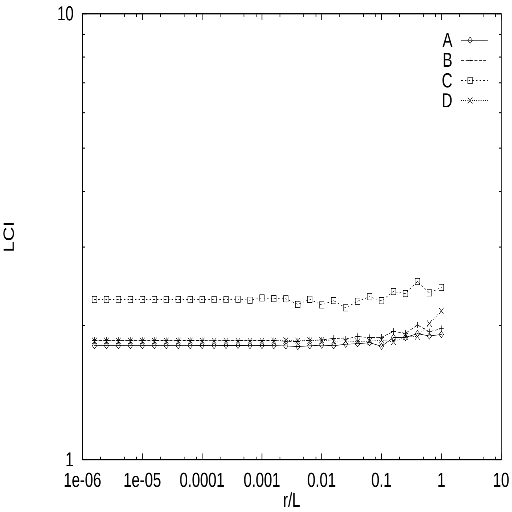
(b) Libration island: $\epsilon = -1.3$, $x_0 = 0.1$, $y_0 = 0.2$



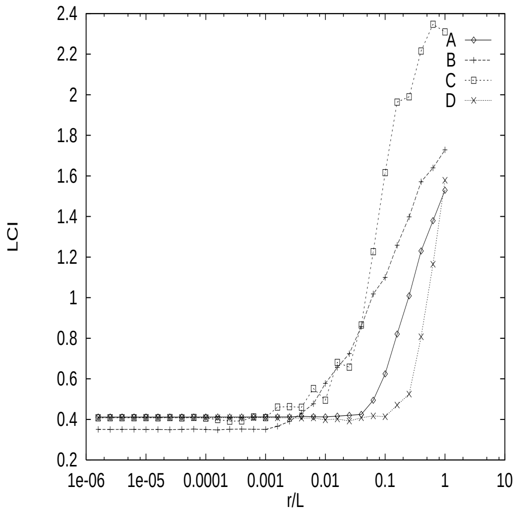
(c) Weak chaos: $\epsilon = 0.9$, $x_0 = 10^{-5}$, $y_0 = 0$



(d) Chaos: $\epsilon = 1.5$, $x_0 = 2$, $y_0 = 0$



(e) Strong chaos: $\epsilon = 10$, $x_0 = 0.1$, $y_0 = 0.1$



(f) Hénon map: $a = 1.4$, $b = 0.3$, $x_0 = 0$, $y_0 = 0$

Figure 6.5: The largest Lyapunov indicator obtained for the different algorithms explained in the text as a function of the noise intensity obtained for the orbits of Fig.6.1; a,b,c,d,e: orbits of the standard map , f) Hénon mapping.

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Chapitre 7

Etude dynamique des series issues du modèle néo-autrichien

Dans ce chapitre nous utilisons une des méthodes entroduites dans le chapitre 6 pour analyser la dynamique des series macroeconomiques. Comme dans le cas des systèmes dynamiques le point de départ est l'analyse des séries issues d'un modèle connu. Nous avons alors utilisé des séries chaotiques obtenues avec le modèle néo-autrichien. Nous montrons que nous pouvons bien établir le déterminisme du système et de façon surprenante nous trouvons que dans le modèle il existe un attracteur dont la dimension est égale au nombre de variables clés. La méthode utlisée pourrait être appliquée dans un travail futur aux series économetriques.

7.1 Chaos déterministe et variables clé.

Bien que l'analyse faite dans l'article presenté au Chapitre 6 permet de determiner de manière assez précise le plus grand exposant de Lyapunov, il est important de préciser que ce type d'étude ne peut être fait que pour des systèmes ayant un nombre petit de degrés de liberté ou bien des systèmes dans lesquels les degrés de liberté qui influencent la dynamique sont de l'ordre de 4, 5 maximum. Non seulement du point de vue dynamique il est interessant de connaître la dimension de l'espace du système étudié mais aussi d'un point de vue économique, car évidemment plus le nombre de degré de liberté est important plus il est difficile d'établir des mesures de réaction aux chocs auquel l'économie est soumise. Toutefois, nous rappellons que pour les systèmes dynamiques c'est la dimension de l'attracteur, quand cette dernière existe, que l'on détermine.

La méthode que nous allons appliquer au séries issues du modèle néo-autrichien et celle décrite dans la section 6.3 (Gao an Zheng 1993 and Celletti et al. 1999a, 1999b). Dans l'article nous n'avons pas appliqué la méthode dans le but de déterminer le nombre de degrés de liberté, but que nous avons pour l'analyse économique, et puisque l'analyse des résultats peut ne pas être immediate nous préférons commencer par l'analyse de deux cas d'école.

Nous rappellons que le plus grand exposant de Lyapunov peut se calculer en utilisant la divergence entre orbites voisines. On utilise un procedure de renormalisation pour approximer le vecteur tangent àu mouvement (Section 6.2.1). Dans le cas des séries on essaye de reproduire cette méthode. Suivant la description donnée dans la section 6.3 soit $\{x_j\}$, $j = 1, ..., N$, une série discrete du temps formé par N points $x_j \in \mathbf{R}$. Un defini un ensemble de coordonnés *retard* dans un espace $\mathcal{E}_m$ de dimension m par:

$$\begin{aligned} y_1 &\equiv (x_1, ..., x_m) \\ y_2 &\equiv (x_2, ..., x_{m+1}) \\ &\dots \\ y_M &\equiv (x_M, ..., x_N) \ , \end{aligned}$$

où $M \equiv N - m + 1$. Puisque on veut calculer la distance entre points voisins (pour mimiquer la distance entre orbites voisines) on doit être sûr que les points soient de vrais voisins, c'est à dire qu'il faut eviter d'avoir des points qui sont voisins car l'orbite est projetée dans un espace de dimension trop petite (Abarbanel 1996).

Pour detecter les points voisins on considere le point $P_j \in \mathcal{E}_m$ ayant coordonnées $P_j \equiv (x_j, ..., x_{j+m-1})$ et une valeur $d_0 > 0$ et on sélectionne tous les couples (P_i, P_j) (soit $n(d_0)$ leur nombre), avec $i > j$, dont la distance est inferieure au seuil d_0.

Soit $P_j^{(k)} \in \mathcal{E}_m$ l'image de P_j après k itérations, i.e. $P_j^{(k)} \equiv (x_{j+k}, ..., x_{j+k+m-1})$. Soit $\|\cdot\|_m$ la distance Euclidienne dans $\mathcal{E}_m$, nous posons $d_{i,j}^{(0)} \equiv \|P_i - P_j\|_m$, $d_{i,j}^{(k)} \equiv \|P_i^{(k)} - P_j^{(k)}\|_m$ et $\alpha_{ij}^{(k)} \equiv \ln \frac{d_{i,j}^{(k)}}{d_{i,j}^{(0)}}$. Une approximation du plus grand exposant de Lyapunov est alors donnée par:

$$\lambda_k(d_0)/k = \frac{1}{n(d_0)} \sum_{\{i,j:d_{i,j}^{(0)} \leq d_0\}} \alpha_{ij}^{(k)} \tag{7.1}$$

Dans la figure 7.1 nous avons dessiné l'évolution au cours du temps d'une variable chaotique stochastique $x(j)$, $j = 1, ..1000$.

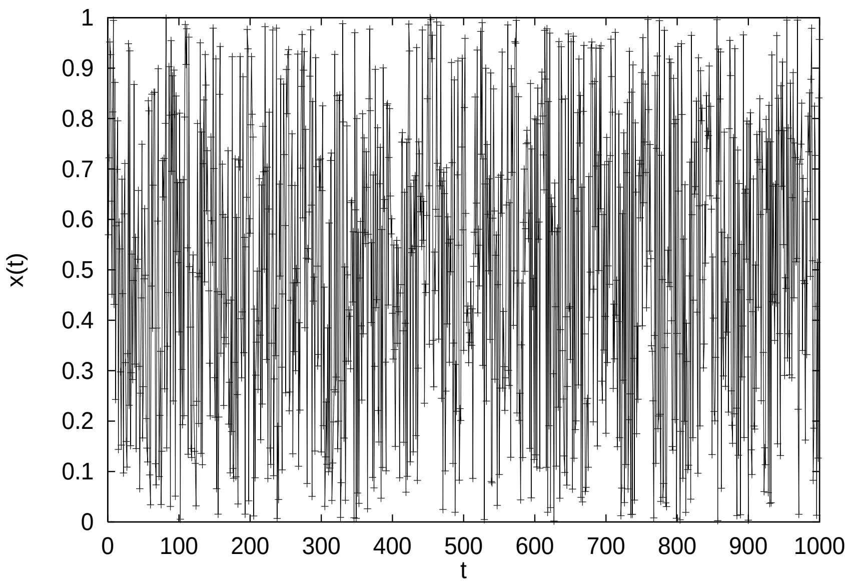

Figure 7.1: Evolution au cours du temps d'une variable chaotique stochastique.

Les quantités $\lambda_k(d_0)/k$ en function de d_0 pour $k = 1, ...6$ ainsi que pour $m = 1, ...5$ sont plottées en Fig.7.2. Dans le cas stochastique il n'existe aucune relation entre la distance $d_{i,j}^{(0)}$ entre deux points et la distance entre les images des deux points. Il est clair que les $d_{i,j}^{(0)}$ étant bornés par le seuil d_0 on obtient que l'indicateur λ décroit avec d_0, les numérateurs $d_{i,j}^{(k)}$ ayant une moyenne constante (Fig.7.2). Le nombre de degrés de liberté (ou la dimension m de l'attracteur) n'interviennent evidemment pas.

La Fig.7.3 montre l'évolution au cours du temps d'une variable chaotique déterministe du mapping standard (Eq.6.1) correspondant à une orbite de conditions initiales $x_0 = 2.$, $y_0 = 0$. Le paramètre perturbatif est $\epsilon = 1.3$.

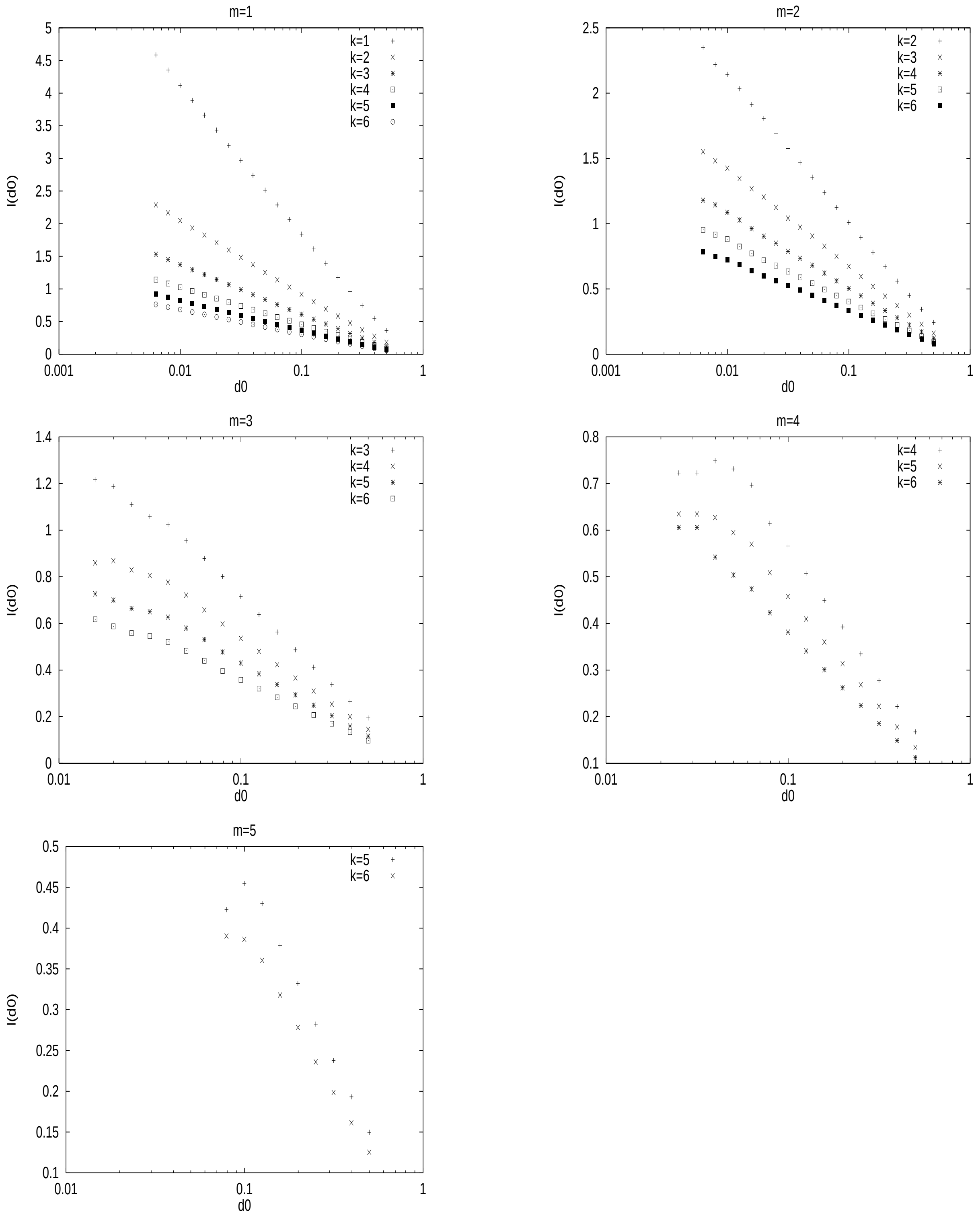

Figure 7.2: Variation de l'indicateur $\lambda_k(d_0)/k$ pour $k = 1, 2...6$ en fonction de la distance maximale d_0 entre les couples des points pour un signal stochastique.

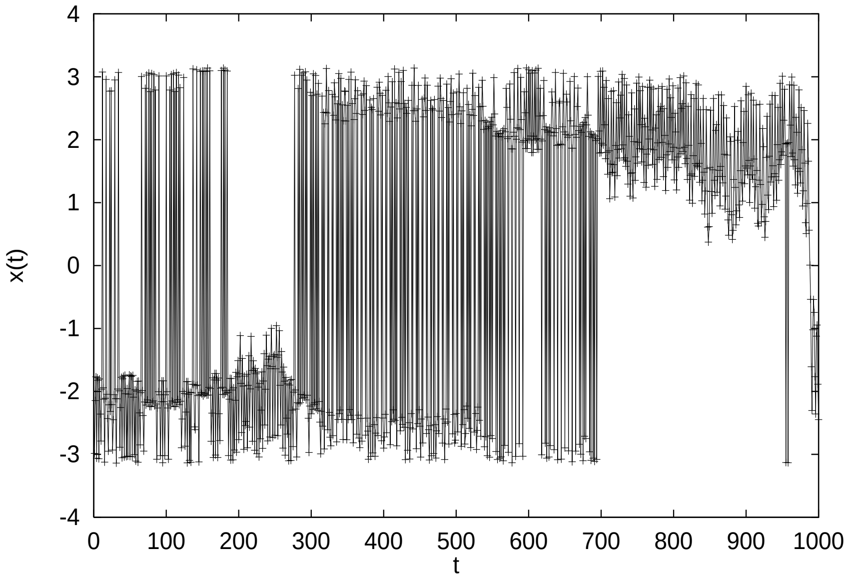

Figure 7.3: Evolution au cours du temps d'une variable chaotique déterministe

Si le signal est déterministe la quantité $\lambda_k(d_0)$ doit être constante pour une certain plage de valeurs d_0 (Fig.7.4) car le chaos déterministe est caracterisé par la divergence exponentielle des orbites. En particulier la quantité $\lambda_k(d_0)$ donne le plus grand exposant de Lyapunov et on a $\lambda_k(d_0)/k = \lambda_1(d_0)$. On observe que le comportement de $\lambda_1(d_0)$ pour $m = 1$ est similaire au cas stochastique. En effet, la dimension du système étant $m = 2$, pour $m = 1$ on detecte de couples de points qui sont voisins dans l'espace de dimension 1 par effet de projection. On appelle ces couples "faux voisins". Les images des faux voisins n'ont aucune relation avec les distances initiales entre le points et par conséquent on a le même phénomène que l'on observe pour le cas stochastique. Grâce à cet effet on détermine la dimension de l'espace ou bien le nombre de degrés de liberté importants pour la dynamique.

7.2 Applications aux séries issues du modèle néo-autrichien.

7.2.1 Le choc financier et les séries chaotiques.

Pour obtenir des séries chaotiques nous avons consideré un choc de type financier. Les firmes pour une seule période ont un comportement pessimiste (Fig.7.5), c'est-à -dire elles accumulent des encaisses desirées modelisées comme suit:

$$h_d^f(t) = \rho(t)(m(t-1) + f(t-1) + h^f(t-1)) \tag{7.2}$$

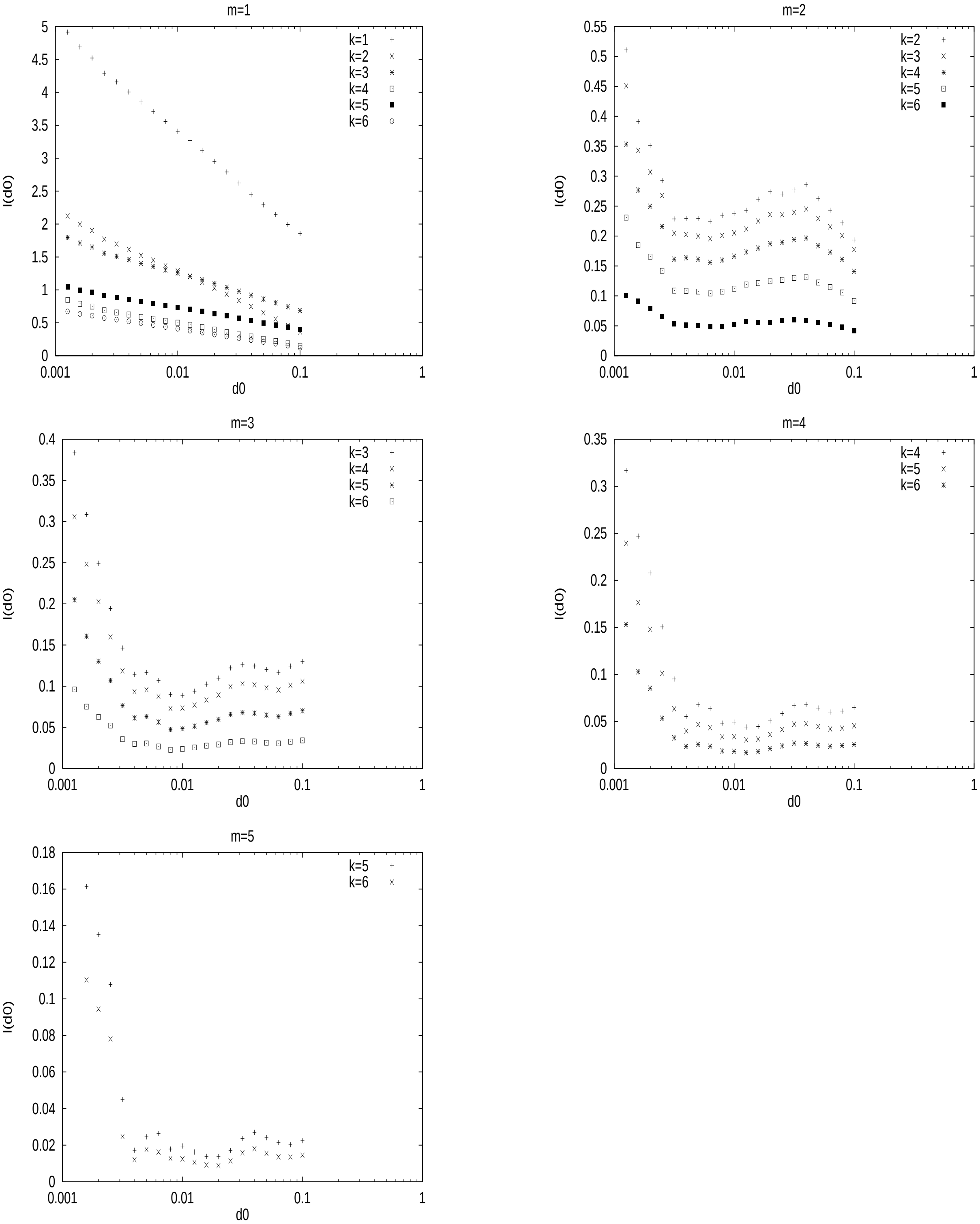

Figure 7.4: Variation de l'indicateur $\lambda_k(d_0)/k$ pour $k = 1, 2...6$ en fonction de la distance maximale d_0 entre les couples des points pour un signal chaotique déterministe.

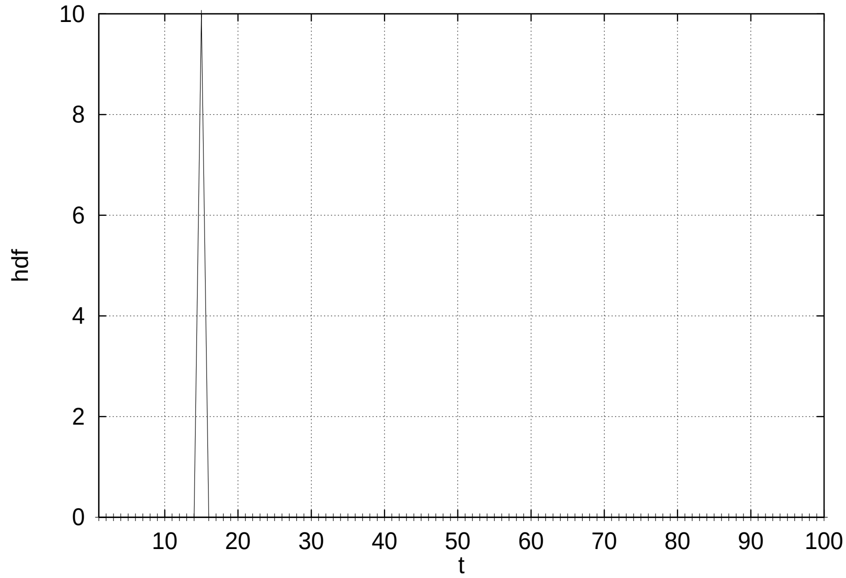

Figure 7.5: Variation au cours du temps des encaisses desirées par le firmes. Le choc a lieu seulement à la période 15.

le nombre des nouveaux processus diminue à la période t et cet effet est compensé par l'augmentation du nombre des nouveaux processus à la période $t+1$ (Fig.7.6).

Malgré le fait qu' à la période $t+1$ le nombre total des processus corresponde à celui du régime régulier (on fait l'hypothèse de croissance zéro) le choc introduit suffit pour casser l'équilibre inter-temporel de la structure productive et pour donner, sous les conditions que l'on verra ci-dessous, une orbite complexe.

Les agents ont un comportement rationnel en ce qui concerne l'offre (on ne fait pas de déclassement de processus) et en ce qui concerne l'objectif de construction. Le financement est demandé pour avoir un taux de natalité $x_1(t)$ sur la base du taux de croissance moyenne $\bar{g}$ observé, pendant toute une phase de construction, c'est-à -dire sur un nombre n_c de périodes:

$$x_1(t) = x_1(t - n_c)(1 + \bar{g})^{n_c} \tag{7.3}$$

Ce type de comportement rend la dynamique instable car les agents ne font qu'amplifier le effets du choc. En effet, en cas de diminution du volume de l'activité, la baisse du taux de croissance aura pour conséquence une diminution du taux de natalité qui elle va engendrer une nouvelle baisse du taux de croissance (instabilité Harrodienne). Du point de vue dynamique, une hausse du volume de l'activité rend aussi la dynamique instable (éloignement definitif du régime régulier) si les contraintes financières et humaines n'interviennent pas, mais les conséquences

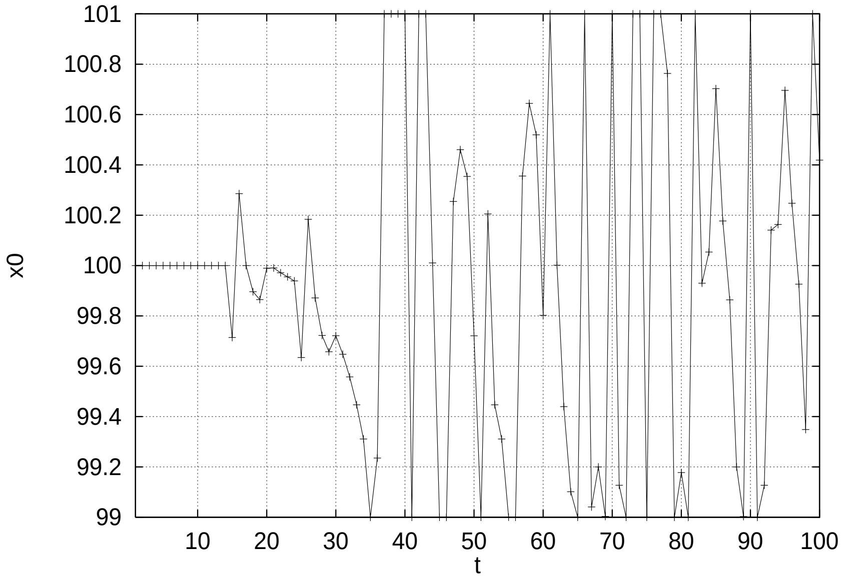

Figure 7.6: Variation au cour du temps du taux de natalité. Le nombre des nouveaux processus diminue à la période 15 et cet effet est compensé par l'augmentation du nombre des nouveaux processus à la période 16.

économiques sont naturellement plus favorables que dans le cas précédent.

Une fois modélisé un comportement qui favorise l'apparition du chaos, il ne reste qu' à limiter les variations du taux de naissance pour que le chaos soit confiné. Pour cela le nombre de $x_1(t)$ est limité dans l'intervalle:

$$x_0(t)(1-\Delta)(1+g)^{(t+n_c+n_u)} < x_1(t) < x_0(t)(1+\Delta)(1+g)^{(t+n_c+n_u)} \tag{7.4}$$

où Δ est un paramètre qui fixe l'amplitude de l'interval de variation du nombre de nouveaux processus par rapport au régime régulier. Dans toutes les simulations qui suivent nous avons choisi $\Delta = 0.01$, c'est à dire un écart maximum de 1% du ré gime régulier.

7.2.2 L'analyse dynamique

La Figure 7.7 montre l'évolution au cours du temps du taux de croissance pour le cas du choc financier précédemment décrit avec prix et salaires qui suivent les déséquilibres du marché avec un coefficient de reaction de 0.5 pour le prix et de 1 pour les salaires. Le prélèvement varie avec le taux de croissance de l'économie.

Nous avons appliqué à cette série la méthode décrite dans la section 6.3 (Gao an Zheng 1993 and Celletti et al. 1999a, 1999b). La figure 7.8 montre la variation de l'indicateur $\lambda_k(d_0)/k$ pour

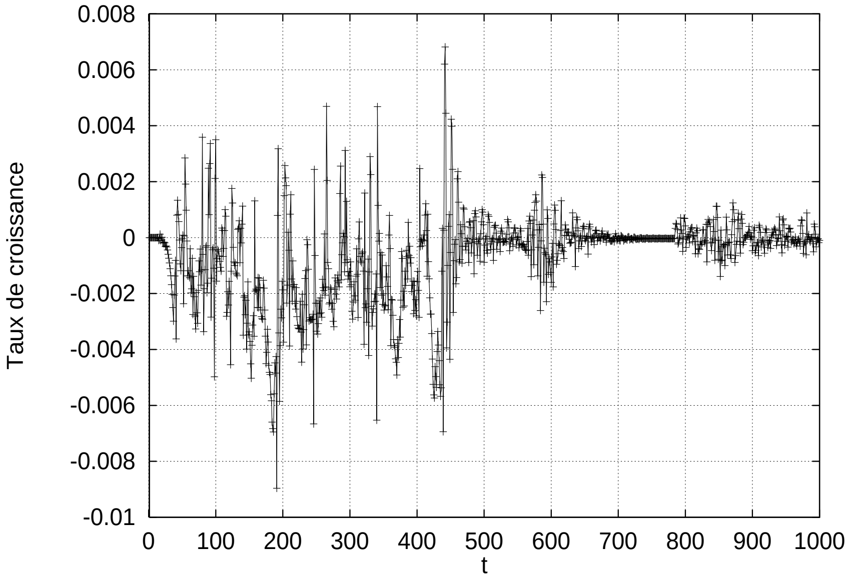

Figure 7.7: Variation au cours du temps du taux de croissance. le coefficient de reaction est de 0.5 pour le prix et de 1 pour les salaires. Le prélèvement varie avec le taux de croissance de l'économie.

$k = 1, 2...6$ en fonction de la distance maximale d_0 entre les couples de points. Nous retrouvons la signature du caractere déterministe à partir de $m = 4$, c'est-à -dire de façon surprenante nous retrouvons les 4 variables clé du modèle. Tout semble se passer comme si le système avait un attracteur de dimension égale au nombre de variables clés du modèle. Naturellement le modèle est deterministe mais le nombre de degrés de liberté est de l'ordre de $n^c + n^u + 4$: 24 dans le cas consideré. Si tous les degrés de liberté avaient le même rôle dans la dynamique cette méthode n'avais aucune expoire de les detecter.

Pour un test ulterieur du determinisme, mais surtout de la determination de la dimension m, nous avons melangé les données de la série et nous retrouvons bien dans ce cas la signature du cas stochastique (Fig.7.9).

Dans la Figure 7.10 la même analyse a été faite sur la série du taux de croissance dans le cas ou le prélèvement n'intervient pas. Les variables clés sont alors au nombre de 3 et c'est pour $m = 3$ que $\lambda_k(d_0)/k$ devient constant. Nous avons ensuite considéré un coefficient de réaction de prix égal à zéro, et nous retrouvons $m = 2$ (Fig.7.11) pour les deux variables clé qui restent: les salaires et le financement externe.

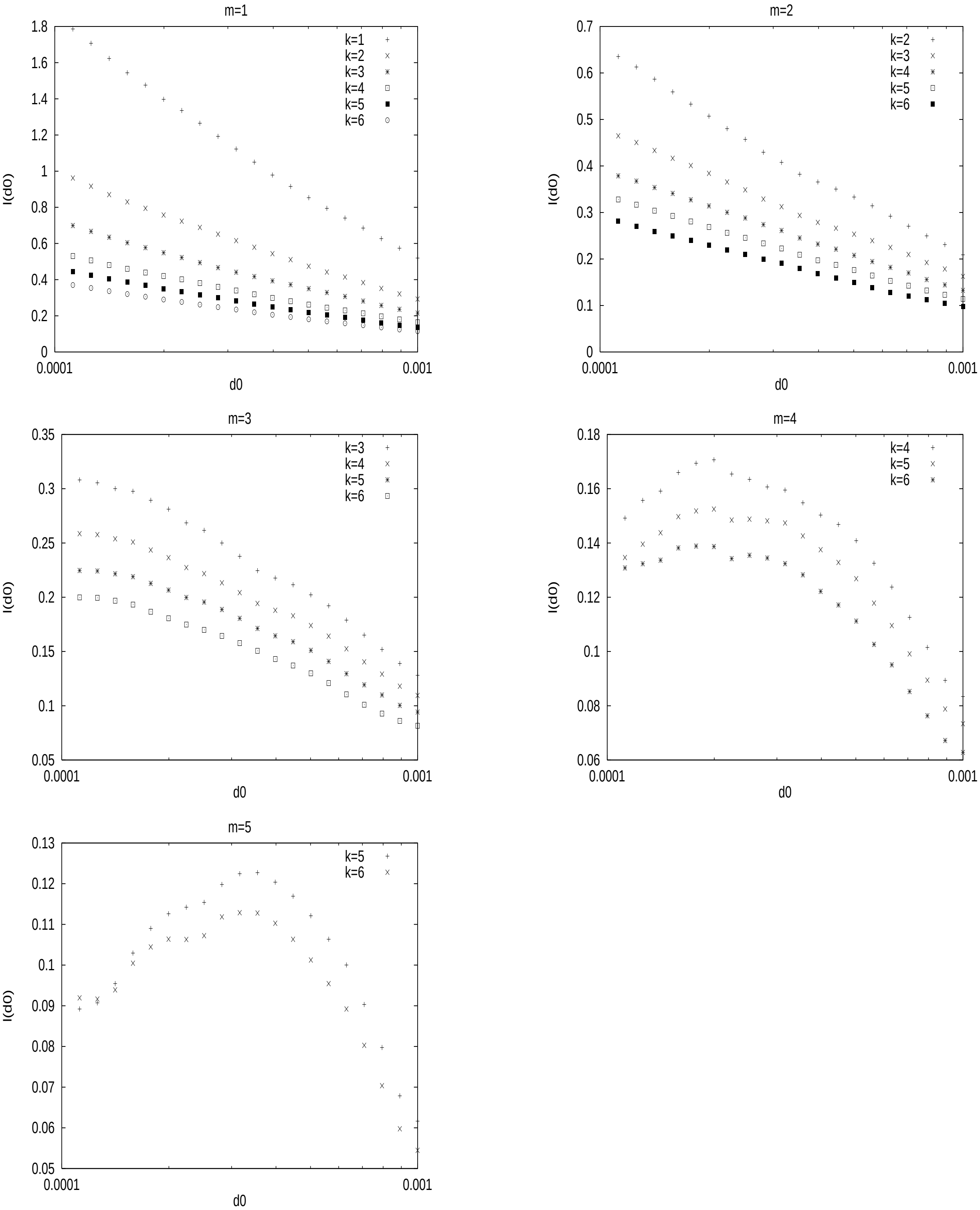

Figure 7.8: Variation de l'indicateur $\lambda_k(d_0)/k$ pour $k = 1, 2...6$ en fonction de la distance maximale d_0 entre les couples des points dans le cas ou le coefficient de reaction est de 0.5 pour le prix et de 1 pour les salaires et le prélèvement varie avec le taux de croissance de l'économie.

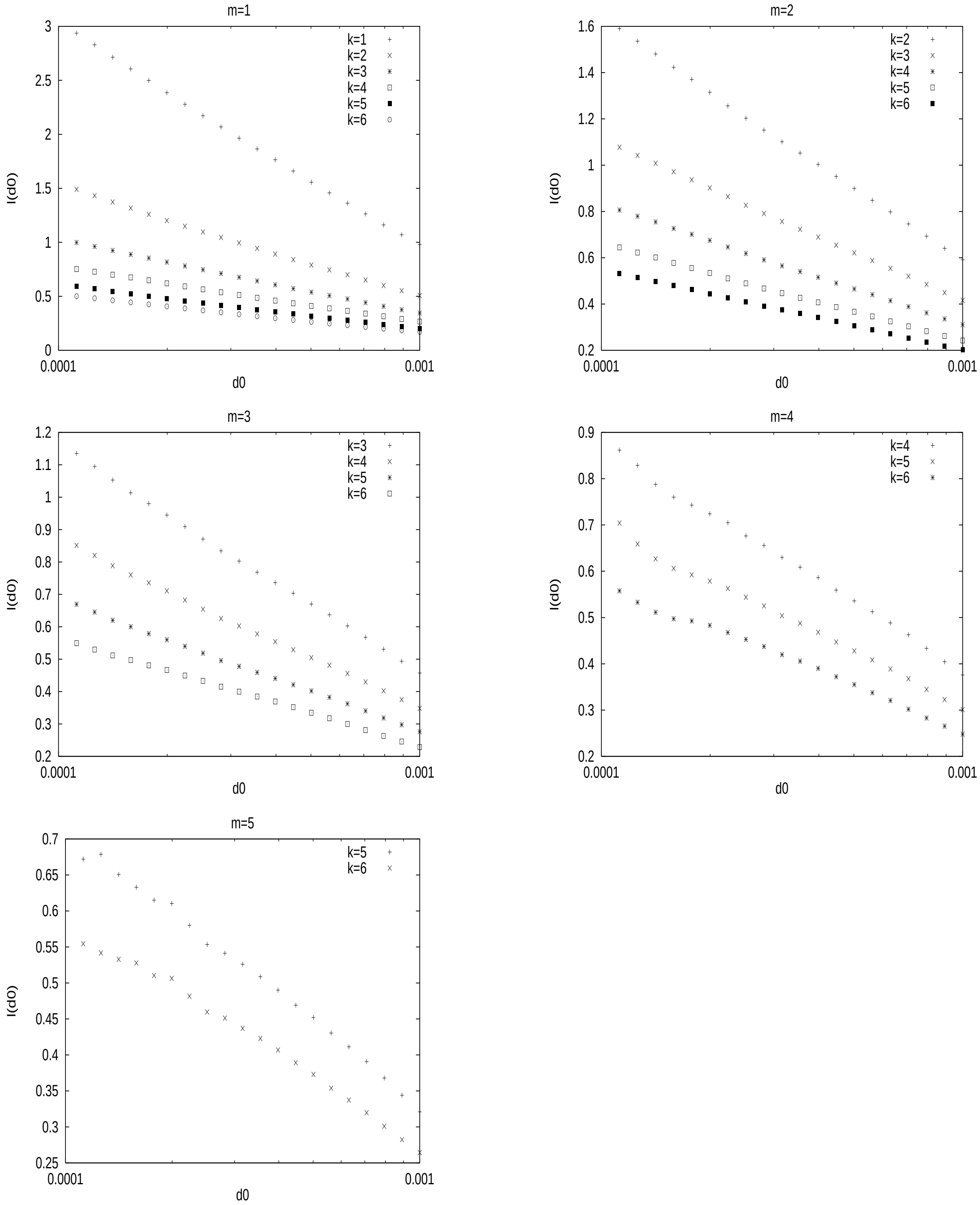

Figure 7.9: Variation de l'indicateur $\lambda_k(d_0)/k$ pour $k = 1, 2...6$ en fonction de la distance maximale d_0 entre les couples des points dans le cas ou on mélange les points de la série analysée dans la Fig.7.8.

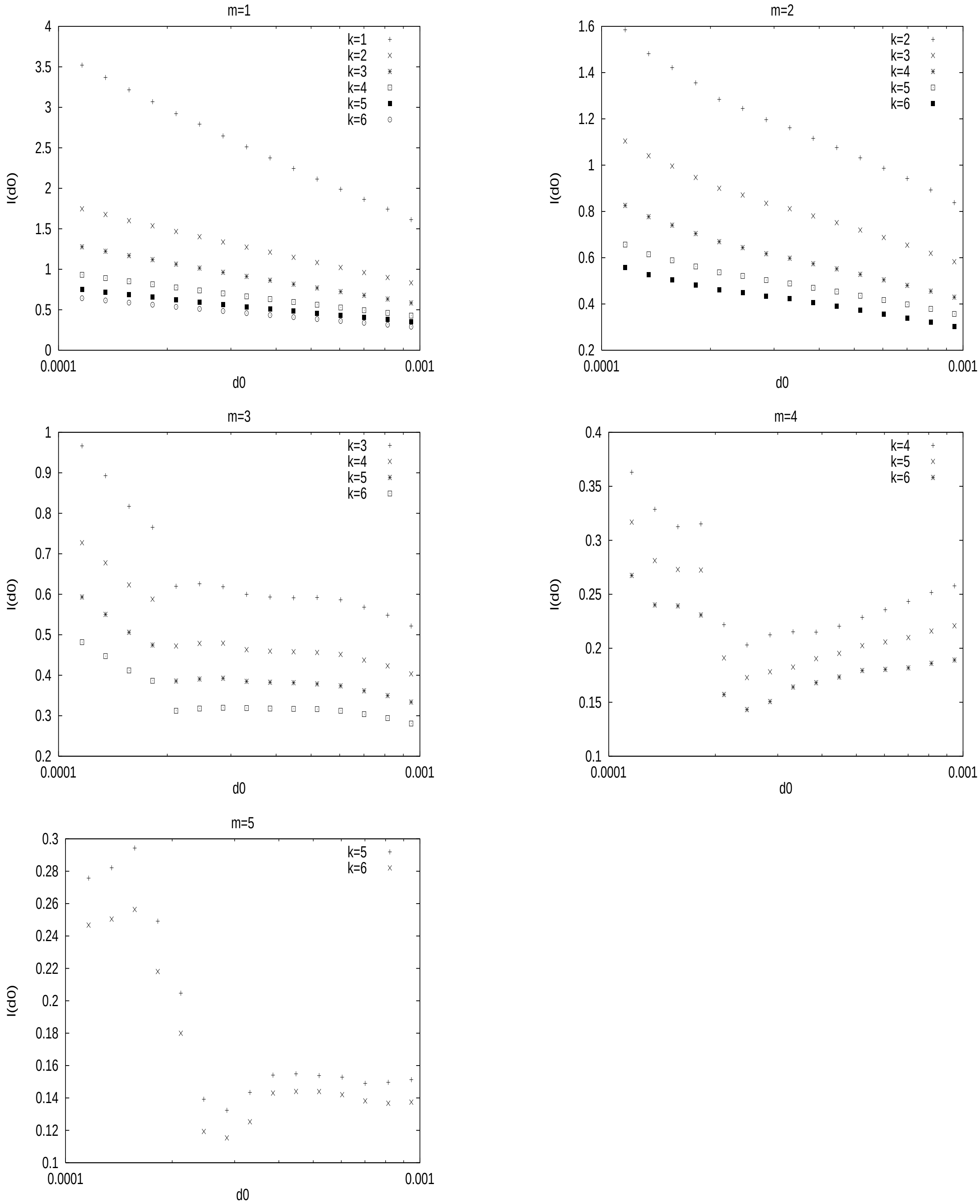

Figure 7.10: Variation de l'indicateur $\lambda_k(d_0)/k$ pour $k = 1, 2...6$ en fonction de la distance maximale d_0 entre les couples des points dans le cas ou le coefficient de réaction est de 0.5 pour le prix et de 1 pour les salaires et le prélèvement ne joue aucun role.

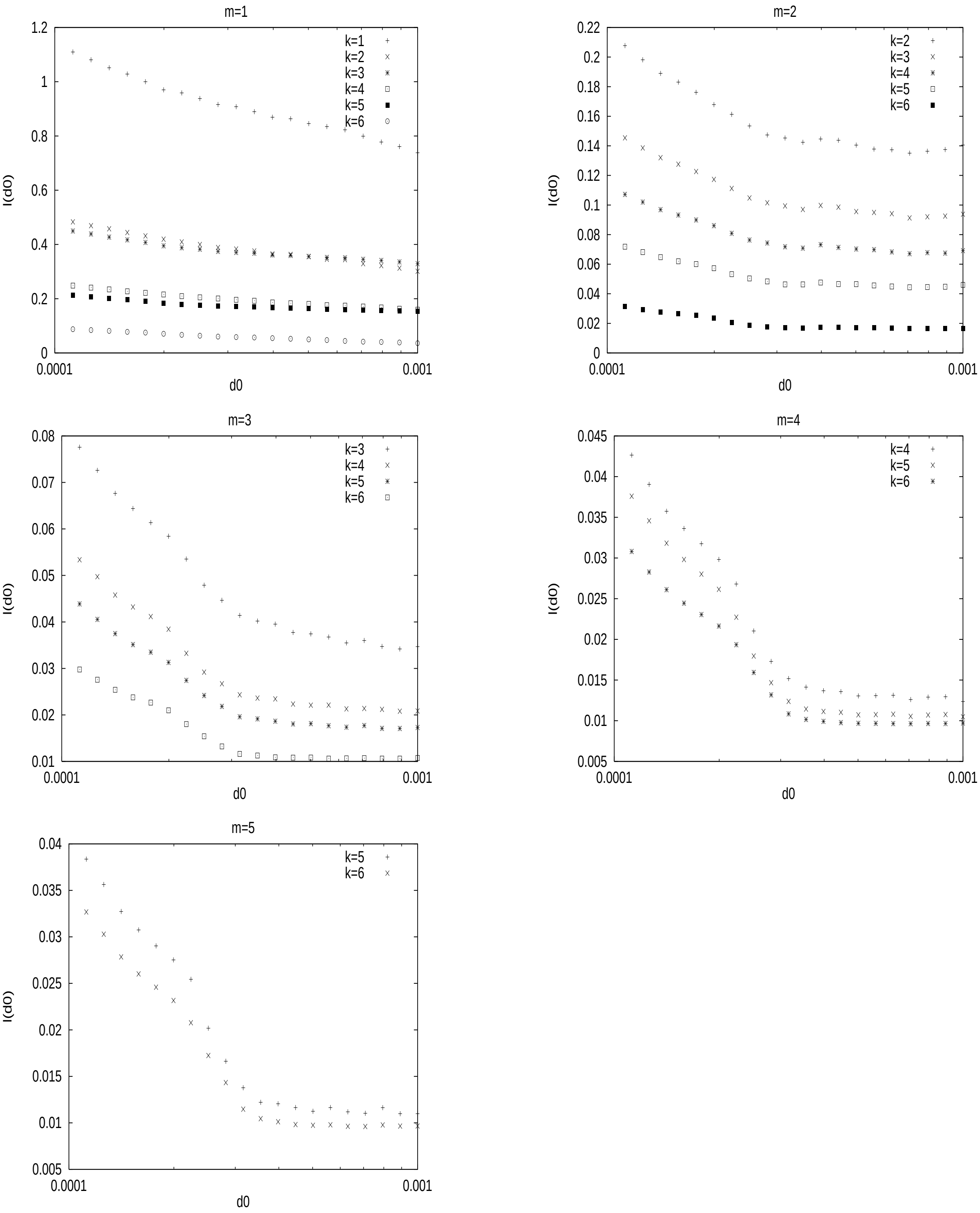

Figure 7.11: Variation de l'indicateur $\lambda_k(d_0)/k$ pour $k = 1, 2...6$ en fonction de la distance maximale d_0 entre les couples des points dans le cas ou seuls les salaires et le financement externe interviennent.

7.3 conclusion

A travers cette analyse il nous semble d'avoir fait un pas de plus dans la compréhension du modèle. Nous pouvons dire que le modèle a un attracteur dont la dimension est égale au nombre de variables clés du modèle. Il s'agit de résultats preliminaires qui, à notre avis, méritent d'être affinés dans un travail ulterieur.

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