Linearized Euler Equations in Aeroacoustic

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Sound generation and propagation in a turbulent flow is a difficult numerical problem [1]. The main difficulty is the occurrence of different scales. Acoustic fluctuations are very small as compared to the aerodynamic fields and tremendous numerical difficulties must be overcome in a direct simulation. While the fluid flow may be affected by small fluid structures containing large energy, such as small vortices in a turbulent flow, the acoustic waves are phenomena of low energy with long wavelengths that may travel over long distances. These different scales and different physical behaviors of fluid flow and sound propagation lead to difficult task to construct numerical methods for their approximation.

Sound propagation is hardly affected by viscosity (that is why noise is so difficult to suppress). Also, sound perturbations are so small that their contribution to the convection velocity of the flow is negligible in many cases. These two facts mean that sound can in essence be described by the Linearized Euler Equations (LEEs). The LEEs are a natural extension to Lighthill's analogy[1] in CAA (Computational Aeroacoustics) and provide accurate numerical solutions by only dealing with perturbations. Refraction effects of sound waves induced by the mean flow can be taken into account and also LEEs are relatively easier to solve numerically.

Non-dimensionalised Euler equation in flux vector form can be written as Eq. 1. For an inviscid flow, the viscous forces are neglected [2].

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + \frac{\partial H}{\partial z} = 0 \tag{1}$$

The Eq. 1 is linearized with the following substitution and solved to compute acoustic propagation.

$$\begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ E \end{bmatrix} = \begin{bmatrix} \overline{\rho} + \rho' \\ \overline{\rho u} + (\rho u)' \\ \overline{\rho v} + (\rho v)' \\ \overline{\rho w} + (\rho w)' \\ \overline{E} + E' \end{bmatrix} \tag{2}$$

The source terms for the LEE are provided from a numerically computed flow field with the help of an in-house LES (Large Eddy Simulation) numerical code. LES is carried for the Forward Facing Step (FFS) with a height $h = 12mm$ and inlet flow velocity $u_x = 10m/s$. LEE solver is coupled with the LES code in time domain to calculate the propagation of the acoustic field[3]. The acoustic pressure field is shown in Fig. 1 where at position $x = 0$ is where the step of height $h = 12mm$ is placed. Grid study on the acoustic side is presented and finally directivity analysis of the acoustic field is carried out. The numerical results are presented and compared with published experimental work.

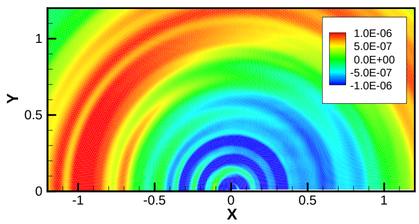

Figure 1: Instantaneous Acoustic Pressure

References

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