On Analyticity of the solutions of $2d$ Boussinesq system.

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Boussinesq system describes the dynamics of homogeneous fluid with temperature transfer. We consider Cauchy problem for two-dimensional Bussinesq system. It has the form

$$\begin{aligned}
&\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nabla p = \nu \Delta \boldsymbol{u} - \vec{e}_2 \theta + F \\
&\nabla \cdot \boldsymbol{u} = 0 \\
&\frac{\partial \theta}{\partial t} + (\boldsymbol{u} \cdot \nabla)\theta = \mu \Delta \theta \\
&\boldsymbol{u}(x,0) = u_0(x),\ \theta(x,0) = \theta_0(x)
\end{aligned} \tag{1}$$

where $\vec{e}_2 = (0,1)$, $x = (x_1, x_2) \in \mathbb{R}^2$, $t \in \mathbb{R}_+$ is time, $\boldsymbol{u}(x,t) = (u_1, u_2) : \mathbb{R}^2 \times \mathbb{R}_+ \mapsto \mathbb{R}^2$ denotes a 2–dimensional velocity vector, $\theta(x,t) : \mathbb{R}^2 \times \mathbb{R}_+ \mapsto \mathbb{R}$ corresponds to heat transport, positive integers ν and μ are the viscosity coefficients, $F : \mathbb{R}^2 \times \mathbb{R}_+ \mapsto \mathbb{R}^2$ is an external forcing and the scalar function $p(x,t)$ denotes pressure.

Mathematical results on the existence and uniqueness of solutions of the system (1) can be found in [1], [2] and references therein.

Using the technique developed by J. Mattingly and Ya.G. Sinai in [3] for $2d$ Navier-Stokes system we obtain analyticity of the solutions of (1) with initial data from the space of Pseudomeasures

$$\Phi(\alpha) = \{f : \sup_k |k|^\alpha |\hat{f}(k)| < \infty\}$$

References

[1] J.R. Cannon and E. DiBenedetto, *The initial value problem for the Boussinesq equations with data in L^p*, Approximation methods for Navier-Stokes problems, 129–144, Lecture Notes in Math., 771, Springer, Berlin, 1980.

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[3] J.C. Mattingly and Ya.G. Sinai. *An Elementary Proof of the Existence and Uniqueness Theorem for the Navier-Stokes Equations*, Commun. Contemp. Math., 1(4):497–516, 1999.