A model of transport in decaying 2D turbulence: Flow and stirring due to a point vortex in time-dependent ambient strain

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Abstract submitted to EE250

The flow in the late stages of the decaying two-dimensional turbulence is dominanted by a small number of strong coherent vortices [2]. Except for their brief and relatively infrequent interactions these *coherent structures* move like interacting point vortices, i.e., the motion of their centres can be reasonably described in terms of point vortex dynamics by a finite set of ODEs. Each vortex 'feels' the presence of other vortices whose combined influence is, in the first approximation, that of an irrotational ambient straining flow with variable rate of strain (1) and rotating principal axes. We consider a simple model of transport of a passive scalar around such a coherent vortex at distances much larger than its size but much smaller than the distance to its nearest neighbours. The vortex is approximated by a point vortex with circulation Γ, the strain rate has harmonic time-dependence,

$$\alpha(t) = B\frac{\Delta + \cos\omega t}{\Delta + 1}\,, \tag{1}$$

and the principal axes rotate with constant angular velocity λ. In the frame of reference rotating together with the axes of strain we then have the following flow

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \frac{\Gamma}{2\pi\,(x^2+y^2)}\begin{bmatrix} -y \\ x \end{bmatrix} + \begin{bmatrix} -\alpha(t) & \lambda \\ -\lambda & \alpha(t) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}. \tag{2}$$

Transport in aperiodically time-dependent flows of this kind is also later discussed. When the strain is steady, i.e. $\lambda = \omega = 0$, the phase portrait is a familiar cats-eye pattern oriented at 45° to the strain axes [1]. In general, we have three dimensionless numbers, $\Omega = \omega/B$, $\Lambda = \lambda/B$ and Δ (related to the amplitude of the strain oscillations, see 1). Exploring this three-parameter family of flows in the Lagrangian representation we find a remarkable variety of different regimes with various degrees of chaos and other interesting geometric features. Different regimes of the parameter space are amenable to analysis.

The straining flow grows with distance while the vortex flow decreases. Therefore, near the origin the vortex is a basic flow and the strain is a small perturbation. In the region where the perturbation is also slow comparing with the turnover time scale of the vortex the perturbation methods of Hamiltonian dynamics can be used to deteremine the

perturbed tori as well as the location of the resonances. In the far field the relation is reversed. The oscillating strain is the basic flow while the vortex is a small perturbation. The basic flow is degenerate in the sense that all orbits have the same period. In presence of a small perturbation, fluid elements follow the iso-surfaces of an adiabatic invariant which is also calculated using non-canonical averaging methods. Various intermediate parameter regimes are explored numerically.

The patterns of both chaotic and regular motion found in this problem show surprising complexity which is further explored and extended to aperiodically time-dependent situation through determination of certain invariant structures in the flow (i.e. distinguished hyperbolic trajectories and their stable and unstable manifolds, [3]) which serve as a template for stirring. The problem has a bearing on the efficiency of transport in a generic irrotational flow. It is also a preliminary to understanding the fate of a weak, small-scale vorticity background in two-dimensional turbulence where the filaments spun of coherent structures are carried by flows of this kind.

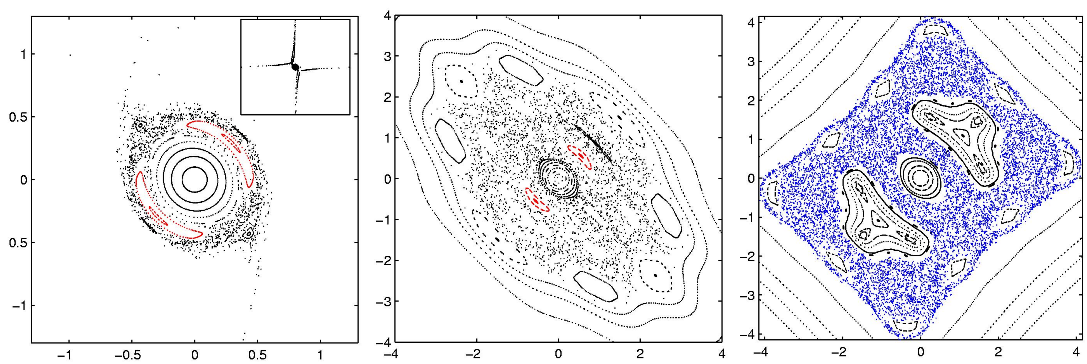

Figure 1: A few examples of Poincare sections for advections by the flow (2) in the frame rotating with the strain axes for different values of the system parameters (time-periodic situation). The geometry of surviving KAM tori can be determined using averaging methods and the mechanism of stirring in the chaotic regions is analysed through determination of so-called distinguished hyperbolic trajectories and their invariant manifolds.

References

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