Spectral energetics in quasi-static MHD turbulence

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Abstract submitted to EE250

Homogeneous turbulence of an electrically conducting fluid, subject to a magnetic field, is studied numerically. The fluid is incompressible and contained in a periodic box, while the Navier–Stokes equations are solved with a pseudo-spectral code with large scales forcing [1]. The effect of a uniform magnetic field **B** is taken into account via the quasi-static approximation, given the low magnetic Reynolds number. As observed in previous works [2, 3], turbulence becomes anisotropic and axisymmetric, under the effect of the additional Joule dissipation. Indeed, this attenuates the velocity fluctuations in the direction parallel to **B**. Some insight into the flow dynamics can be obtained from the kinetic energy equation in spectral space

$$F\left(\mathbf{k}\right) = T\left(\mathbf{k}\right) - 2\nu k^2 E\left(\mathbf{k}\right) - \frac{\sigma B_0^2}{\rho}\cos^2\left(\psi\right) E\left(\mathbf{k}\right), \tag{1}$$

where $\mathbf{k}$ is the wavenumber vector with magnitude k, $F\left(\mathbf{k}\right)$ is the forcing term, $T\left(\mathbf{k}\right)$ is the nonlinear transfer, ν is the kinematic viscosity, ρ is the fluid density, σ is the electric conductivity, and ψ is the angle between $\mathbf{k}$ and $\mathbf{B} = B_0\mathbf{e}_3$. The last term on the right side of (1) accounts for the Joule damping. Unlike isotropic turbulence — where the energy equation can be conveniently averaged over thin spherical shells — the axisymmetric character of MHD turbulence requires accounting for the angular dependence

on ψ. To retain this, (1) is averaged over thin spherical rings whose centre lies on the $\mathbf{e}_3$ axis. Two-dimensional energy spectra $E\left(k_\perp, k_\parallel\right)$, where $k_\perp = k\sin(\psi)$ and $k_\parallel = k\cos(\psi)$, indicate that the Joule dissipation attenuates the energy in a region close to the $k_\parallel$ axis, compared to the case $N = 0$, fig.1 (N is the interaction parameter, proportional to B_0). For small N, fig.2, the energy is still distributed rather evenly, but when N is large, fig.3, the region where the energy is dissipated takes the shape of a cone [4]. The actual formation of the cone is in part controlled by the nonlinear transfer, which tends to replenish the energy in the region where the Joule dissipation is dominant. This work focuses on the distribution of transfer and its detailed build-up, in order to clarify the degree of locality of the wavevectors triads involved in the energy exchanges in the $\left(k_\perp, k_\parallel\right)$ space.

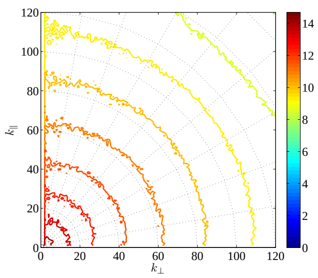

Figure 1: Contours of $\log(E(k_\perp, k_\parallel))$ for $N = 0$.

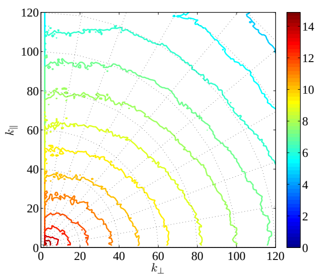

Figure 2: Contours of $\log(E(k_\perp, k_\parallel))$ for $N = 1$.

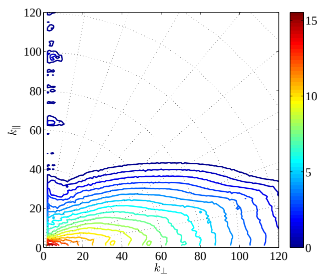

Figure 3: Contours of $\log(E(k_\perp, k_\parallel))$ for $N = 10$.

References

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