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0.0.1 FIRST PART : ANISOTROPIE IN KRAICHNAN FLOWS

The Kraichnan random ensemble of velocity has been extensively used to model various phenomena related to turbulent transport both in the inertial interval of scales that develops at hight Reynold numbers and at moderate reynolds numbers wher the viscosity effects play an important role. The passive transport of scalar or vector quantities in a velocity field is governed by the Lagrangian flow descibing the evolution of the trajectories of fluid particles. From the mathematical point of view, such flow provides an example of random dynamical system which in the case of the Kraichnan velocities is descibe by a stochastic differential equation. For the Kraichnan flows corresponding to moderate Reynold numbers, the methods borrowed from the theory of random dynamical systems or stochastic differential equation appear to provide important information about the transport properties of the flows. To start with , the values of the Lyapunov nexponents of the flow, whose existence is asserted by the multiplicative ergodique theorem, allow to decide wheter the flow is chaotic or not, leading to different directions of the cascades of passively advected scalars. More detailled information about the transport properties of the flow may be extracted from the knowledge of the fluctuations of the exponential streching rates aroud their limiting long time values equal to the Lyapunovs exponents.

In the generic case where all the lyapunovs exponents are differents, the statistics of the steching exponents may be expected to exhibit at long but finite

time a large deviation regime captured quantitatively by a single function of the vector of the streching rates. Since the existence of such multiplicative large deviation regime is not assured by general mathematical theorems, it is interesting to have at our disposal models where it may be established and studied in detail.One such example that has been known for some time is the homogeneous isotopic Kraichnan flows.The corresponding stochastic differential equation has benn studied in the mathematical literature in the eighties and nineties of the last century. In paricular , the lyapunov exponents have been found by Y Le Jan ("On Isotropic brownian motion" 1985). With the regain of interest of physicist in the Kraichnan model in the mid-nineties , the same stochastic equation resurfaced as the model for the lagrangian flow at moderate Reynold numbers with the motivations, the accents and the language proper to the turbulence theory community. In particular, it was realised that many propetries of the turbulent transport require more information about the flow than the spectrum of lyapunovs exponentsand may be expressed in terms of the rate function of the large deviations of the streching exponents. In the homogeneous and isotropic

Kraichnan flow, the large deviation regime of the streching exponents is gaussian and the corresponding rate function is a quadratic polynomial .It's simple form form has permits to extact analytic answers for many characterixtics of passive advection in such flows.

The simplicity of the multiplicative large deviation regime in the homogeneous isotropic Kraichnan flow is due to the decoupling of the dynamics of the stretching exponents from that of the eigen-directions for streching and contraction . As a result also the exact distribution of the stretching exponents may be found analytically in this case as it appears to be related to the heat kernel of the quantum Calogero-Sutherland Hamiltonian for particles on the line with the attractive pair patential proportional to the function $\sinh^{-2}$ of the inter-particle distance.Here, we analyse the two-dimensional Kraichnan flow in a periodic square often used in numerical simulation.The large scale anisotropy due to the shape of the flow volume generically persists on small scales inducing isotropy breaking terms in the distribution of strain that drives the evolution of the stretching exponents. Due to the presence of such terms, the streching exponents dynamics does not decoupleanymore fom that of the unstable eigen-directions. The Lyapunovs exponents may nevertheless be still computed analytically and their difference expressed in term of elleiptic integrals. The distribution of the sum of the streching exponents is still gaussian for all times (this is general fact for the homogeneous Kraichnan flows). As for the rate function of the large deviation of the difference of the streching exponents, its calculation may be reduced to that of the ground-state energy of the integrable one-dimensional periodic quantum Lamé operator.For general values of the coupling constant, the eigenvalus of the latter may be found by numerical by numerical diagonalization of infinite tridiagonal matrices. This approach permit to obtain the large deviation rate function for the streching exponents out to be non-quadratic altough with quadratic exponents.The analysis of the spectral gap of the Lamé operator permits to assess the time scale at which the multiplicative large deviation regime sets in.

0.0.2 SECOND PART : FLUCTUATIONS DISSIPATION OF DIFFUSION PROCESS

In the last few years, a group of relation were derived involving the distribution of work done on a system by non-conservative and/or time dependant forces. It seems they have not been noticed before the nineties, even though in most case the proofs require that have been around for many decades. These relations are Jarzynsky relation, Fluctuation detailed relation and Gallavoti-cohen relation.

Here, we show all the known formula by a single and rigorous formalisme. We present also another way to put the system out equilibrium, the occurence of flux in the system, and we proves Jarzynsky relation and Fluctuation detailed relation in this case.