On the long-time evolution of inviscid thin jets

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Abstract submitted to EE250

Free surface thin jets of inviscid fluids occur in many different areas of fluid dynamics, including surface waves, asymmetric bubble collapse, bursting of bubbles at a free surface, drop impact onto a solid surface, or even ink-jet printing. Fundamental instabilities involving jets have been widely studied, among which are the Rayleigh–Taylor (RT) and Richtmyer–Meshkov (RM) instabilities. These two instabilities correspond to the situation where a fluid is accelerated towards another one, the boundary between the two fluids being initially perturbed. In the case of Rayleigh–Taylor instability, a heavy fluid is subject to a constant body force like gravity acting in the direction of a lighter fluid, whereas in the case of Richtmyer–Meshkov instability, the perturbed interface is impulsively accelerated by a shock wave, a situation which can be well approximated by imposing an impulsive acceleration perpendicular to the interface.

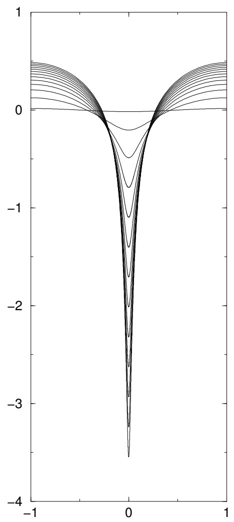

Figure 1:

Here, we concentrate on the special case of infinite density ratio, neglect the effect of surface tension, and assume that the fluid is incompressible. We consider a vertical inviscid thin jet subject to a constant body force $\mathbf{g}$ pointing either upwards or downwards and we define the alternate situation where the jet is described in a galilean frame of reference and the fluid far from the jet is pulled away with constant acceleration $-\mathbf{g}$. Moreover, we postulate that the flow in the jet in well approximated in this galilean frame of reference by a velocity potential of the form :

$$\varphi = \frac{1}{2}\left(\frac{\dot{a}}{a}x^2 + \frac{\dot{b}}{b}y^2 + \frac{\dot{c}}{c}z^2\right), \tag{1}$$

where a, b and c are functions of time only. This notation is the one used in [1] and allows a natural way of finding the free surface shape. Incorporating this velocity potential into Euler equations and using asymptotic expansions of functions a, b, and c, we find that, at long-time in three dimensions, $c(t) = c_0 t - M/8c_0^2 t^2 + o(t^{-2})$, where M and c_0 are constants. Then it follows that the curvature of the spike asymptotically scales like $\kappa_0 \sim 2t^2/M$. The jet encounters an overshoot in acceleration scaling like t^{-4} :

$$a_t = -\frac{3M}{4t^4} + o(t^{-4}),$$

and the self-similar equation of the free surface reads :

$$-\frac{X^2 + Y^2}{M^2} + Z^2 = 1, \qquad (2)$$

where $X = xt^{1/2}$, $Y = yt^{1/2}$ and $Z = z/t$. Therefore, knowing the constant M from the long-time evolution of the curvature of the spike, we deduce without adjustable parameter the overshoot in acceleration and the self-similar equation of the free surface.

In two dimensions, *i.e.* for $b = 0$, we have $c(t) = c_0 t - M^2/6c_0^3 t^3 + o(t^{-3})$, the curvature scales like $\kappa_0 \sim t^3/M^2$ and the self-similar equation reads :

$$-\frac{X^2}{M^2} + Z^2 = 1,$$

where $X = xt$ and $Z = z/t$.

The theory for two-dimensional flows has been tested using an accurate boundary integral method. Figure 1 shows the successive computed profiles of the free surface during the non-linear stage of RM instability. Using this numerical method, we were able to check the asymptotic evolution of the spike and the consistancy between the two prefactors in the curvature of the spike and in the overshoot in acceleration. Moreover, the self-similar equation of the free surface is found to be in very good agreement with the computations.

A detailed version of this work is currently under consideration for publication in Physical Review Letters [3].

References

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[2] L. Duchemin, C. Josserand and P. Clavin, Phys. Rev. Lett. **94**, 224501, (2005).

[3] L. Duchemin, *On the long-time evolution of inviscid thin jets*, submitted to Phys. Rev. Lett., (2006).