STATISTICAL THEORY OF EXTREME EVENTS IN OCEANIC WAVE TURBULENCE

Francesco Fedele

Dept. of Civil Engineering, Georgia Institute of Technology, Savannah Campus, Georgia USA

Waves in open ocean are modeled by Gaussian seas, as the linear superimposition of a very large number of elementary waves having amplitudes related to a given spectrum and random phases. If the elementary waves nonlinearly interact among them exchanging energy by resonance interaction, then the dynamics of the wave field is governed by the well known Zakharov equation for wave turbulence (Zakharov 1999).

In the context of Gaussian seas, it is well known that the occurrence of large waves is due to the dynamics of a stochastic wave group (Fedele 2006, Fedele & Tayfun 2006) whose structure is described by the space-time covariance of the sea surface (Lindgren, 1972; Boccotti 1989, 2000), with a Rayleigh-distributed amplitude (crest) h, in the limit h approaching infinity. Drawing on the theory of quasi-determinism of Boccotti (2000) and the regression approximation approach of Rychlik (1987), we derive under the same limit conditions, a new coherent structure consisting of a large stochastic wave group of O(h), evolving non-interactively in the presence of a finite random background of O(1).

We then investigate the nonlinear dynamics of this new wave structure in oceanic wave turbulence according to the Zakharov equation describing weakly nonlinear random seas. We shall consider both second order interactions due to bound waves and third order effects due to four wave quasi-resonance interactions. As a corollary, we then propose new theoretical upper bounds for the statistical distribution of crest heights and crest-to-trough heights over large waves. Comparisons based on the nonlinear simulations of the Zakharov equation (see also Socket-Juglard et al. 2005), with wave data collected at the Tern platform in the northern North Sea during an extreme storm, and the results of the wave tank experiment of Onorato et al. (2006), show good agreement with the proposed theoretical wave distributions.

The numerical simulations showed that the time evolution of an initial Gaussian random field consists of three distinct phases: an initial phase where the field is weakly Gaussian and nonlinearities are not developed yet. This phase is followed by a strong non-Gaussian transient, during which the wave field becomes non-Gaussian, indicating the failure of the central limit theorem due to the building up of space-time correlation. Stronger deviations from Gaussianity occur, an evidence of the strong intermittency characteristic of the transient field. Then, a third and final phase kicks in, during which the wave field reaches a steady state. In particular, for initial wave fields consisting of narrow-band seas (unrealistic oceanic conditions), the steady state consists of a non-gaussian field with kurtosis greater than the Gaussian value. Very narrow band initial fields yield more intense modulation instability and very strong transients which are followed by a less intense steady state. Instead, moderately narrow band initial wave fields, tend to a steady state monotonically with a very smooth transient.

Finally, for initial broadband wave fields (realistic oceanic conditions), during the transient the field becomes non-Guassian but at the steady state the Gaussianity is restored again and the Tayfun distribution (Tayfun 1980,1986) seems to fit very well the crest data, whereas the Rayleigh law fits very well the wave height data.

REFERENCES

Boccotti P.(1989) On mechanics of irregular gravity waves. Atti Acc. Naz. Lincei, Memorie;19:11-170.

Boccotti P. (2000) Wave mechanics for ocean engineering. Elsevier Science, Oxford.

Fedele F. (2006) On wave groups in a Gaussian sea. Ocean Engineering Volume 33:17-18, pp. 2225-2239.

Fedele F. & Tayfun A. (2006) Extreme waves and Stochastic wave groups. Proceedings of the 25th International Conference on Offshore Mechanics and Arctic Engineering (OMAE), Hamburg, Germany

Lindgren G. (1972) Local maxima of Gaussian fields. Ark. Mat.;10:195--218.

Onorato M, Osborne AM, Serio M,Cavaleri L, Brandini C, Stansberg CT, 2006. Extreme waves, modulational instability and second order theory: wave flume experiments on irregular waves. European Journal of Mechanics B/Fluids Vol 25 586-601

Rychlik, I. (1987) Regression approximations of wavelength and amplitude distributions. Adv. Appl. Prob. 19. 396-430

Socquet-Juglard, H., Dysthe, K., Trulsen, K., Krogstad, H. E. & Liu, J. 2005. Probability distributions of surface gravity waves during spectral changes. J. Fluid Mech. 542, 195-216.

Tayfun, M.A. (1980) Narrow-band nonlinear sea waves. J. Gephys. Res.; 85 (C3), 1548-1552.

Tayfun, M.A. (1986) On Narrow-Band Representation of Ocean Waves. Part I: Theory. J. Geophys. Res.;91(C6):7743-7752.

Zakharov V. (1999) , Statistical theory of gravity and capillary waves on the surface of a finite-depth fluid, Eur. J. Mech. B 18 (3) 327-344