DETERMINATION OF VORTEX RING CIRCULATION USING POTENTIAL FLOW ANALYSIS

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[Note on Prior Work: This abstract is based on previously published work by the author (Krueger, 2005 and 2006).]

The sudden ejection of a finite-duration jet from a nozzle or orifice is a frequently observed unsteady flow. It is a distinguishing feature of many important systems ranging from aquatic propulsion of squid and salps (Siekmann, 1963; Weihs 1977) to synthetic jet actuators (Glezer and Amitay, 2002). A piston-cylinder mechanism is commonly used to generate such flows in the laboratory where the generation of the jet is due to the piston motion as illustrated in Figure 1.

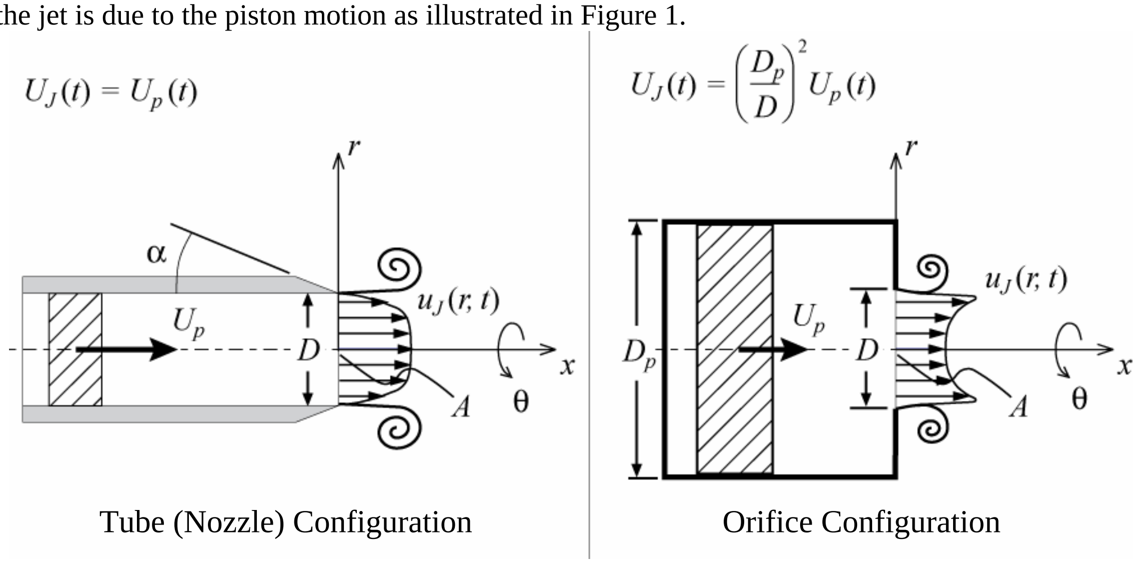

Figure 1. Schematic of Two Configurations of Piston-Cylinder Vortex Ring Generators.

The formation and evolution of the vortex ring that results from the roll-up of the jet shear layer has been the subject of a substantial body of research (see Shariff and Leonard, 1992 for a review). A key vortex ring characteristic that is directly related to the formation process is the total circulation of the ring, namely $\Gamma_T = \int_{ring} \omega_\theta \, dr dx$, where ω_θ is the azimuthal vorticity. Two common methods for determining Γ_T in terms of the formation parameters involve consideration of the flux of vorticity in the jet (Didden, 1979) and dynamics of the vortex sheet roll-up (Pullin, 1979). The various assumptions made in these two approaches limits their applicability to short time behavior in the case of the later (Nitsche, 1996) and long-time behavior in the case of the former. A different approach, developed and refined by the author (Krueger, 2005 and 2006), considers the integral of the incompressible vorticity transport equation over the domain external to the vortex ring generator. The present discussion summarizes the approach developed by the author in the specific context of inviscid, incompressible flow as described by Euler's equations.

For vortex ring formation by high Reynolds number jets, vorticity diffusion across the centerline may be ignored. Then integrating the vorticity transport equation over the flow external to the vortex ring generator and in time yields

$$\Gamma_T = \underbrace{\frac{1}{2}\int_0^{t_p} u_{cl}^2(t)\,dt}_{\Gamma_U} + \underbrace{\frac{1}{\rho}\int_0^{\infty}\left(p_{cl}(t) - p_{\infty}\right)dt}_{\Gamma_p}$$

where t_p is the pulse duration and u_{cl} and p_{cl} are the velocity and pressure at (x, r) = (0, 0). The compact nature of the vorticity field for high jet Reynolds number results in irrotational flow at (x, r) = (0, 0) so that Γ_T may be determined by potential flow analysis. For rapidly initiated jets, Γ_p is determined by the integral of the unsteady Bernoulli equation for flow in front of the forming ring, giving $\Gamma_p \approx U_0 D / C_p$ where $C_p = \pi$ for the tube geometry and 2.00 for the orifice geometry. Here U_0 is the maximum value achieved by the jet velocity, U_J, during the jet pulse. The solution for u_{cl}, on the other hand, is obtained from a potential flow solution inside the piston-cylinder mechanism using a boundary condition at the exit that matches the evolving flow for $x > 0$. To this end, a semi-empirical boundary condition was constructed for both the tube and orifice geometries using the fact that the jet must transition to a free jet for t_p sufficiently large. The final result is not repeated hear in the interest of space, but it is noted that the use of potential flow analysis allowed the specific geometry characteristics for both tube and orifice generators to be accounted for analytically in a straight-forward manner. In the tube case, a boundary layer correction may be added for improved fidelity, but no boundary layer correction is required in the orifice case because the contraction of the flow as the jet exit plane is approached minimizes boundary layer growth in this configuration.

Comparison of the results of this analysis with experimental and numerical results of vortex ring formation over a wide range of jet Reynolds number and pulse durations gives excellent agreement. The agreement for Γ_T is within 10% for the tube geometry and 20% for the orifice geometry (with highest error for short pulses), whereas other models give errors greater than 20% and 65% for the tube and orifice cases respectively. The interesting aspect of this approach is that the solution is based on inviscid techniques, even though the quantity of interest is related to the vortical portion of the flow where all of the viscous action occurs. The fidelity of the results is due in part to accounting for the actual (i.e., viscous) flow evolution through the semi-empirical boundary condition employed for the potential flow solution inside the piston-cylinder mechanism. In this regard, the analysis is analogous to classical airfoil theory, where the appropriate bound vortex circulation is determined by the empirically observed condition that the flow leaves the trailing edge smoothly.

References

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