Infinite-dimensional geometry of optimal mass transport and the Burgers equation

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In the 60's, Arnold showed that the Euler equation can be thought of as the geodesic flow on the group of volume preserving diffeomorphism. In a similar spirit, Otto recently showed in [2] that the optimal transport problem can also be thought of as geodesic flow on the space W of all volume forms with total volume 1. In particular the space W can be regarded as the quotient of the group of all diffeomorphisms by the subgroup of volume-preserving ones, while and the geodesic flow on the diffeomorphism group, given by the Burgers equation, is closely related to that on the Wasserstein space of densities W. Our work is an extension to this observation. We showed that this relation between diffeomorphism group and W can be understood using Hamiltonian reduction. Moreover, the relation between Hamilton-Jacobi theory and optimal mass transport recently shown in [1] can also be understood in this framework. We also consider the following non-holonomic version of the classical Moser theorem: given a bracket-generating distribution on a manifold two volume forms of equal total volume can be isotoped by the flow of a vector field tangent to this distribution. We discuss these results from the point of view of an infinite-dimensional non-holonomic distribution on the diffeomorphism groups. This is a work in progress.

References

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