Vanishing viscosity limit for 2D flows in an unsteadily rotating circle

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We consider 2D viscous incompressible flows in a disk with rotating boundary. We assume that the prescribed angular velocity is bounded variation (BV) in time, which includes impulsively started and stopped rotations.

We study the vanishing viscosity limit for this problem in the case that the initial velocity has radial symmetry, but it is not necessarily compatible with the boundary condition. In this situation, the solution to the Navier-Stokes equations has an initial layer, while there is a boundary layer in the vanishing viscosity limit.

We prove [1] that for any circularly symmetric initial velocity in L^2, the solution of the Navier-Stokes equations converges strongly in $L^\infty([0,T],L^2)$ to the corresponding stationary solution of the Euler equations. This result generalizes work of S. Matsui, J. Bona and J. Wu, and is related to work of X. Wang. Our proof relies on a symmetry reduction of the equations and semigroup methods for the reduced problem. In reduced form, the Navier-Stokes equations become a heat equation with potential that is singular the origin, and forcing that is measure-valued in time.

We also analyze vorticity production at the boundary, and asymptotically derive the Prandtl equations, which describe the behavior of the fluid in the boundary layer.

References

[1] M. C. Lopes, A. L. Mazzucato, H. J. Nussenzveig Lopes, "Vanishing viscosity limit for incompressible flow inside a rotating circle". Submitted to *Indiana Univ. Math J.*.