The Universality of Dynamic Multiscaling in Homogeneous, Isotropic Turbulence [1]

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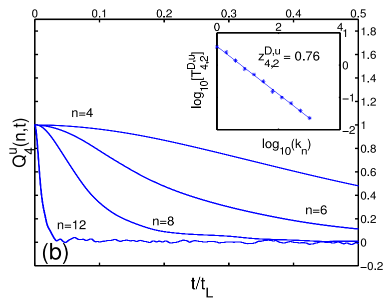
The elucidation of the universal scaling properties of equal-time and time-dependent correlation functions in the vicinity of a critical point was one of the most important achievements of statistical mechanics over the past forty years. The analogous systematization of the power laws and associated exponents that govern the behaviour of structure functions in a turbulent fluid, or in a passive-scalar advected by such a fluid, is a major challenge in the areas of nonequilibrium statistical mechanics, fluid mechanics, and nonlinear dynamics. We develop here the systematics of the multiscaling of time-dependent structure functions for the case of *decaying* fluid and passive-scalar turbulence.

The nature of multiscaling of *time-dependent* structure functions has been examined recently[4, 5, 6] but only for the case of *statistically steady* turbulence. Does it have an analogue in the case of decaying turbulence, since time-dependent structure functions must, in this case, depend on the origin of time t_0 at which we start our measurements? This question has not been addressed hitherto. We show here how to answer it in decaying fluid and passive-scalar turbulence. In particular, we propose suitable normalisations of time-dependent structure functions that eliminate their dependence on t_0; we demonstrate this *analytically* for the Kraichnan version of the passive-scalar problem and its shell-model analogue and *numerically* for the GOY shell model[2, 7, 8, 9] for fluids and a shell-model version of the advection-diffusion equation. In these models we then analyse the normalised time-dependent structure functions for the case of decaying turbulence like their statistically steady counterparts [5, 6]. This requires a generalisation of the multifractal formalism[2] that finally yields the same bridge relations between dynamic and equal-time multiscaling exponents as for statistically steady turbulence[5, 6]. Studies show that, if dynamic

multiscaling exists, time-dependent structure functions must be characterised by an infinity of time scales and associated dynamic multiscaling exponents[5]. We show that the dynamic exponents depend on how we extract time scales from time-dependent structure functions; e.g., for fluid turbulence, time scales obtained from integrals (superscript I and subscript 1) and second derivatives (superscript D and subscript 2) of order-p time-dependent structure functions yield the *different* dynamic exponents $z_{p,1}^{I,u}$ and $z_{p,2}^{D,u}$. Finally, we demonstrate how the different dynamic multiscaling exponents are related to the equal-time multiscaling exponents via *different classes of linear bridge relations* for decaying turbulence. In addition, we find numerically for shell models of fluid and passive-scalar turbulence that dynamic-multiscaling exponents have the same values for both statistically steady and decaying turbulence.

To summarise, our work provides strong evidence for the *universality* (i.e., the equality of dynamic scaling exponents in decaying and statistically steady turbulence) of the multiscaling of time-dependent structure functions in turbulence of fluids and passive-scalars.



Representative plot of the normalised, GOY-model, fourth-order, time-dependent velocity structure function in decaying fluid turbulence for different shells n versus the nondimensional time t/t_L, where t_L is the large-eddy-turnover time. The inset shows a log-log plot of the derivative-time versus the wavevector; the slope yields the dynamic multiscaling exponent $z_{4,2}^{D,u} = 0.76$.

References

[1] S. S. Ray, D. Mitra, and R. Pandit, to be submitted in December 2006.

[2] U. Frisch, *Turbulence: The Legacy of A.N. Kolmogorov* (Cambridge University, Cambridge, England, 1996).

[3] G. Falkovich, K. Gawedzki and M. Vergassola, Rev. Mod. Phys. **73**, 913 (2001).

[4] V.S. L'vov, E. Podivilov, and I. Procaccia, Phys. Rev. E **55**,7030 (1997).

[5] D. Mitra and R. Pandit, Phys. Rev. Lett. **93**, 2 (2004).

[6] D. Mitra and R. Pandit, Phys. Rev. Lett. **95**, 144501 (2005).

[7] T. Bohr, M.H. Jensen, G. Paladin, and A. Vulpiani, *Dynamical Systems Approach to Turbulence* (Cambridge University, Cambridge, England, 1998).

[8] L.P. Kadanoff, D. Lohse, J. Wang, and R. Benzi, Phys. Fluids. **7**, 617 (1995).

[9] E. Gledzer, Sov. Phys. Dokl. **18**, 216 (1973).