Hydrodynamics of Fish Turns

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Swimming in fish is characterised by a travelling wave passing down the length of the body [1, 2, 3]. Symmetric transverse oscillations of the fish body will generate thrust and propel the fish forward. Turns are effected by generating an asymmetric transverse movement of the fish body, frequently as a C-shaped bend [4]. Typical fish swimming speeds allow for simplifying assumptions of incompressible and inviscid flow.

The objective of the current work is to use existing theoretical models to analyse fish turns. Lighthill's classical 'elongated-body theory' [1] on fish swimming is the fundamental basis for the three dimensional flow model and recoil correction concept implemented here. For the two-dimensional flow case, Wu's approach to swimming of a waving plate [2] is used. Hill developed a numerical boundary element method for large amplitude swimming for two and three-dimensional flow [5]. His approach has been used to validate the theoretical models for fish turns.

Lighthill's theory for fish swimming in three dimensions assumes transverse dimensions and deflections are small compared to the body length. Flow around the body is the sum of steady fluid flow (steady longitudinal swimming velocity U) and flow due to transverse body displacement, $h(x,t)$. Pressure force is generated by the added mass accelerating sideways due to body undulation, while moving along the body length with wave-speed U_o. Force and moment equations balance inertial components due to added mass with the pressure components. Non-zero force and moment balance is manifest as rigid body translation and rotation about the centre of mass. Thus the net transverse motion of the body can be represented as a linear combination of prescribed motion and rigid-body motion or recoil correction (Eqn. 1).

$$h(x,t) = f(x)g(t) + R(t) + x\Theta(t) \tag{1}$$

Wu's theory examines fish propulsion as a two dimensional potential flow problem for a waving plate of finite chord and negligible thickness. The body profile is represented as a swimming plate with infinite degrees of freedom. The Kussner-Schwarz [6] general solutions for oscillating rigid airfoils with finite DOFs were modified for a deformable plate. Wu's approach accounts for discontinuities due to the wake and the thin plate by introducing an analytic acceleration potential function to solve the Euler equations. The momentum equation is given by Eqn 2, ψ is the acceleration potential function.

$$-\frac{1}{\rho}\nabla p = -\frac{\partial\psi}{\partial x} = \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)^2 h(x,t) \tag{2}$$

In the method developed for turning in two dimensional flow, Wu's theory is extended for analysing turns. Transverse motion of the flat waving plate is prescribed by a displacement signal as given by Eqn 3 for finite time t_o. Lighthill's approach to calculating recoil correction is implemented to ensure zero net force and moment. Accordingly, angular and transverse motion of the body are computed and final orientation of the plate after the manoeuvre is calculated.

$$f(x)g(t) = (x-x_{cg})^2(-7.63t^6+22.905t^5-20.92t^4+3.66t^3+1.96t^2+.0257t) \quad (3)$$

A two dimensional numerical model employing a boundary element method has been developed to validate the theoretical model. The numerical method uses a discrete point vortex distribution to represent the infinitesimally thin plate. Using Green's Identity, unsteady potential flow solutions are developed to construct general solutions to the Laplace equation. Bernoulli's equation is used to calculate pressure over the plate and thereby, instantaneous force and moment. Lighthill's recoil correction approach is implemented to compute angular and transverse recoil at the end of the applied signal. Numerical results for angular and transverse recoil are compared with results based on Wu's theoretical model in Figure 1. These results compare quite well suggesting Wu's model adapts well to studying turns of the form of Eqn 3.

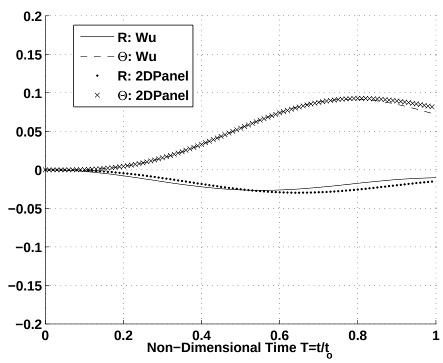

Figure 1: 2D flow recoil correction calculations for prescribed signal f(x)g)t) applied from time $t = 0$ to $t = t_o$

A similar approach has been developed in three dimensions using Lighthill's model and three-dimensional boundary element methods. Details on methodology and results will be discussed in the poster.

References

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