Mixing in a Simple Viscoelastic Flow

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In the past several years it has come to be appreciated that in low Reynolds number flow the nonlinearities provided by non-Newtonian stresses of a complex fluid can provide a richness of dynamical behaviors more commonly associated with high Reynolds number Newtonian flow. For example, experiments by V. Steinberg and collaborators have shown that dilute polymer suspensions being sheared in simple flow geometries can exhibit highly time dependent dynamics and show efficient mixing [1, 2, 3]. The corresponding experiments using Newtonian fluids do not, and indeed cannot, show such nontrivial dynamics. To better understand these phenomena we study numerically the 2D Oldroyd-B viscoelastic model at low Reynolds number. A background force is used to create a periodic cell with four-roll mill vortical structure around a hyperbolic fixed point. We consider both steady and time-periodic forcing. For low Weissenberg ($\mathcal{W}i$) number the elastic stresses are bounded and slaved to the forcing, with mixing confined to small sets near the hyperbolic point. At larger $\mathcal{W}i$ an analog to the coil-stretch transition occurs, yielding large stresses and stress gradients concentrated on sets of small measure, perhaps indicating the development of singularities. The flow then becomes very sensitive to perturbations in the forcing and there is a transition to global mixing in the fluid.

Figure 1 (a) - (b) shows contour plots of tr $\mathbf{S}$ (in color), where S is the polymer stress, with vorticity contour lines overlayed on top. Figure 1(a) and 1(a') are simulations for $\mathcal{W}i = 0.6$, and Fig. 1(b) and 1(b') show results for $\mathcal{W}i = 5.0$. The vortical lines for lower Weissenberg number (including $\mathcal{W}i = 0.6$) are not changed qualitatively by the addition of the polymer stress, i.e. the four vortex flow persists and no additional features are created. Above a critical Weissenberg number this begins to change, and we see for $\mathcal{W}i = 5.0$ that additional vortices, which are oppositely signed, are generated along the stable and unstable manifolds of the hyperbolic point. Increasing the Weissenberg number decreases the overall magnitude of vortex strength. As $\mathcal{W}i$ increases tr $\mathbf{S}$ concentrates on thinner sets along the unstable manifold of the hyperbolic points in the flow. At the central hyperbolic point tr $\mathbf{S}$ is dominated by S_{11}, the first component of the stress tensor. Figure 1(a') shows slices of $S_{11}(\pi, y, t)$ along the stable manifold of the central hyperbolic point for $t = 1, 2, ..., 10$, increasing in time. It appears that S_{11} is approaching a cusp-singularity exponentially in time for $\mathcal{W}i = 0.6$. For $\mathcal{W}i = 5.0$, tr S appears to become unbounded (exponentially) in time, see Fig. 1(b) and 1(b'). A local solution can be constructed at the hyperbolic point which agrees very well with the simulations. This local solution predicts exponential in time approach to a solution whose smoothness depends critically on the Weissenberg number. This work has been submitted for publication [4].

When a small time periodic perturbation from the four vortex background force is added (along with a small amount of numerical viscosity to control the large stress gradients) the flow

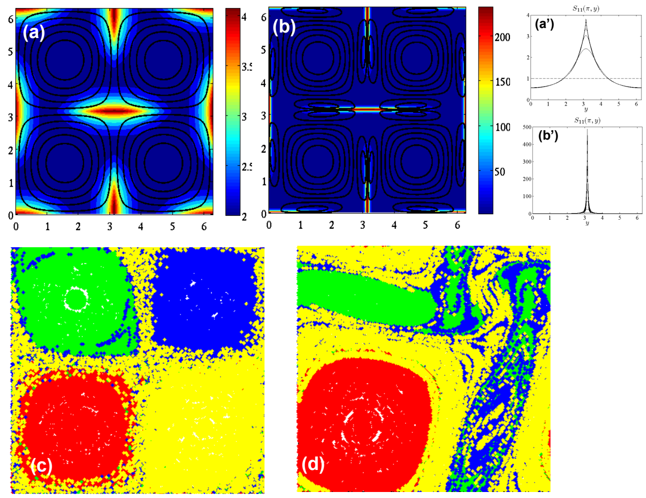

Figure 1: (a) - (b) Color contours represent tr $\mathbf{S}$ and overlayed contour lines are vorticity lines (at $t = 6$). (a) $\mathcal{W}i = 0.6$, (b) $\mathcal{W}i = 5.0$, Note the difference in scale. (a') Plot of $S_{11}(\pi, y, t)$ (first component of stress along stable manifold of hyperbolic point) for $t = 1, 2, ..., 10$, (increasing in time) and $\mathcal{W}i = 0.6$. (b') $S_{11}(\pi, y, t)$ for $t = 1, 2, ..., 10$, $\mathcal{W}i = 5.0$. (c) - (d) Particle tracers after $t = 400$. Initial configuration of particles was a single color in each quadrant. (c) $\mathcal{W}i = 0.5$, very similar to solution for a pure Newtonian solvent. (d) $\mathcal{W}i = 10.0$.

demonstrates large scale mixing for sufficiently large $\mathcal{W}i$. Figure 1 (c) - (d) show particle tracers at time $t = 400$. The initial configuration of the particle tracers is a single color in each of the four vortex cells. With a steady force the particles remain entirely in their respective cells. Figure 1(c) shows the effect of the time periodic perturbation for low Weissenberg number (which is qualitatively similar to the behavior for a very low Reynolds number Newtonian fluid). There is some mixing along the stable and unstable manifold, but the four vortices remain largely separated. Figure 1(d), for sufficiently large $\mathcal{W}i$, shows very different behavior. The blue, green, and yellow vortices have mixed quite significantly - one can see the effect of stretching and folding which interweaves the fluid particles from different vortices. However, the vortex on the lower left remains largely undisturbed.

References

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