Chaotic hypothesis in fluids and possible applications

Chaotic hypothesis (CH): *Motions developing on the attracting set of a chaotic system can be regarded as motions of a transitive hyperbolic (also called "Anosov") system.*

Besides the physical meaning it has a few implications
(1) Existence of time averages and their statistics μ (SRB statistics)

$$\frac{1}{T}\int_0^T F(S_t x)\,dt \xrightarrow[T\to\infty]{} \int_{\mathcal{A}} F(y)\mu(dy) \stackrel{def}{=} \langle F\rangle$$

(2) Coarse graining is rigorously definable and SRB is, consequently, interpretable as equidistribution on the attracting set ($\Rightarrow$ variational principle and existence of Lyapunov function)
(3) μ admits an explicit representation so that averages can be written and compared (without computing them)
(4) large deviations law holds: $f_j \stackrel{def}{=} \frac{1}{\tau}\int_0^\tau F_j(S_t x)dt$

$$Prob(\mathbf{f}\in\Delta) = Prob((f_1,\ldots,f_n)\in\Delta) \propto_{\tau\to\infty} e^{\tau \max_{\mathbf{f}\in\Delta}\zeta(\mathbf{f})}$$

ζ defined in a convex open set Γ and analytic there and $\zeta = -\infty$ outside $\overline{\Gamma}$
(5) mechanical interpretation of entropy creation rate as *entropy increase of the thermostats*: it is the divergence of the equations of motion

$$\varepsilon(x) = \sum_j \frac{Q_j}{T_j} + \text{"total derivative"}$$

(6) in *time reversal invariant* cases FT: (F_j odd)

$$\zeta(-\mathbf{f}) = \zeta(\mathbf{f}) - \langle\varepsilon\rangle$$

provided $\sigma = \varphi(\mathbf{F})$ and $\langle\varepsilon\rangle > 0$. No free parameters. More surprisingly

$$\frac{Prob_\mu(F_j(S_t x) = \varphi(t),\, t\in[0,\tau])}{Prob_\mu(F_j(S_t x) = -\varphi(\tau - t),\, t\in[0,\tau])} \propto e^{\int_0^t \sigma(S_t x)dt}$$

2. Fluids

(1) Fluid equations are not reversible. Equivalence conjecture:

$$\dot{\mathbf{u}} + \utilde{\mathbf{u}} \cdot \utilde{\partial}\,\mathbf{u} = \nu\Delta\mathbf{u} - \partial\, p + \mathbf{g},$$

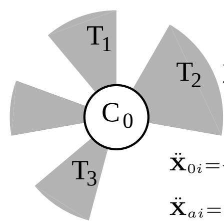
$$\dot{\mathbf{u}} + \utilde{\mathbf{u}} \cdot \utilde{\partial}\,\mathbf{u} = \alpha(\mathbf{u})\Delta\mathbf{u} - \partial\, p + \mathbf{g}, \qquad \alpha = \frac{\int \mathbf{u}\cdot\mathbf{g}}{\int(\partial\,\mathbf{u})^2} \;\Rightarrow\; \int \mathbf{u}^2 = \mathcal{E} = const$$

Same statistics for “local observables”: F local $\Rightarrow$ F depends on finitely many Fourier comp. of $\mathbf{u}$.

Same statistics $\Rightarrow$ as $R \to \infty$ if $\mathcal{E}$ is chosen $= \langle \int \mathbf{u}^2\rangle_{\mu_\nu}$ (equivalence): “Gaussian NS eq.” or “GNS”. So far *only numerical tests in strongly cut off equations and* $d = 2$ (Rondoni,Segre).

(2) If so CH→FT may be applicable: BUT necessary a local version

(3) Model (reversible)



$\mathbf{X}_0, \mathbf{X}_1, \ldots, \mathbf{X}_n$

$\ddot{\mathbf{X}}_{0i} = -\partial_i U_0(\mathbf{X}_0) - \sum_a \partial_i U_a(\mathbf{X}_0, \mathbf{X}_i) + \mathbf{F}_i$

$\ddot{\mathbf{X}}_{ai} = -\partial_i U_a(\mathbf{X}_a) - \partial_i U_a(\mathbf{X}_0, \mathbf{X}_a) - \alpha_a \dot{\mathbf{X}}_{ai}$

with
$\alpha_a \equiv \frac{W_a - \dot{U}_a}{3N_a k_B T_a} \;\Rightarrow\; K_a = \frac{1}{2}\sum_i \dot{\mathbf{X}}_{ia}^2 = \frac{3}{2} k_B T_a\, N_a, \qquad a = 1, \ldots, n$
System works on thermostats: $W_a = Q_a =$ **heat**

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$$W_a = -\dot{\mathbf{X}}_a \cdot \partial_{\mathbf{X}_a} U_a(\mathbf{X}_0, \mathbf{X}_a)$$

Divergence $= \sigma(\mathbf{X}, \dot{\mathbf{X}}) = \varepsilon(\mathbf{X}, \dot{\mathbf{X}}) + \dot{R}(\mathbf{X}, \dot{\mathbf{X}})$ with:

$$\varepsilon(x) = \sum_{i>0} \frac{Q_i}{k_B T_i}, \quad R = \sum_{i>0} \frac{U_i}{k_B T_i}$$

Compatibility? near equil. entropy creation independently defined (DeGroot-Mazur)

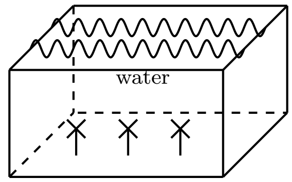
Idea: regard fluid composed by vol. elements each as system thermostatted by neighbors? In the continuum approx.

Volume elements $E = d\mathbf{x}$ are not separated by walls, hence exchange particles and move.

Size λ macrosc. small, microsc. large: diffusion, in characteristic evolution time, must be $\ll \lambda$. $\varepsilon(x) = \sum_{E,E'} \frac{Q_{E,E'}}{k_B T_{E'}} \qquad \Rightarrow \qquad \langle \varepsilon \rangle = \varepsilon_{classic} + \frac{\dot{S}}{k_B}$

$$k_B \langle \varepsilon \rangle = k_B \varepsilon_{classic} + \dot{S}, \qquad k_B \varepsilon_{classic} = \int_{\mathcal{C}_0} \Big(\kappa \left(\frac{\partial T}{T} \right)^2 + \eta \, \frac{1}{T} \underset{\sim}{\boldsymbol{\tau}}' \cdot \underset{\sim}{\partial} \mathbf{u} \Big) \, d\mathbf{x}$$

3. Bandi-Cressman-Goldburg experiment



Lagrangian flow *on a 2D surface*
$\dot{\mathbf{x}} = \mathbf{u}(\mathbf{x}, t)$
with $\mathbf{u}$ turbulent
"Kraichnan flow (space-time colored)"

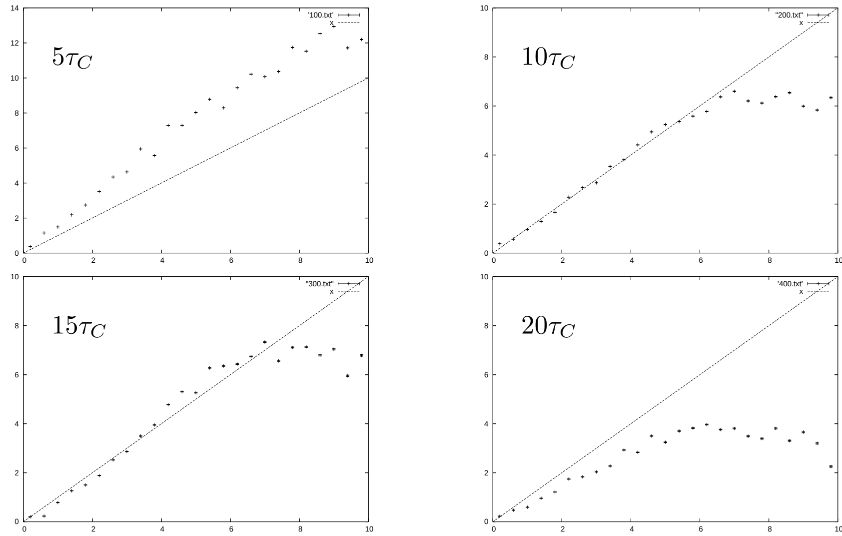
Turbulent water in $1m \times 1m \times 0.3m$. Generate "Lagrangian trajectories" on surface

$$\dot{\mathbf{x}} = \mathbf{u}(\mathbf{x}, t)$$

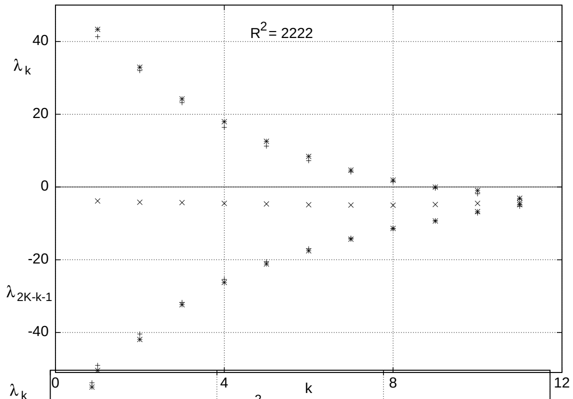
$\sigma(\mathbf{x}(t), t) = -\mathrm{div}\mathbf{u}(\mathbf{x}(t), t)$ whose time av. is exper. measured to be $\sigma_+ = \Omega > 0$.

$$p = \frac{1}{\tau}\int_0^\tau \frac{\sigma(\mathbf{x}(t), t)}{\Omega}\, dt, \qquad ? \qquad \zeta(-p) = \zeta(p) - p\,\Omega$$

for $\tau \gg \tau_c =$ "characteristic time of turbulence evolution".
(1) a definitely non Gaussian statistics for the variable $p = \frac{1}{\tau}\int_0^\tau \frac{\sigma(\mathbf{x}(t),t)}{\Omega} dt$.
(2) remarkably large fluctuations of p for values of τ up to $800ms$.
(3) the obstacle of not knowing the quantity p because the equations of motion are usually not known is not present: FR prediction, no free parameters.
(4) the statistics is quite large as about 8×10^4 Lagrangian trajectory staying in the field of vision for a time length $T = 6$s are observed. Dividing T into $k = T/\tau$ "segments", with time duration τ, $8 \times 10^4\, k$ averages of velocity divergences are obtained, with k varying between 60 and 15 (where 60 corresponds to $\tau = 100$ms). Small statistics so far had been a major obstacle to FR tests.
Theory: Chetrite,Delannoy,Gawedzki, Bonetto,Gentile,G.

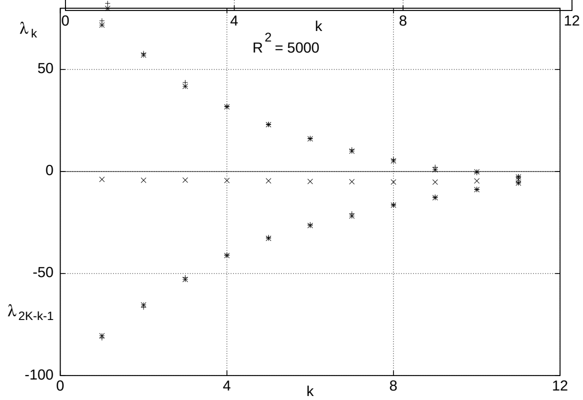

5τ_C
'100.txt'
x
10τ_C
'200.txt'
x
15τ_C
'300.txt'
x
20τ_C
'400.txt'
x

.



R^2= 2222
λ_k
λ_{2K-k-1}
40
20
0
-20
-40
0
4
8
12
k

.



R^2 = 5000
λ_k
λ_{2K-k-1}
50
0
-50
-100
0
4
8
12
k

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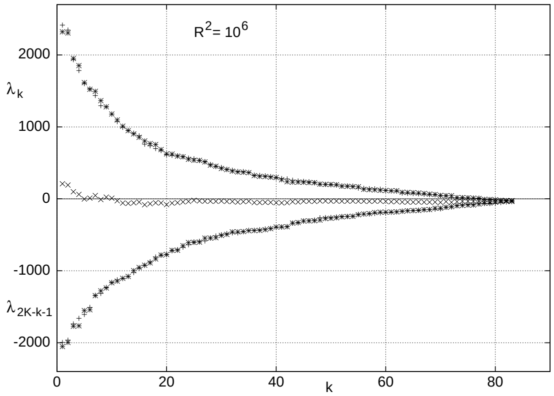
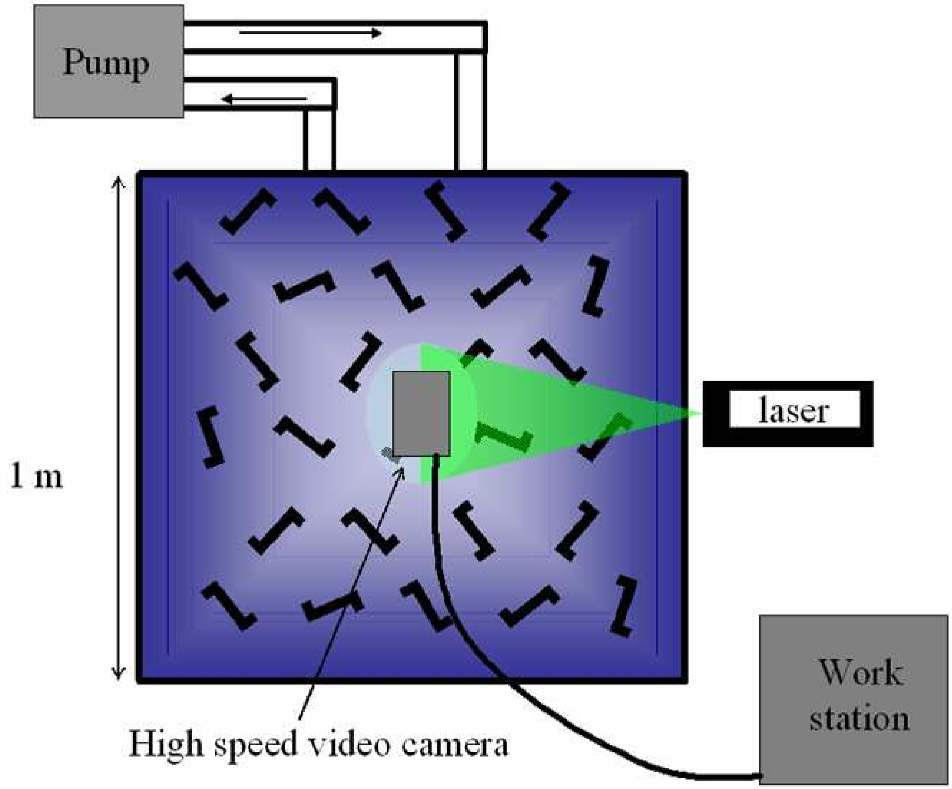


$R^2 = 10^6$
2000
λ_k
1000
0
-1000
λ_{2K-k-1}
-2000
0
20
40
k
60
80


Pump
laser
1 m
Work
station
High speed video camera