

# experimental particle tracking in turbulent flows

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# why would one track fluid particles ?



Scalar dispersion:

how does it spread from the source ?

$$\langle \theta(\mathbf{x}, t) \rangle = \int_{s \leq t} ds \int_V d\mathbf{y} p_1(\mathbf{x}, t; \mathbf{y}, s) S(\mathbf{y}, s)$$

➔ one particle statistics

Mixing:

how fast two chemicals become close together?

what is the magnitude of concentration fluctuations

$$\langle \theta(\mathbf{x}_1, t_1) \theta(\mathbf{x}_2, t_2) \rangle = \int_{s_1 \leq t_1} ds_1 \int_{s_2 \leq t_2} ds_2 \int_V d\mathbf{y}_1 \int_V d\mathbf{y}_2 \\ \times p_2(\mathbf{x}_1, \mathbf{x}_2, t_1, t_2; \mathbf{y}_1, \mathbf{y}_2, s_1, s_2) S(\mathbf{y}_1, s_1) S(\mathbf{y}_2, s_2)$$

➔ two particle statistics

# issues in particle tracking in turbulent flows

- single out individual particles
  - track individual trajectories
  - enough spatial resolution
  - enough time resolution
- 
- single particle statistics  
(trajectories, velocity, acceleration)
  - multi-particle statistics  
(relative dispersion, shape evolution...)

# Lagrangian tracking methods

- Doppler techniques:
  - ultrasound Doppler (Lyon, Grenoble)
  - Laser Doppler (Göttingen, Lyon)

➔ gives the velocity from a frequency modulation
- Imaging techniques: 3D particle tracking velocimetry
  - 1D silicon strip detectors (Cornell)
  - 2D cameras (Risoe, Göttingen, Cornell, Zurich)

➔ gives the trajectories



# resolution requirements

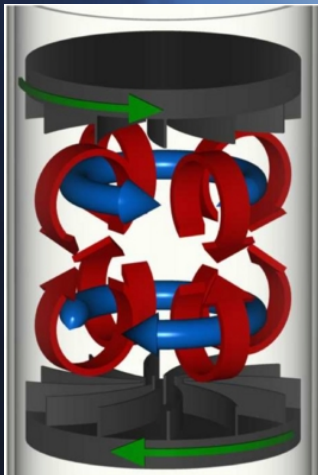
ex: to achieve a measurement in a  $R_\lambda=1000$  lab flow:

- temporal resolution (required for both methods):  
 $T_L/\tau_\eta \sim 1000$  and in experiments  $\tau_\eta \sim 1\text{ms}$
- spatial resolution (for imaging only):  
 $L/\eta \sim R_\lambda^{3/2} \sim 4000$  with  $\eta \sim 30\ \mu\text{m}$   
+ measure over  $2L$   
+ resolve scales sub  $\eta$   
with 1/10 sub pixel resolution: several thousand pixels
- frequency resolution (Doppler)  
resolution in frequency is the resolution in velocity  
maybe in competition with temporal resolution  
(time-frequency analysis)  
in general: parametric estimation

# Doppler techniques

acoustic and optical versions developed in ENS Lyon:

- acoustics: ultrasound  $\lambda \sim 0.6$  mm  
large measurement volume ( $2L \sim 10$  cm)  
not so small particles
- optics: Laser  $\lambda \sim 0.532$   $\mu\text{m}$   
small measurement volume ( $\sim 5$  mm)  
small particles  
very good time resolution

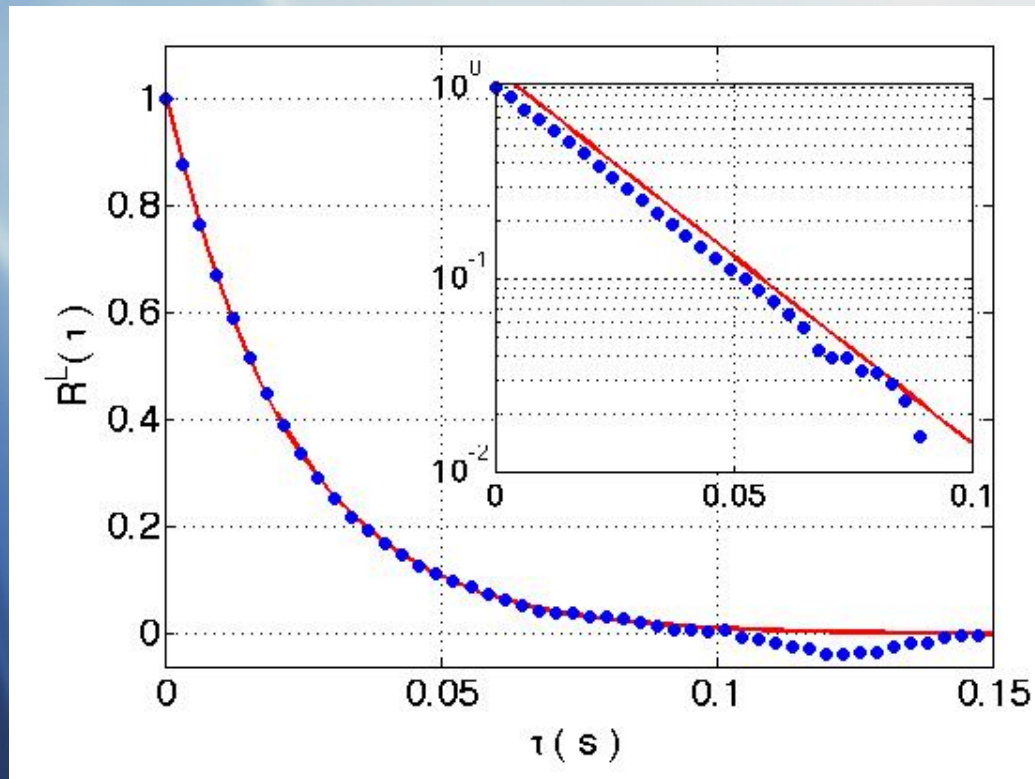


most presented results measured in variations  
on a French washing machine theme

# ultrasound Doppler technique

Lyon: Mordant, Michel, Metz, Pinton

large measurement volume: ultrasounds (about 2 times L)



velocity  
autocorrelation

*Mordant, Metz, Michel & Pinton PRL 2001*

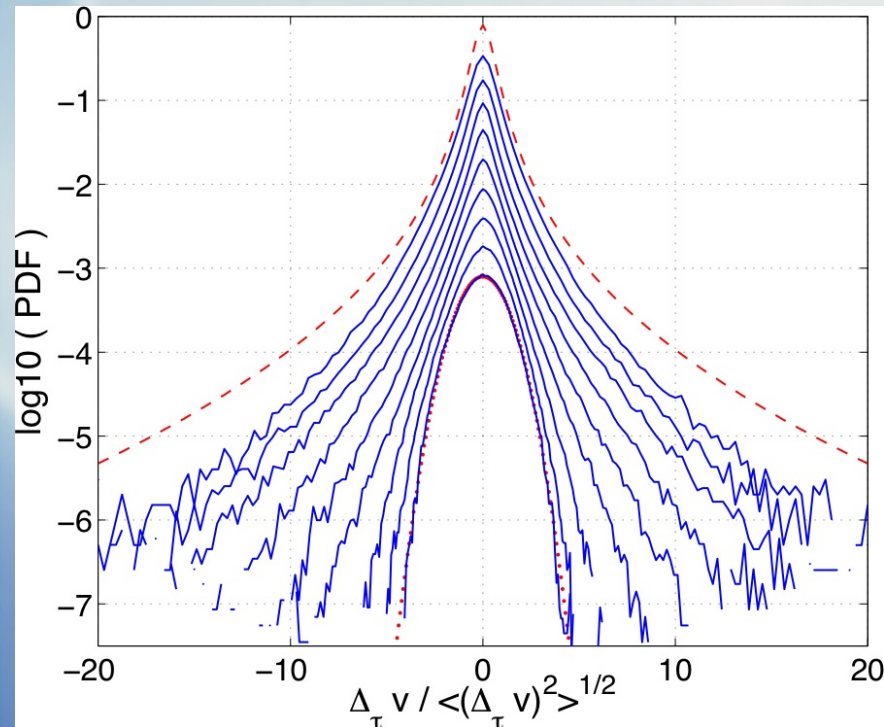
also in a air jet in Grenoble: *Gervais, Baudet, Gagne Exp Fluids 2007*

# ultrasound Doppler technique

velocity time increments

$$\Delta_{\tau} v = v(t + \tau) - v(t)$$

intermittency



*Mordant, Metz, Michel & Pinton PRL 2001*

multifractal description by Chevillard et al. *PRL* 2003



# ultrasound Doppler technique

Kolmogorov constant  $C_0$  :

$$\langle (v(t + \tau) - v(t))^2 \rangle = C_0 \epsilon \tau$$

and

$$\langle v(t + \tau)v(t) \rangle = \sigma^2 \exp\left(-\frac{\tau}{T_L}\right)$$

then 
$$T_L = \frac{2\sigma^2}{C_0 \epsilon}$$

here  $C_0 \sim 4$  at  $R_\lambda = 800$

other recent estimate: Göttingen  $C_0 \sim 6$  at a similar  $R_\lambda$   
(Xu, Ouellette & Bodenschatz *ETC11 proc.*)

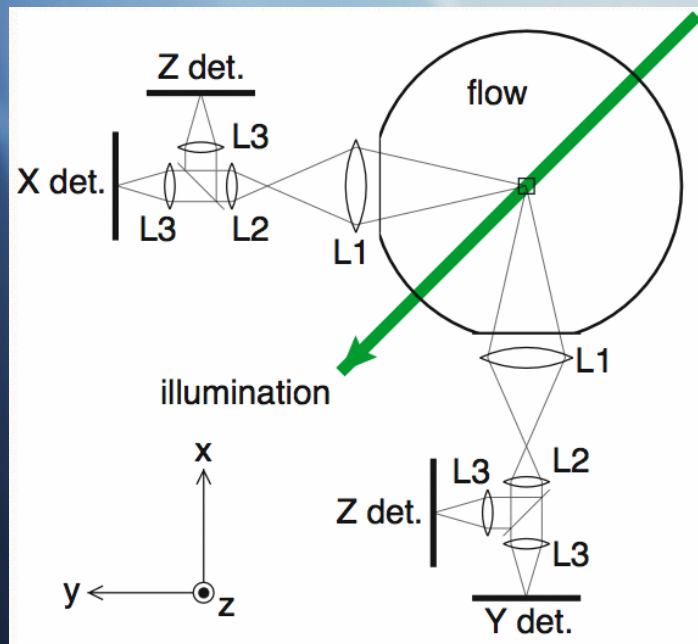
important for stochastic modeling of dispersion:

$$dv = -\frac{v}{T_L} dt + \sqrt{C_0 \epsilon} dW(t)$$

# fluid particle acceleration

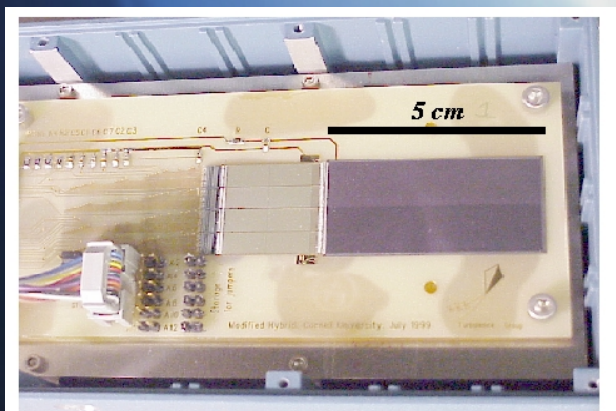
Cornell University (E. Bodenschatz)

to get the acceleration:  
resolve the smallest temporal  
and spatial scales



imaging technique:  
4 linear cameras (silicon strip detectors)  
trajectory of single particles

resolution:  
70,000 frames/s  
8  $\mu\text{m}$ /pixel ( $<\eta$ )



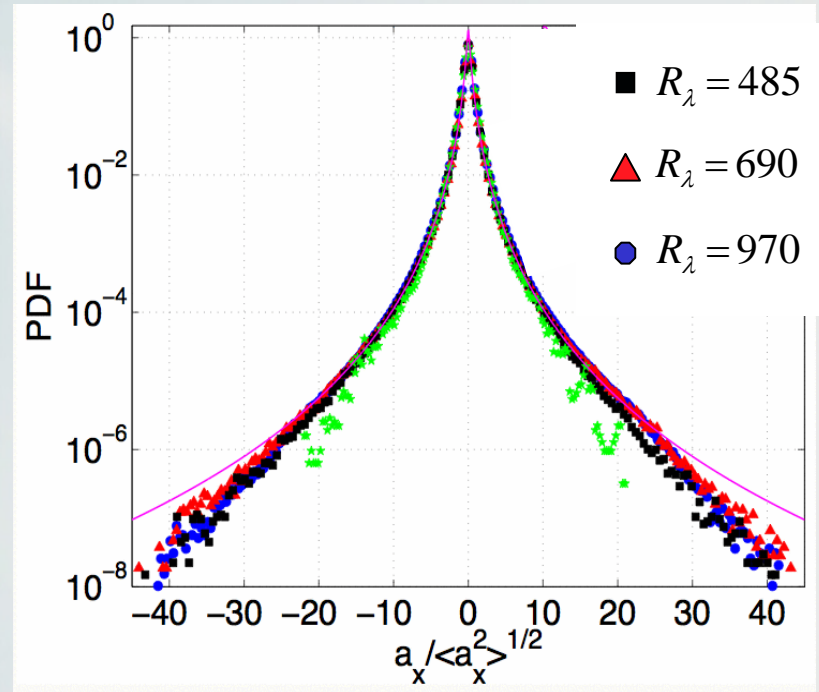
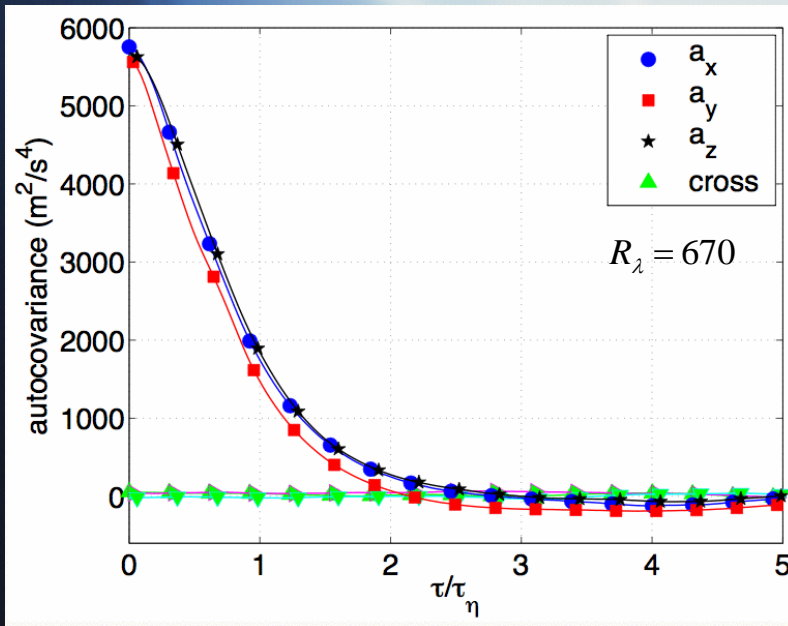
Voth, La Porta, Crawford, Alexander & Bodenschatz  
*J. Fluid Mech.* **469** (2002)

Mordant, Crawford & Bodenschatz *PRL* **93** (2004)

# fluid particle acceleration

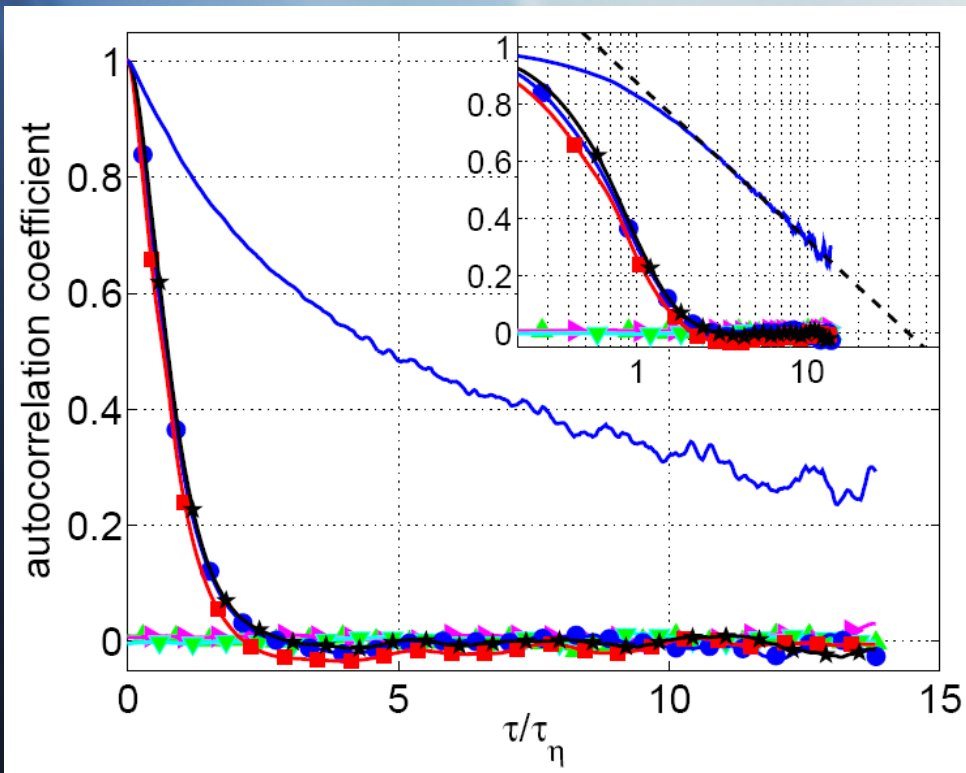
highly non Gaussian distribution  
of the acceleration components

very large accelerations ( $>1000g$ )



time dynamics related to  
the Kolmogorov time scale

# fluid particle acceleration



acceleration magnitude  
correlated over  
integral time scales  
  
trapping in vortices

Mordant, Crawford & Bodenschatz *PRL* **93** (2004)

see also Mordant, L ev eque, Pinton *NJP* 2004, *PRL* 2002

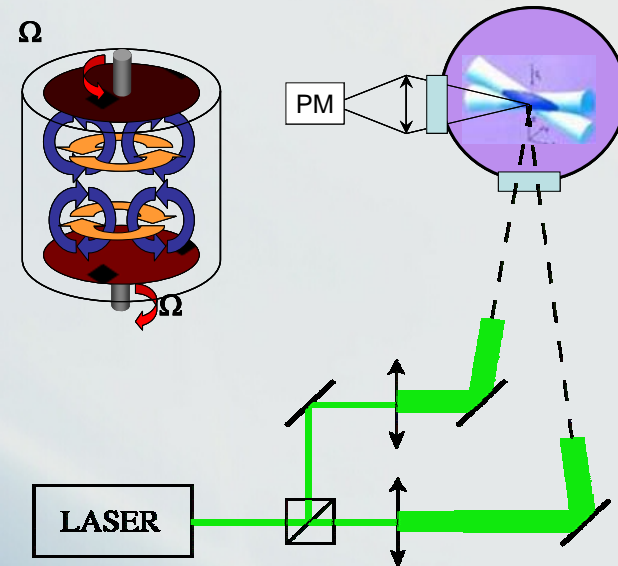
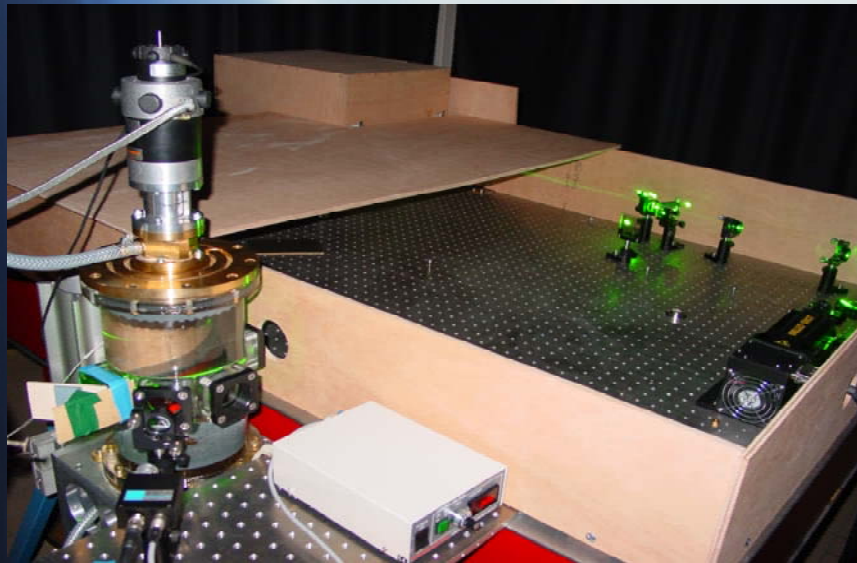


# fluid particle acceleration using Laser Doppler

new experiment in Lyon

R. Volk, G. Verhille, N. Mordant, J.-F. Pinton

based on classical LDV but with wide beams  
1W laser, measurement volume  $\sim (5 \text{ mm})^3$



# fluid particle acceleration using Laser Doppler

reproduces Cornell data very accurately

	$R_\lambda$	$a_0$
Lyon	690	$\sim 6.2$
Cornell	680	$6.2 \pm 0.4$

$$\langle a^2 \rangle = a_0 \epsilon^{3/2} \nu^{-1/2}$$

# inertial particles (preliminary results)

$$\frac{dv_p}{dt} = \beta \frac{Du}{Dt} - \frac{v_p - u}{\tau_s} \quad (\text{very small particles})$$

$$\tau_s = \frac{a^2}{3\nu\beta}$$

$$\beta = \frac{3\rho_f}{2\rho_p + \rho_f}$$

effective inertia  
of the particle

## 3 types of particles:

• latex:	d=1.06	$\beta \sim 1$	a/ $\eta$ =1	neutral
• plastic:	d=1.4	$\beta = 0.8$	a/ $\eta$ =1.5	heavy
• bubbles	d=1.3 10 <sup>-3</sup>	$\beta \sim 3$	a/ $\eta \sim 2.5$	light

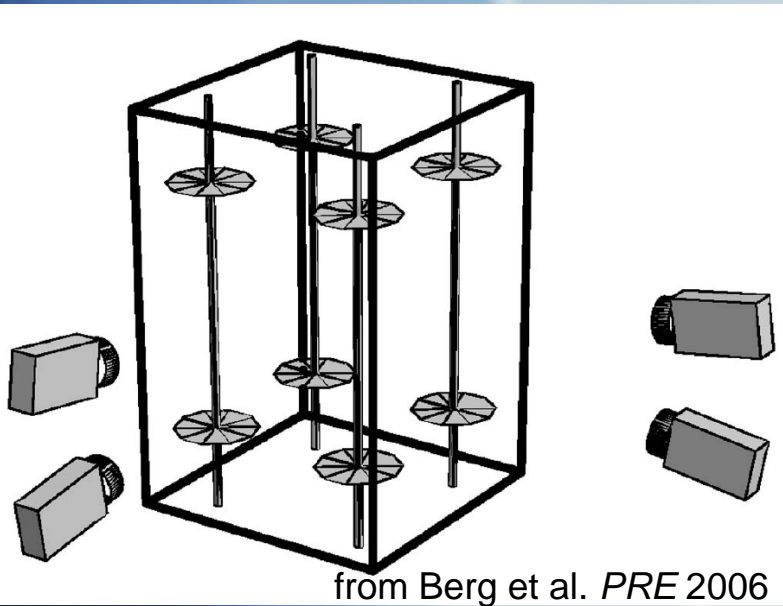
# inertial particles (preliminary results)

no clear change in  
acceleration distribution

clear change in  
acceleration  
variance



# multi-particles measurements



typical measurement:  
relative dispersion of 2 particles

technique: 3D PTV using  
multiple cameras

experimental issues:

- have pairs close enough at the initial time:  
large seeding density and resolution of the small scales
- track particles for a long time ( $\sim T_L$ ):  
low enough seeding density (max 1000 particles in view)
- observe large separations ( $\sim L$ ):  
large field of view (about  $3L$ )

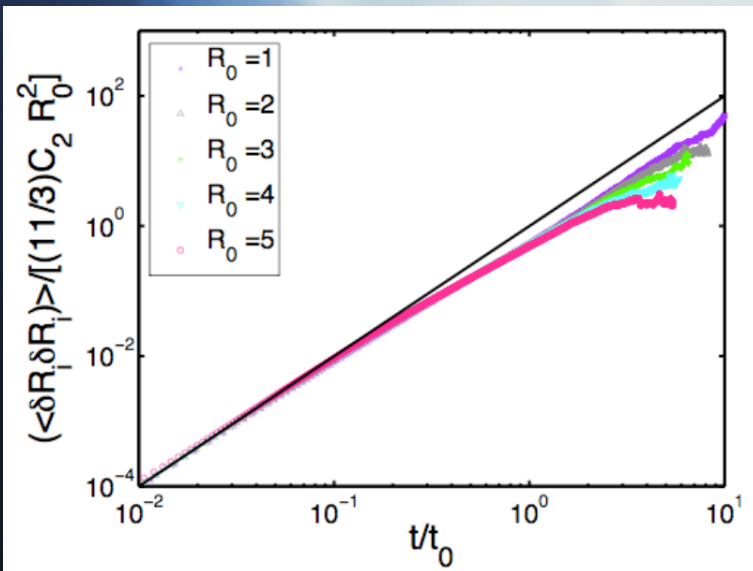
with current cameras moderate Reynolds number

# particles pairs

expected separation following the Richardson prediction

$$\langle r^2 \rangle = g\epsilon t^3$$

for a initial position in the inertial range



for the current exp. ( $R_\lambda \sim 600$ )  
the particles do not forget  
their initial separation  
before reaching the integral length scale

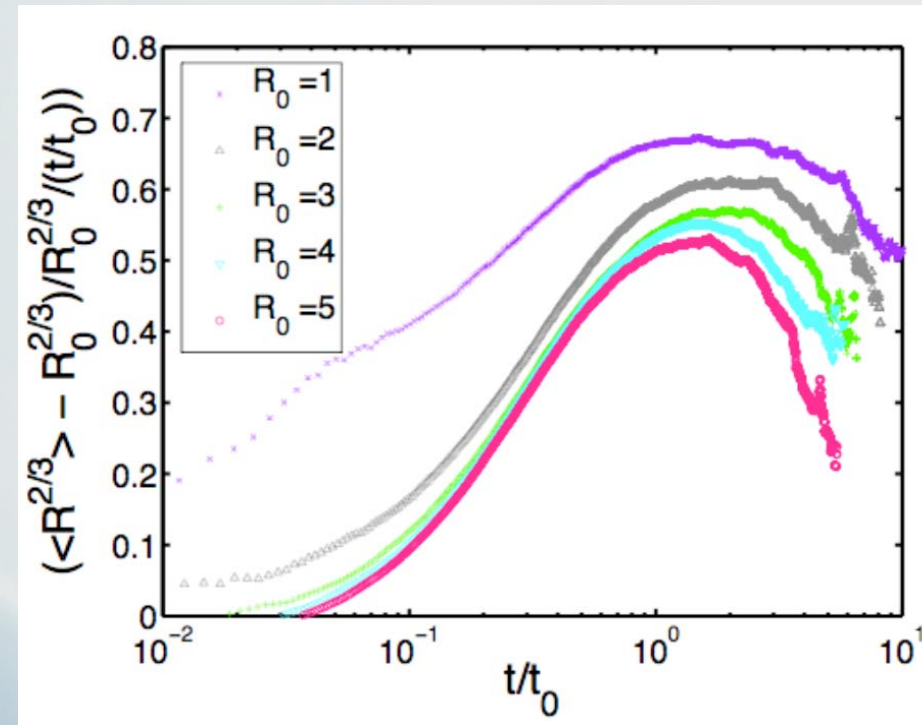
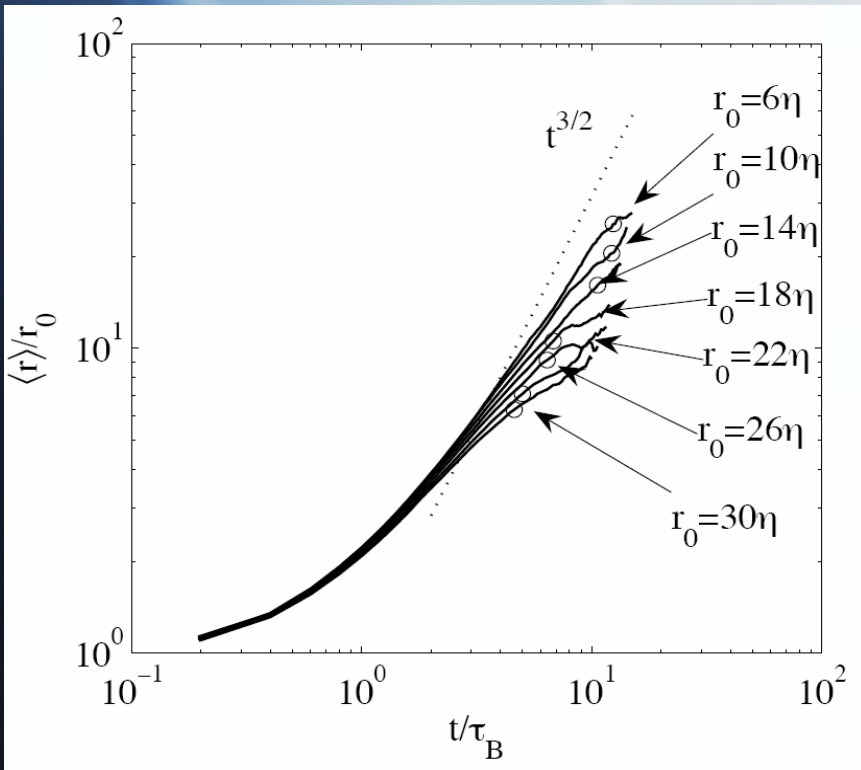
$$\left\langle \left( \frac{r}{r_0} \right)^2 \right\rangle = \frac{11}{3} C_2 r_0^2 \left( \frac{t}{t_0} \right)^2$$

Batchelor  
scaling

$$t_0 = \left( \frac{r_0^2}{\epsilon} \right)^{1/3}$$

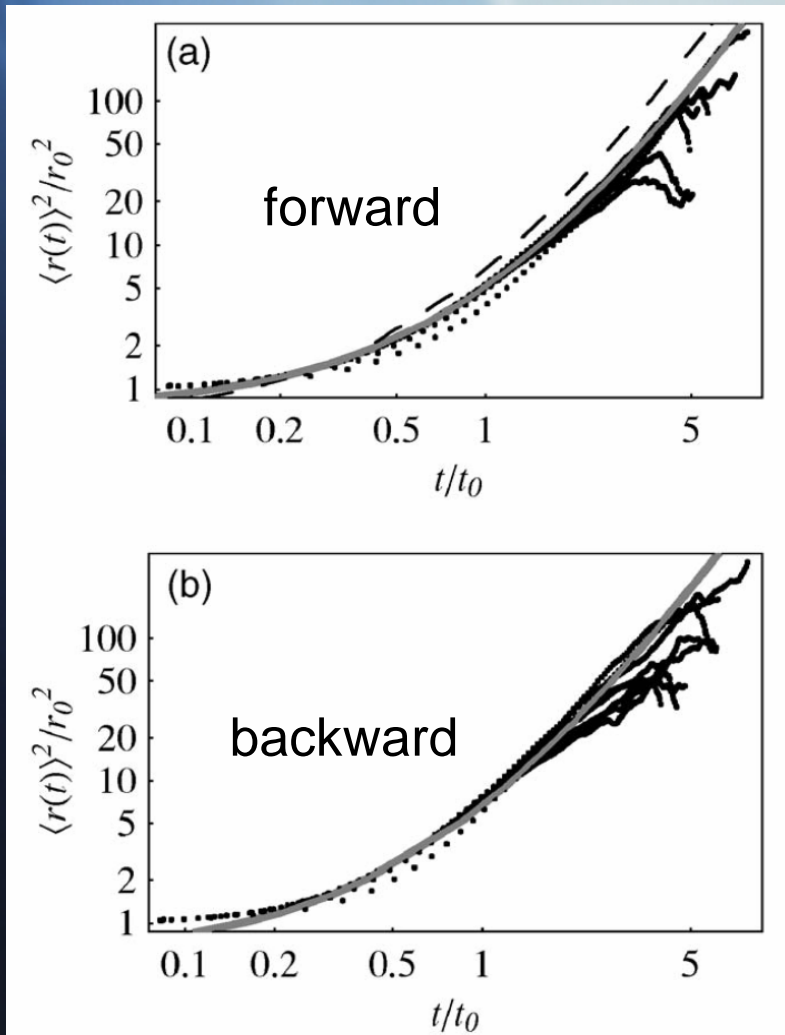
# particles pairs

no clear Richardson scaling:  
requires a very large Reynolds number  
and small initial separations ( $\sim 10\eta$ )



# backward vs forward dispersion

mixing is concerned rather with backwards scattering ie  
how two particles come close together



backward dispersion  
is faster (2x)  
than forward dispersion

link with coarse grained  
velocity gradient tensor

Berg, Lüthi, Mann & Ott *PRE* 2006

see also Xu, Ouellette,  
Nobach & Bodenschatz *ETC11 proc.*



# tetrahedra

matrix of inertia

$$[\rho_1, \rho_2, \rho_3]$$

$$\rho_1 = (X_1 - X_2)/\sqrt{2}$$

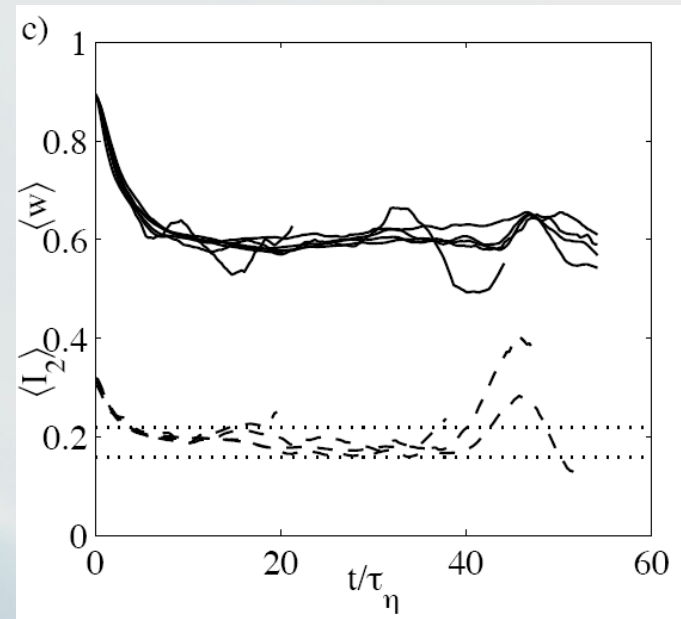
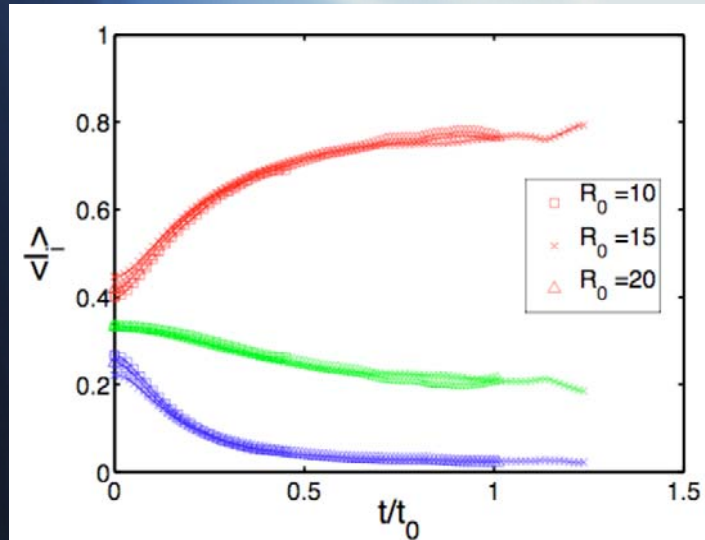
$$\rho_2 = (2X_3 - X_2 - X_1)/\sqrt{6}$$

$$\rho_3 = (3X_4 - X_3 - X_2 - X_1)/\sqrt{12}$$

normalized eigenvalues

$$I_i = \frac{g_i}{g_1 + g_2 + g_3}$$

shape evolution toward planar shapes



# Lagrangian stats

- well resolved 1 particle measurements
- acceleration measurement:
  - strongly non Gaussian distribution
  - long time correlations
- inertial particles / bubbles measurements
- multiple particle measurements in progress