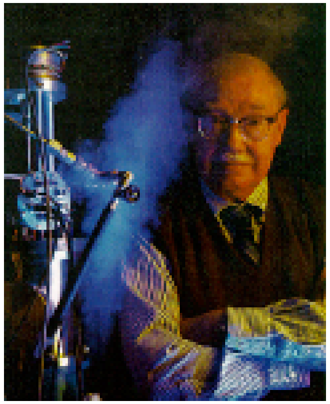


Russell Donnelly

Leonhard Euler (1707-1783)

Joe Niemela

*Euler Equations: 250 years on*

Aussois, France

June 18, 2007

# Huge Reynolds numbers in small apparatus

**K.R. Sreenivasan**

**ICTP, Trieste**

Greg Bewley

Dan Lathrop

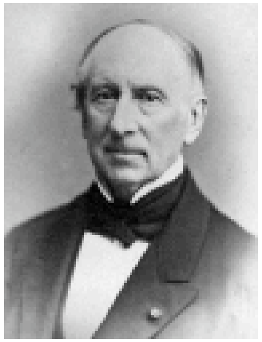
# Euler himself on his equations

“However sublime are the researches on fluids for which we are indebted to Messrs. Bernoulli, Clairault, and d’Alembert, they follow so naturally from our two general formula that one cannot admire sufficiently this accord between their profound meditations and the simplicity of the principles from which I derived my two equations and to which I was led immediately by the first axioms of mechanics.”

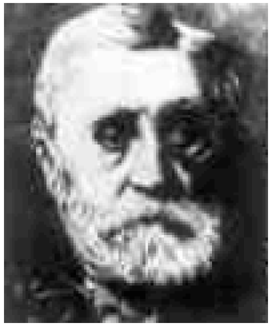
“…everything that the Theory of Fluids contains is embodied in the two equations formulated above…so that it is not the laws of motion that we lack in order to pursue this research but only the Analysis, which has not yet been sufficiently developed for the purpose. It is therefore clearly apparent what discoveries we still need to make in this branch of Science before we can arrive at a more perfect Theory of Motion of Fluids.”

Claude-Louis
Navier
(1785-1836)

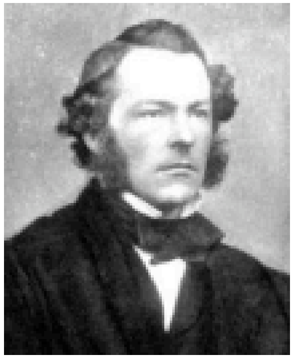
Siméon Denis
Poisson
(1781-1840)



Augustin
Louis Cauchy
(1789-1857)



Adhémar Jean
Claude
Barré de
Saint-Venant
(1797–1886)



Sir George
Gabriel Stokes
(1819-1903)

**Viscosity effects were gradually incorporated into Euler equations**

20th century fluid mechanics

$\nu = 0$ and $\nu$ “very small” are different

By dynamical similarity

$Re \equiv UL/\nu = \infty$ and “very large” are different

## SEVERAL SUBTLETIES EXIST

It has always been clear that the limit $\nu \rightarrow 0$ (or $Re \rightarrow \infty$) is very important, for practical as well as fundamental reasons.

$$Re \equiv UL/\nu$$

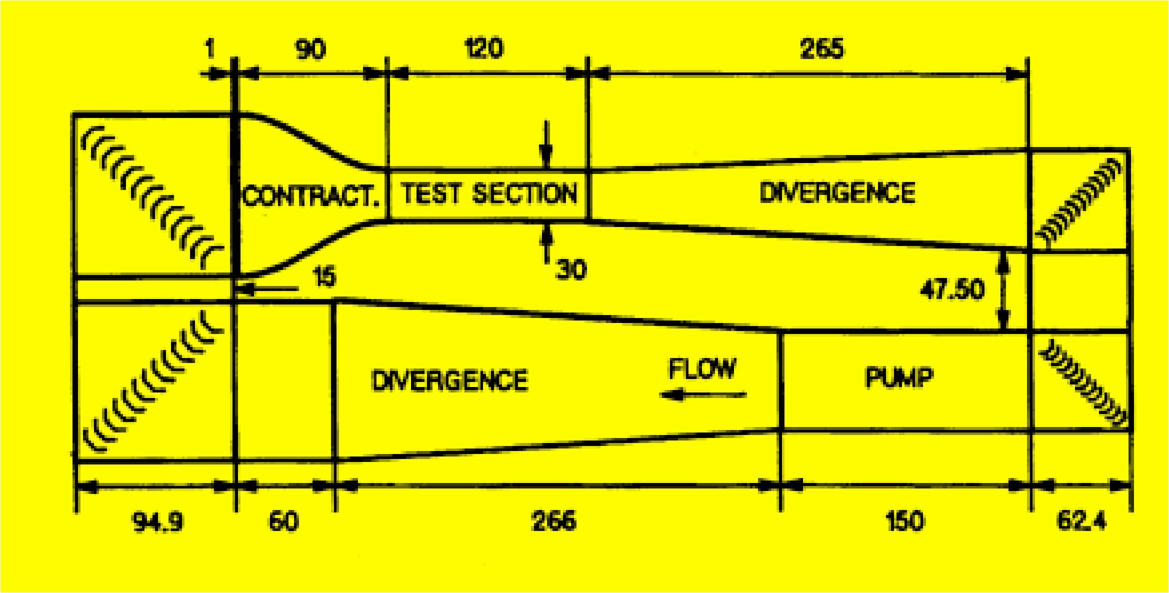
The largest wind tunnel in the world at NASA Ames. This subsonic tunnel can test planes with wing spans of up to 100 feet. It is about 430 m long and 55 m high. Air is driven through these test sections by six 15-bladed fans. Each fan has a diameter equal to the height of a four-story building. The total power 100 MW.

The National Transonic Facility (NTF) at the NASA Langley Research Center, operating since 1984. Cryogenic liquid nitrogen is sprayed and evaporated into a gas that is accelerated through the tunnel's test section up to a Mach number of 1.2. The 150-m long tunnel is powered by a 100 MW turbine motor. The figure on the right shows the giant vanes that help air flow around a corner.

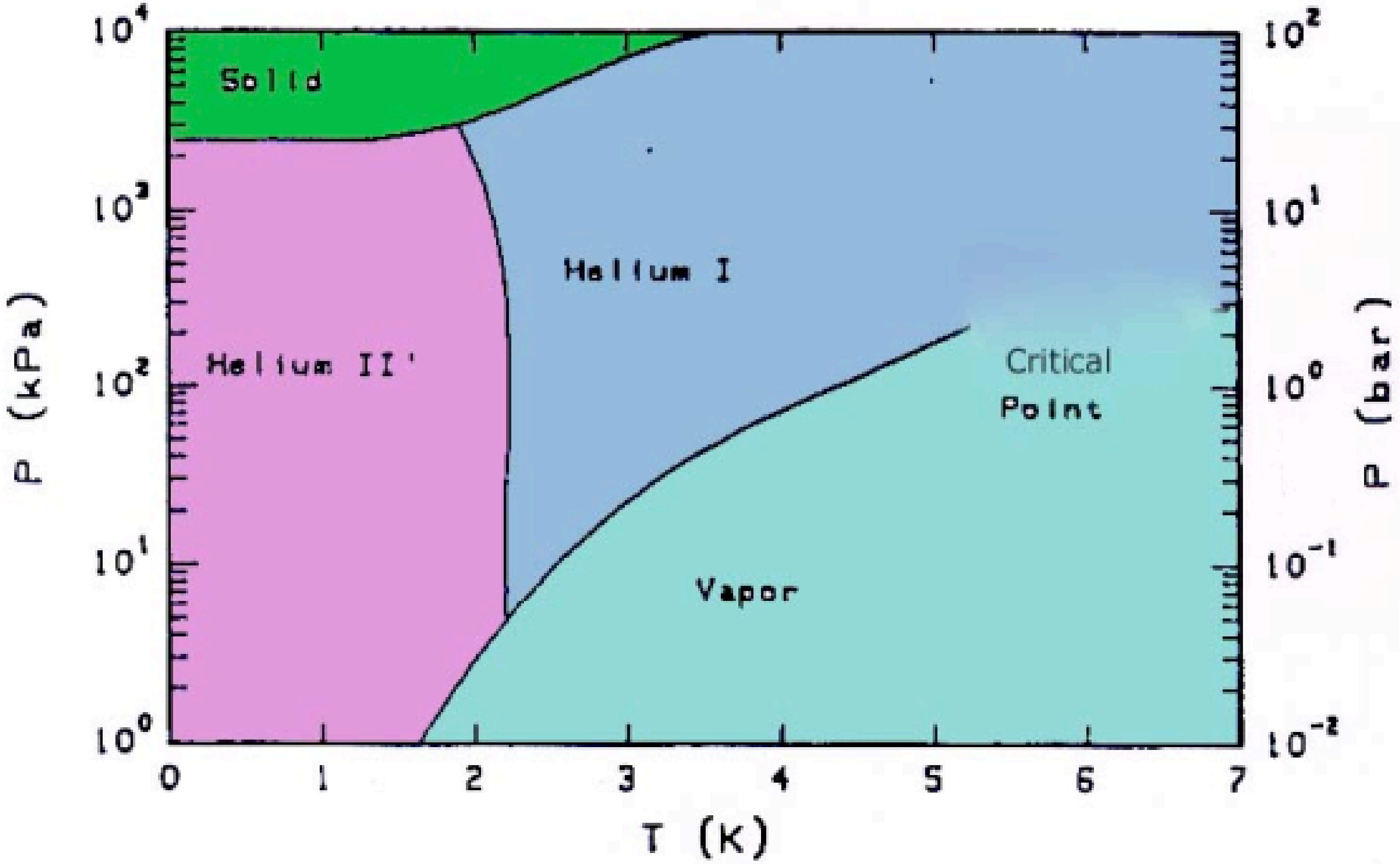
$$Re \equiv UL/\nu$$

# Why not go to the limit?

Helium at cryogenic temperatures has the lowest kinematic viscosity of all fluids (say, 1000 times smaller than that of air at NTP).



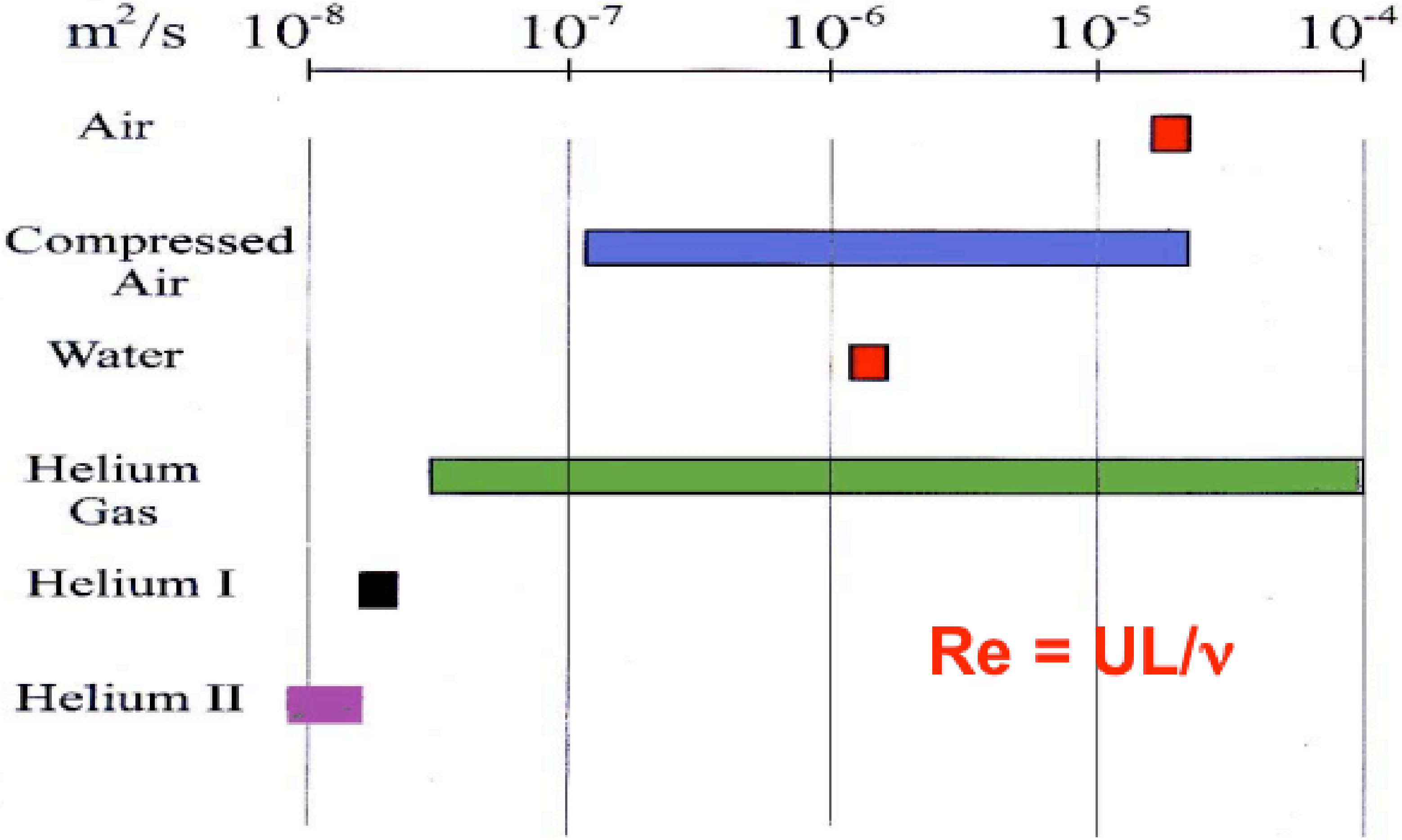
Schematic of helium tunnel capable of generating an Re of about six times that of NASA AMES tunnel



Phase diagram of helium

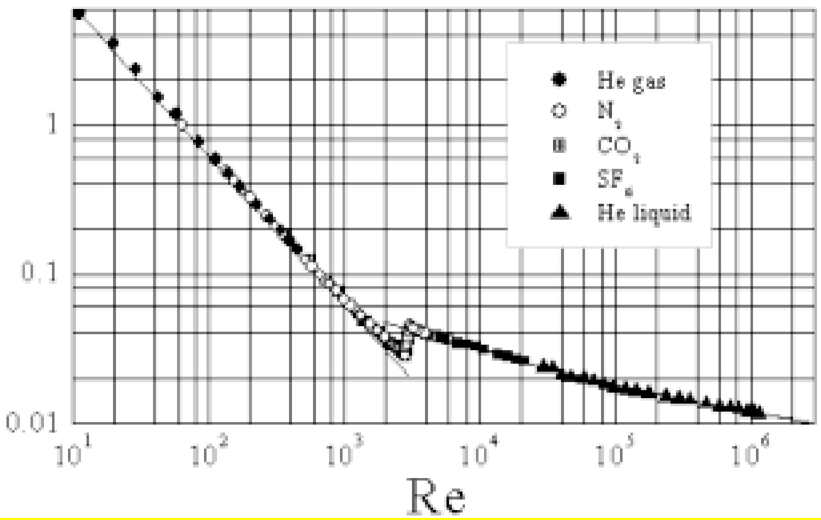
**Helium II has a superfluid component (Kapitza 1938). This prompted Craig & Pellam, *Phys. Rev* 108, 1109 (1957), to remark: "… liquid helium provides the possibility for investigating experimentally true perfect fluid flow."**

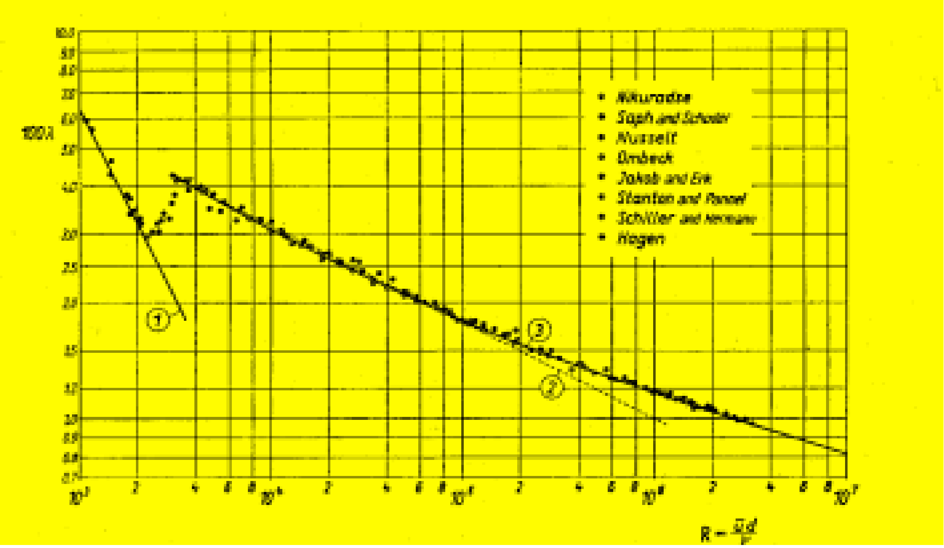
# Kinematic Viscosity of Fluids for Turbulence Research



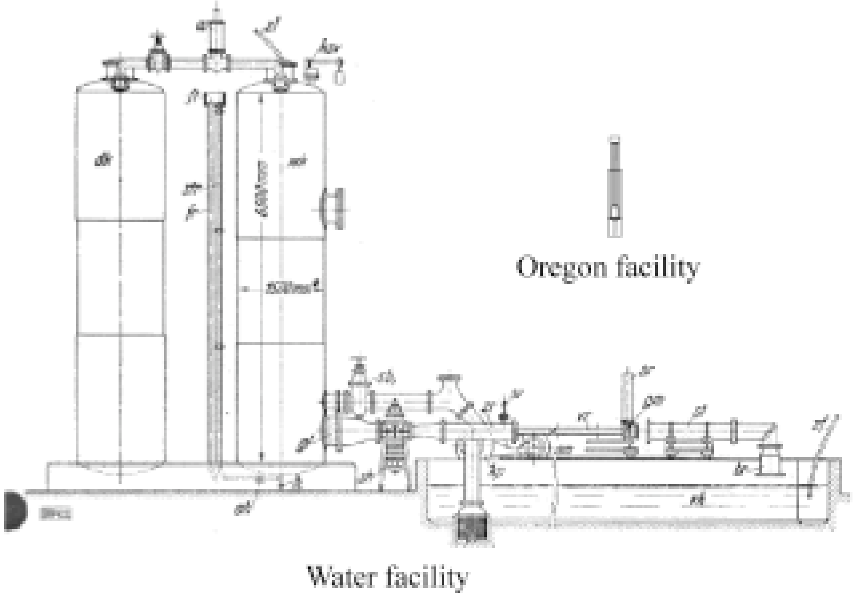
$Re = UL/\nu$

## Oregon measurements

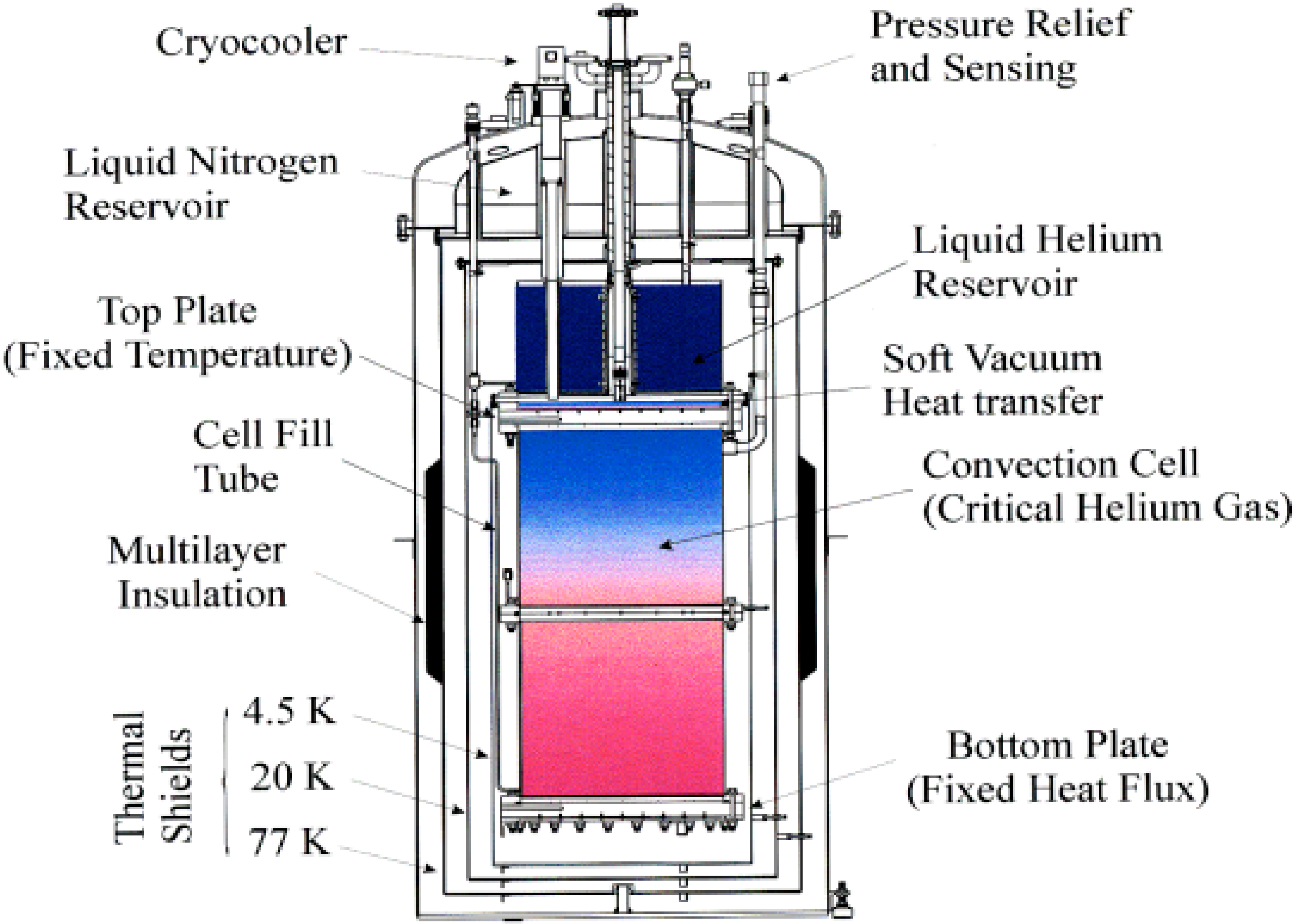




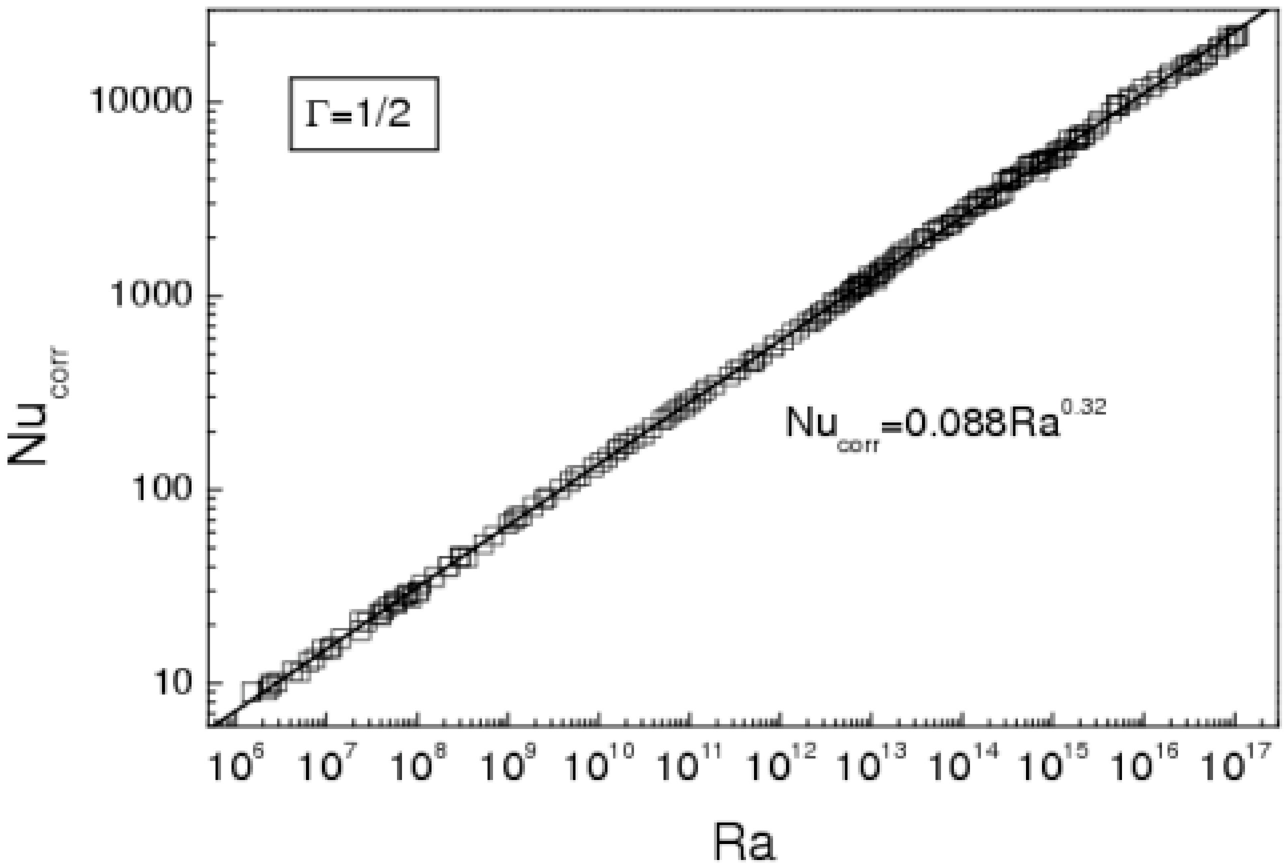
Nikuradse from Schlichting's Boundary layer theory



**comparison of sizes of water and helium apparatus at Oregon**



Schematic view of the convection apparatus



Niemela, Skrbek, Sreenivasan & Donnelly, *Nature* **404**, 837 (2000)
Slightly revised: Niemela & Sreenivasan, *J. Low Temp. Phys.* **143**, 163 (2006)

[Pioneers: Threlfall (Cambridge); Libchaber, Kadanoff and coworkers (Chicago)]

Latest theoretical bound for the exponent (X. Wang, 2007): 1/3 for Pr/Ra = O(1)

# Summary of apparatus size and mesh Reynolds numbers, $R_M$, in a few grid turbulence experiments

| Source | test section | max $R_M$ |
|---|---|---|
| **Kistler & Vrebalovich (1966)<br>(air at 4 atmospheres)** | **2.6 m × 3.5 m** | **2.3 million** |
| Comte-Bellot & Corrsin (1971)<br>(atmospheric air) | 1 m × 1.3 m | 34,000 |
| Oregon towed grid (He II) | 1 cm × 1 cm | 0.5 million |
| **Yale towed grid (He I)** | **5 cm × 5 cm** | **0.8 million** |
| Oregon helium tunnel (He gas) | 6 cm × 6 cm | 75,000 |

# What turbulence properties can we measure?

Particle Image Velocimetry (PIV) using hollow glass spheres

White, Karpetis & Sreenivasan, *J. Fluid Mech.* **452**, 189 (2002)

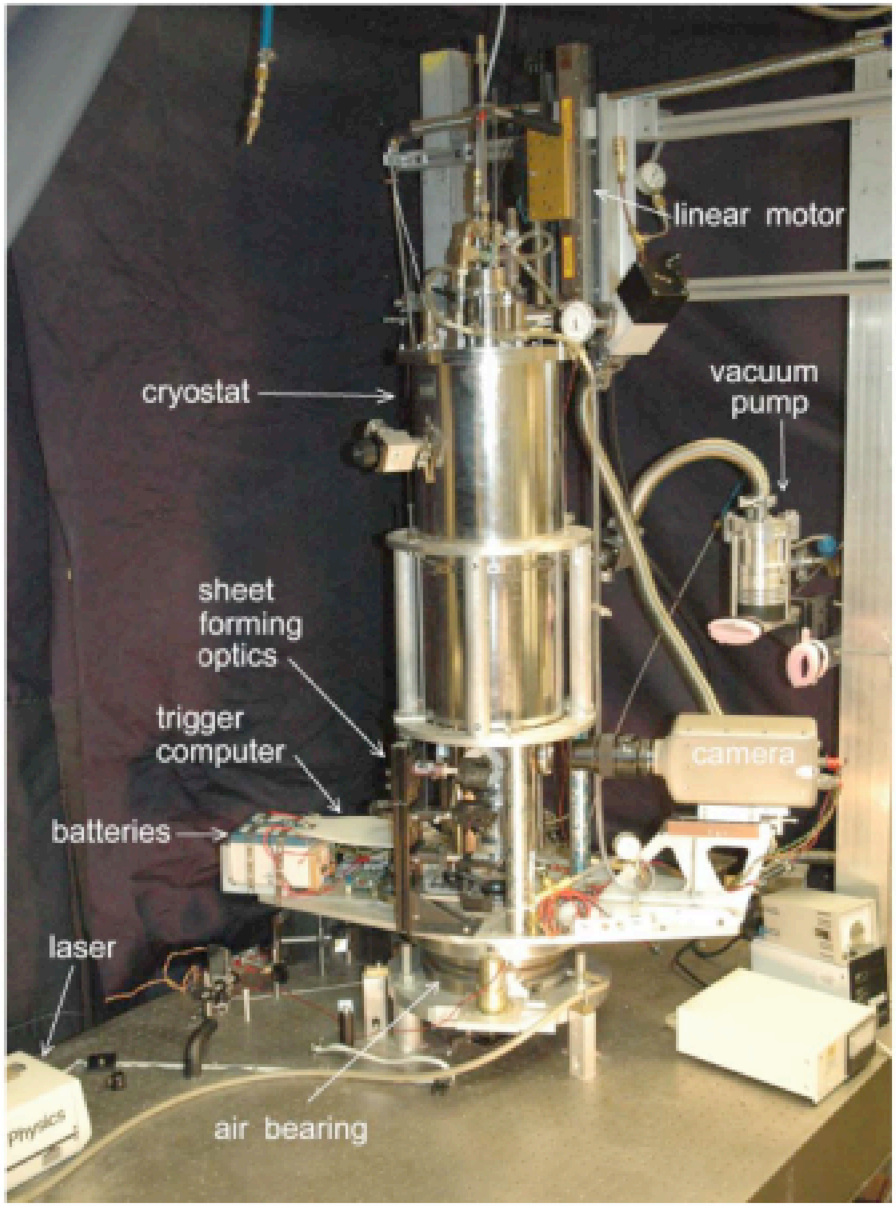
Donnelly, Karpetis, Niemela, Sreenivasan, Vinen, White, *J. Low Temp. Phys.* **126**, 327 (2002)

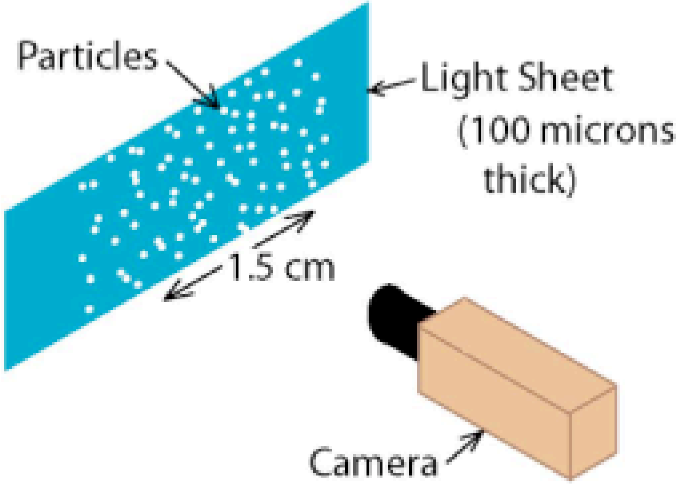
PIV using neutral particles of frozen mixtures of He and H

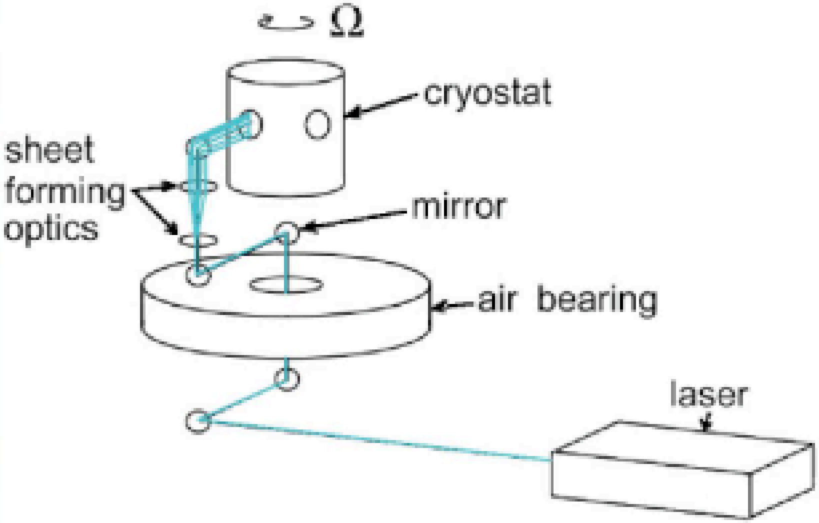
G.P. Bewley, D.P. Lathrop & K.R. Sreenivasan, *Nature* **441**, 558 (2006)
also *Experiments in Fluids* (submitted, 2006)

For a discussion of the interaction between the fluid and particles in He II, see Sergeev, Barenghi & Kivotides, *Phys. Rev. B* **74**, 184506 (2006)

# Apparatus



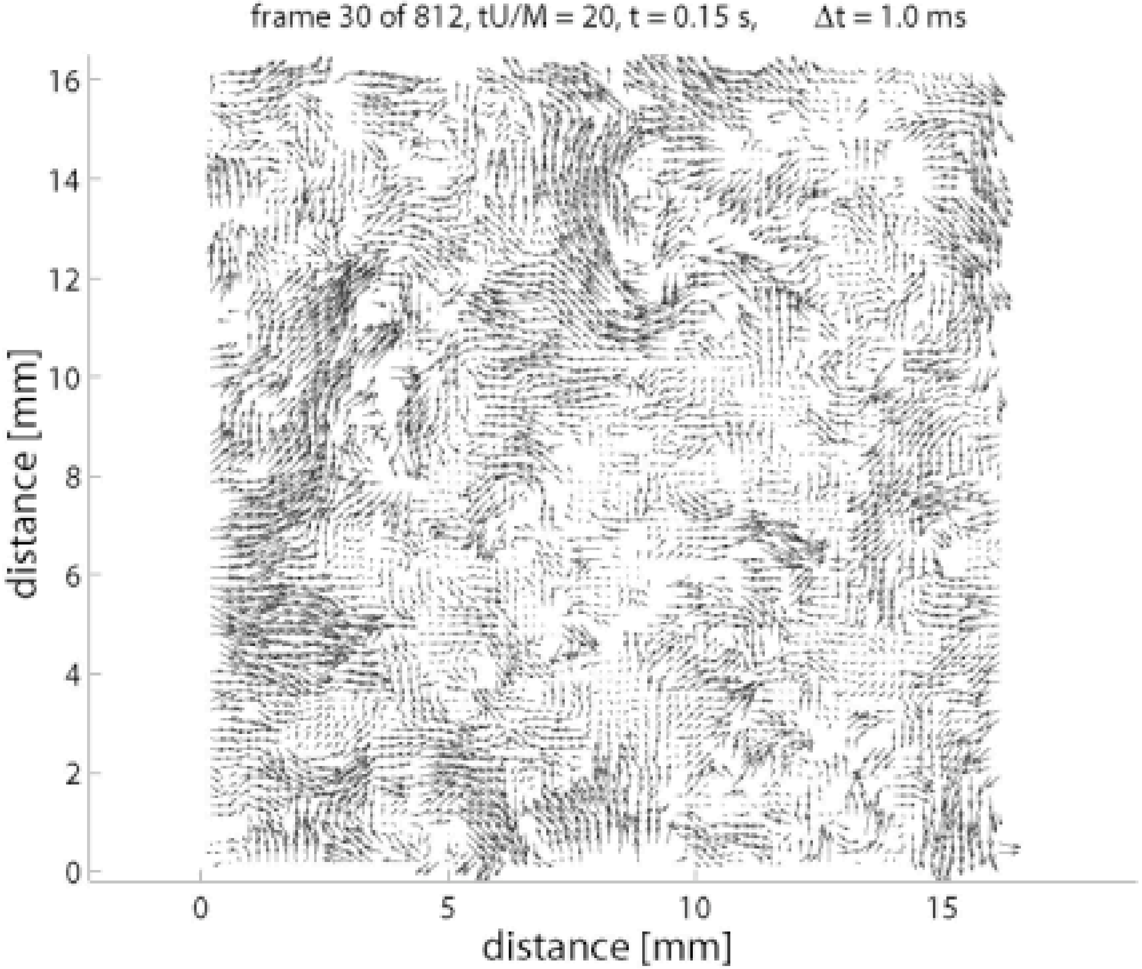




Laser Beam routing

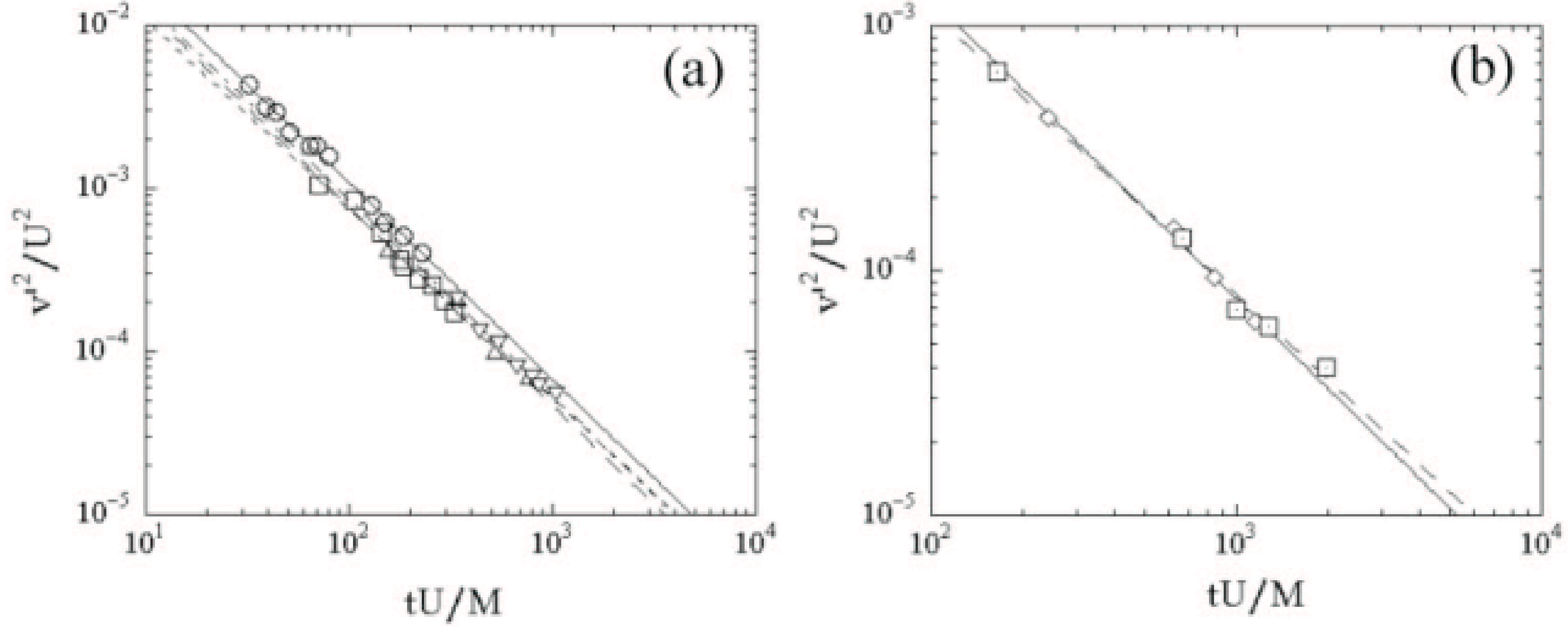
# What happens if we pull a grid through helium?

standard grid turbulence above $T_\lambda$

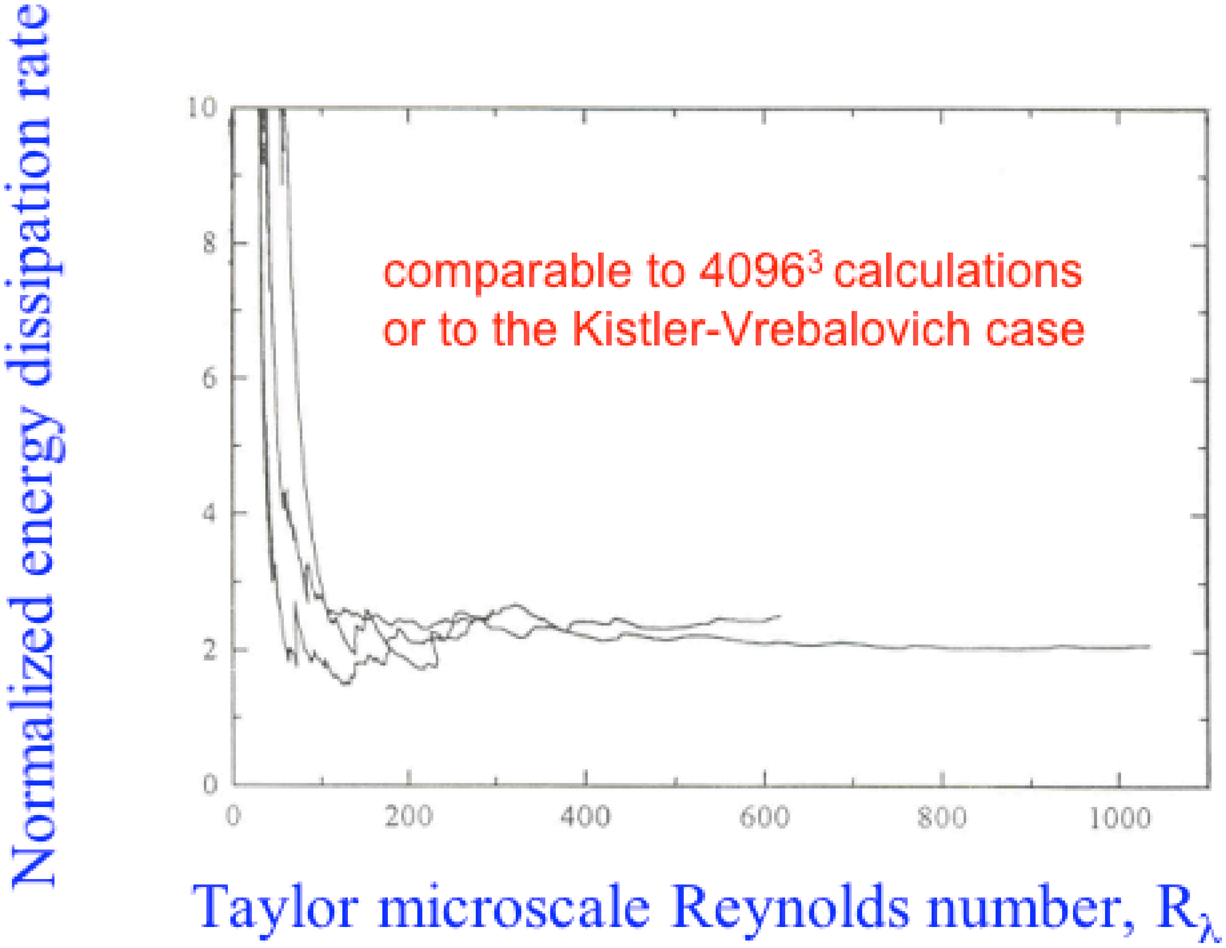


White, Karpetis & Sreenivasan, *J. Fluid Mech.* (2002)

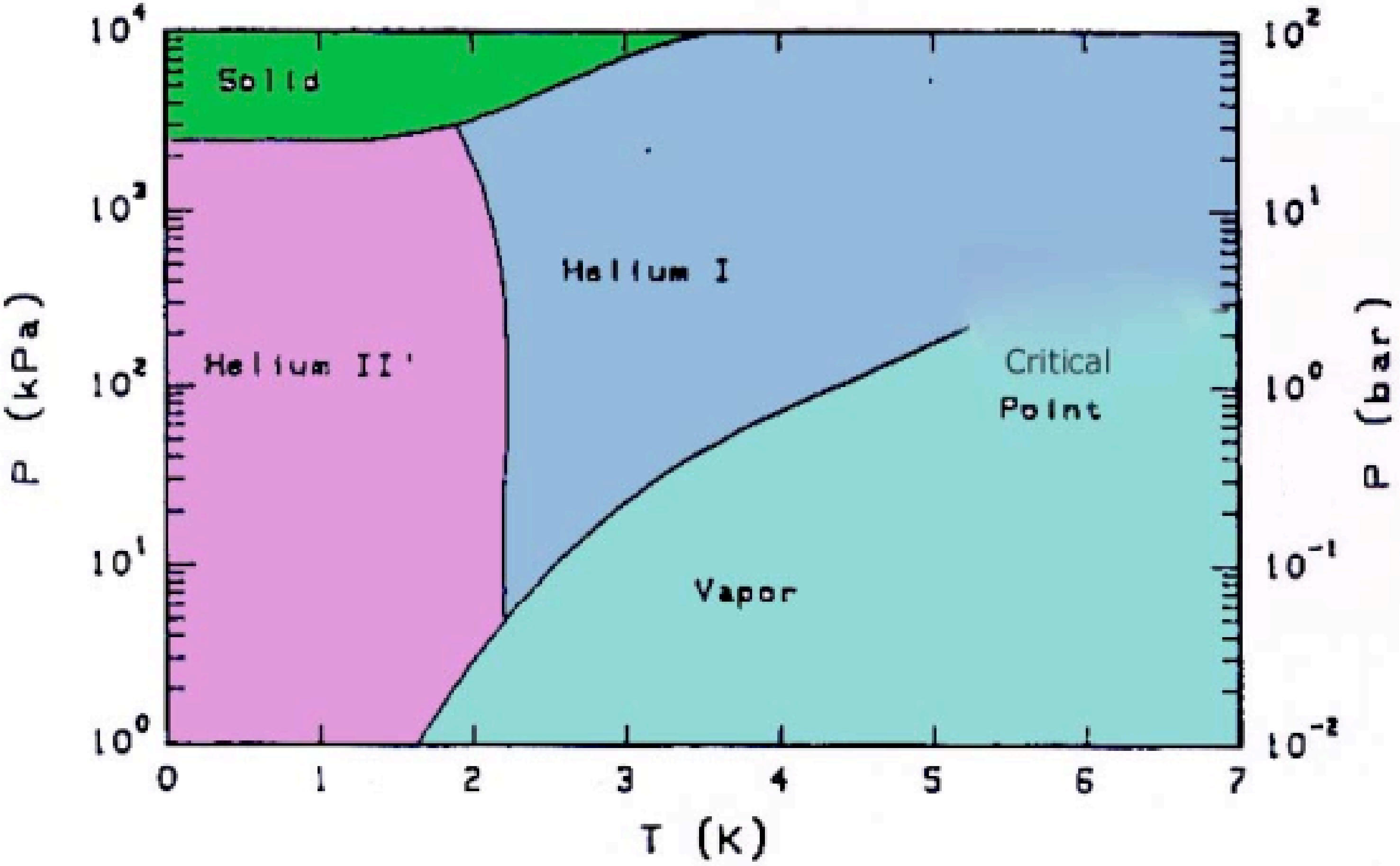
**power-law exponent ≃ 1.1**



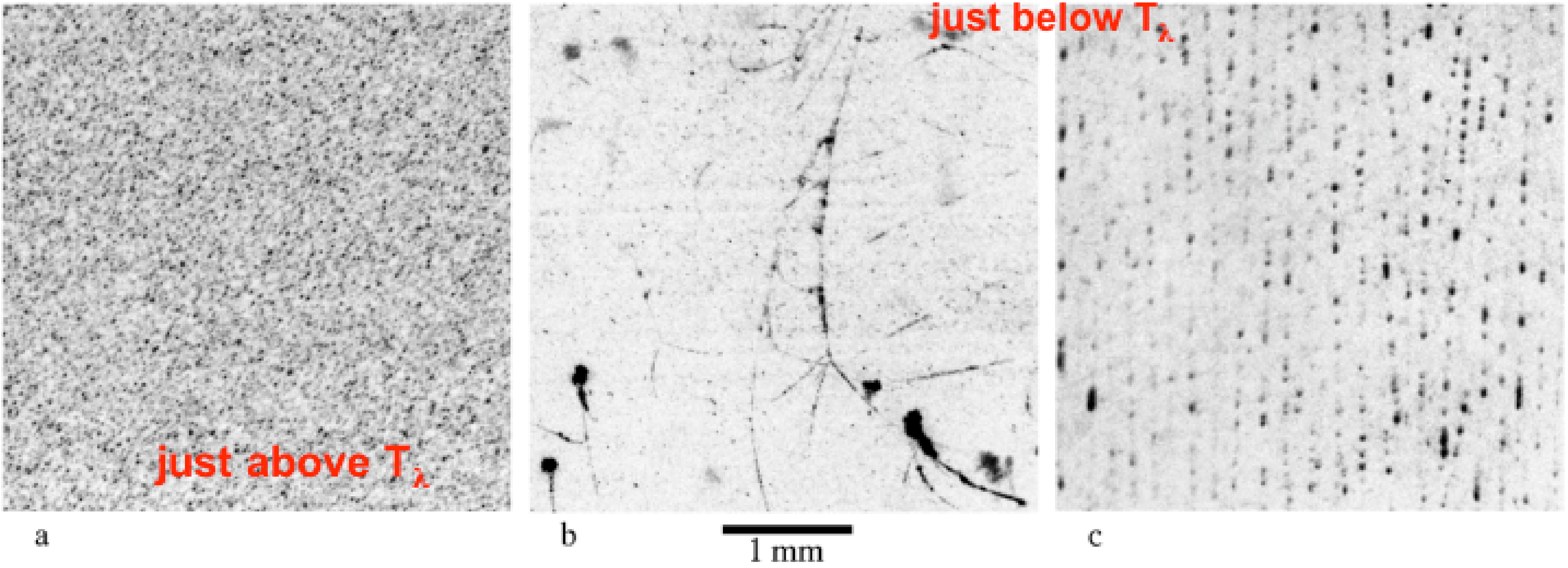
decay of turbulent energy behind towed grids in
(a) liquid helium and (b) liquid nitrogen



comparable to $4096^3$ calculations
or to the Kistler-Vrebalovich case

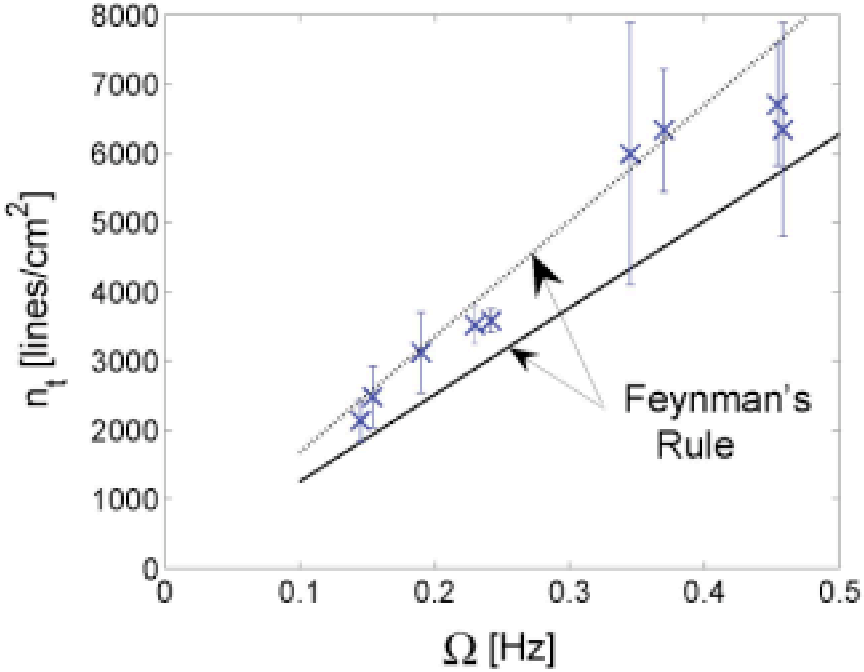


Phase diagram of helium



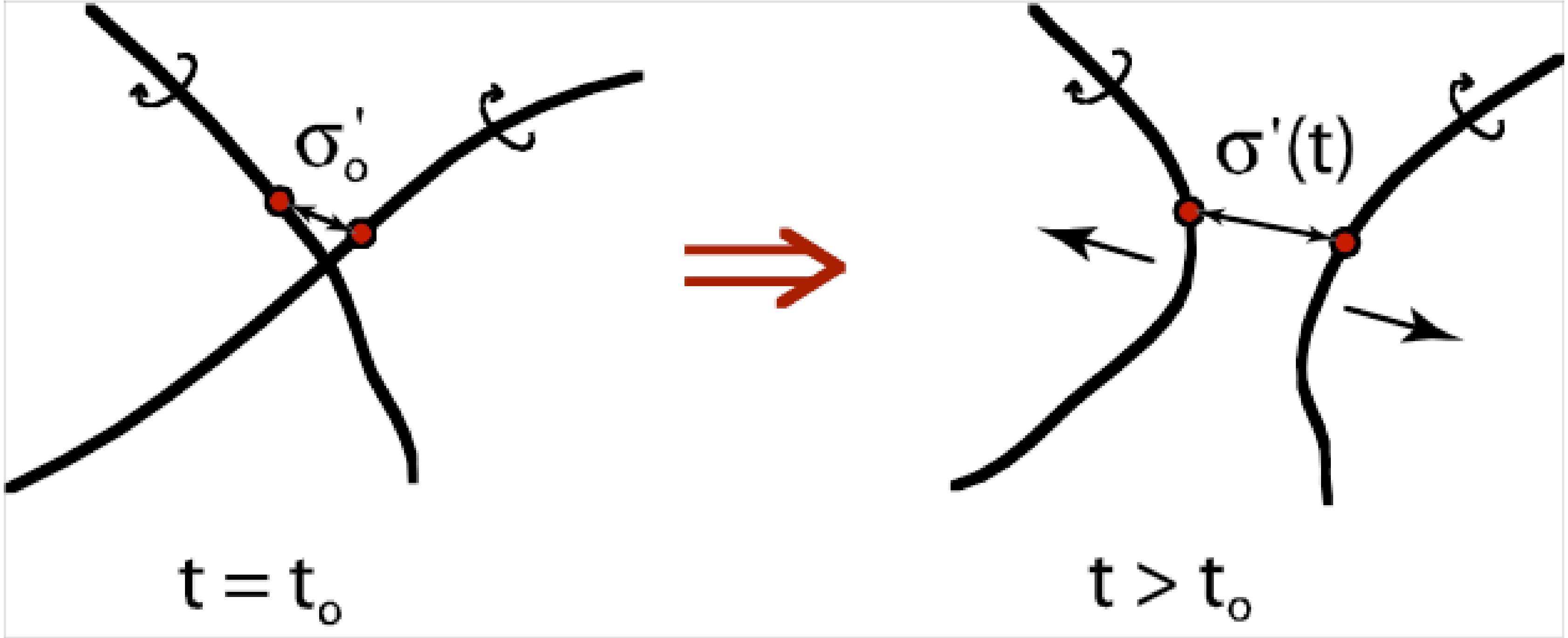
Panel (a) shows a suspension of hydrogen particles just above the transition temperature. Panel (b) shows similar hydrogen particles after the fluid is cooled below the lambda point. Some particles have collected along branching filaments, while other are randomly distributed as before. Fewer free particles are apparent in (b) only because the light intensity is reduced to highlight the brighter filaments in the image. Panel (c) shows an example of particles arranged along vertical lines when the system is rotating steadily about the vertical axis. The spacing of lines is remarkably uniform, although there are occasional distortions of the lattice and possible points of intersection. G.P. Bewley, D.P. Lathrop & K.R. Sreenivasan, Nature 441, 558 (2006)

# Lattice density

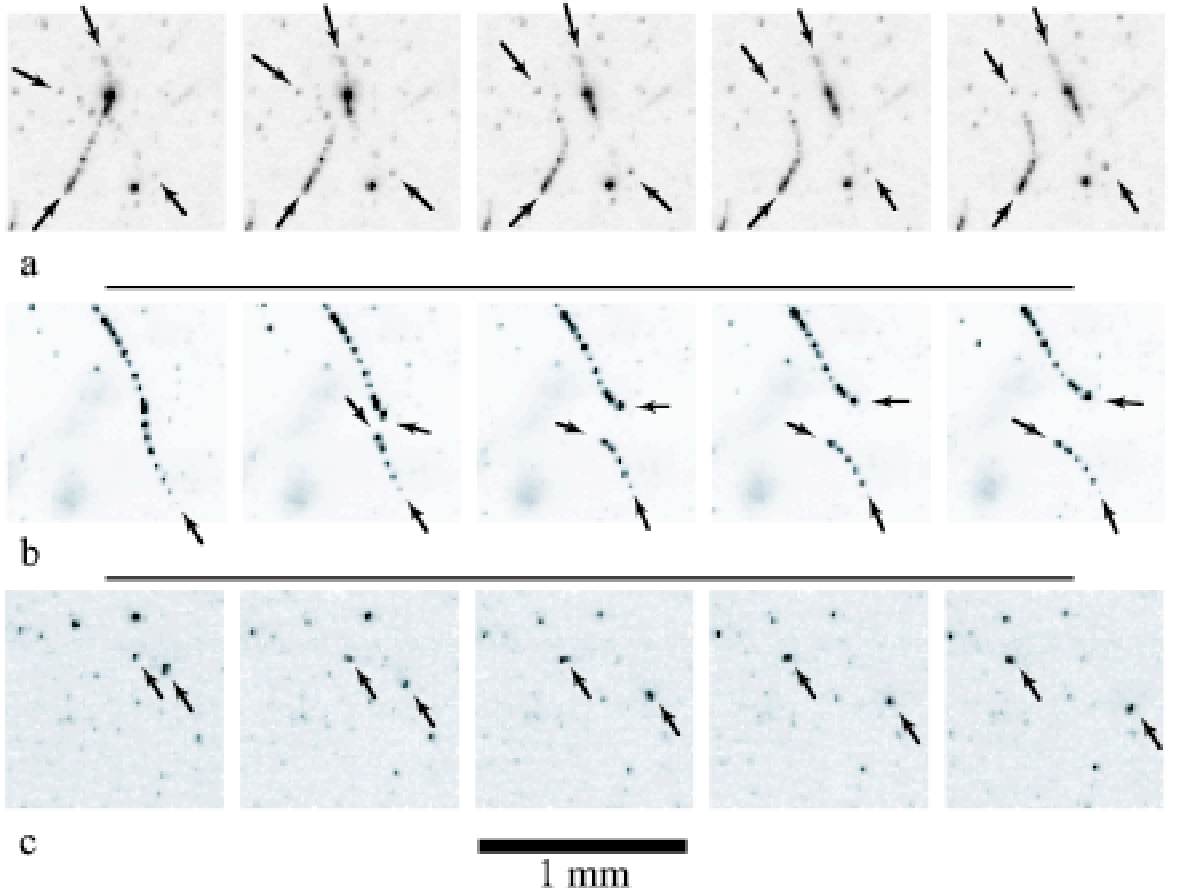


**R.P. Feynman (1955)**
***Prog. Low Temp. Phys.*** **1, 17**

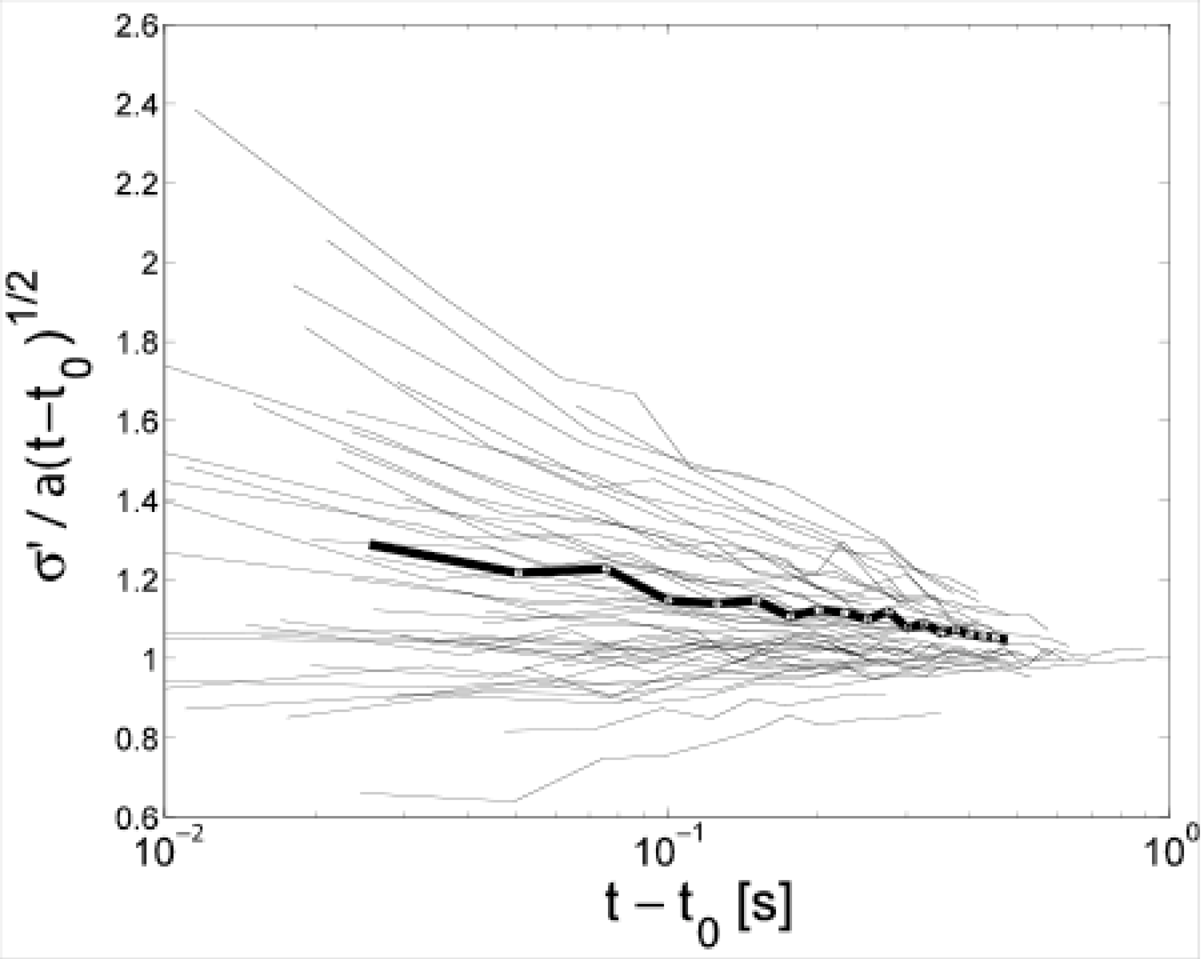
$n_v \approx 2000\Omega$ lines/cm$^2$



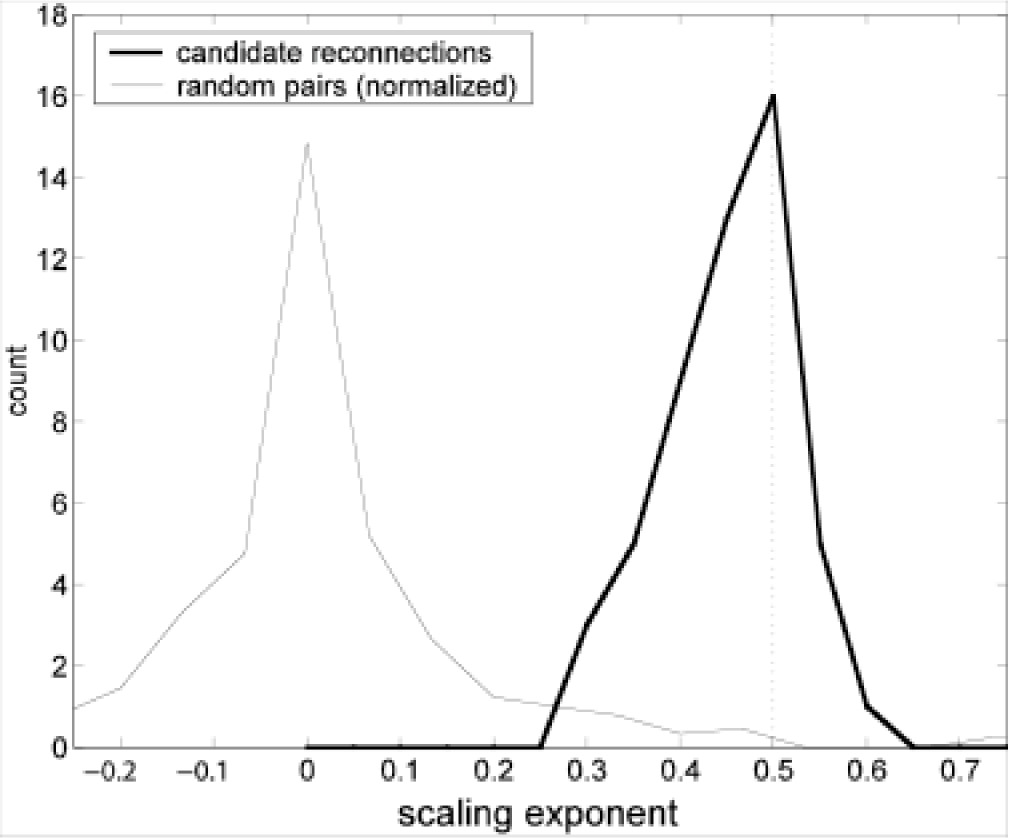
The cores of reconnecting vortices at the moment of reconnection, $t_0$, and after reconnection, $t > t_0$. The small circles mark the positions of particles trapped on the cores of the vortices. The arrows indicate the motion of the vortices and particles. We measure the distance between two particles at different times. Critical element in numerical simulations; for the classical case, see Kerr & Hussain, Physica D **37**, 474 (1989).



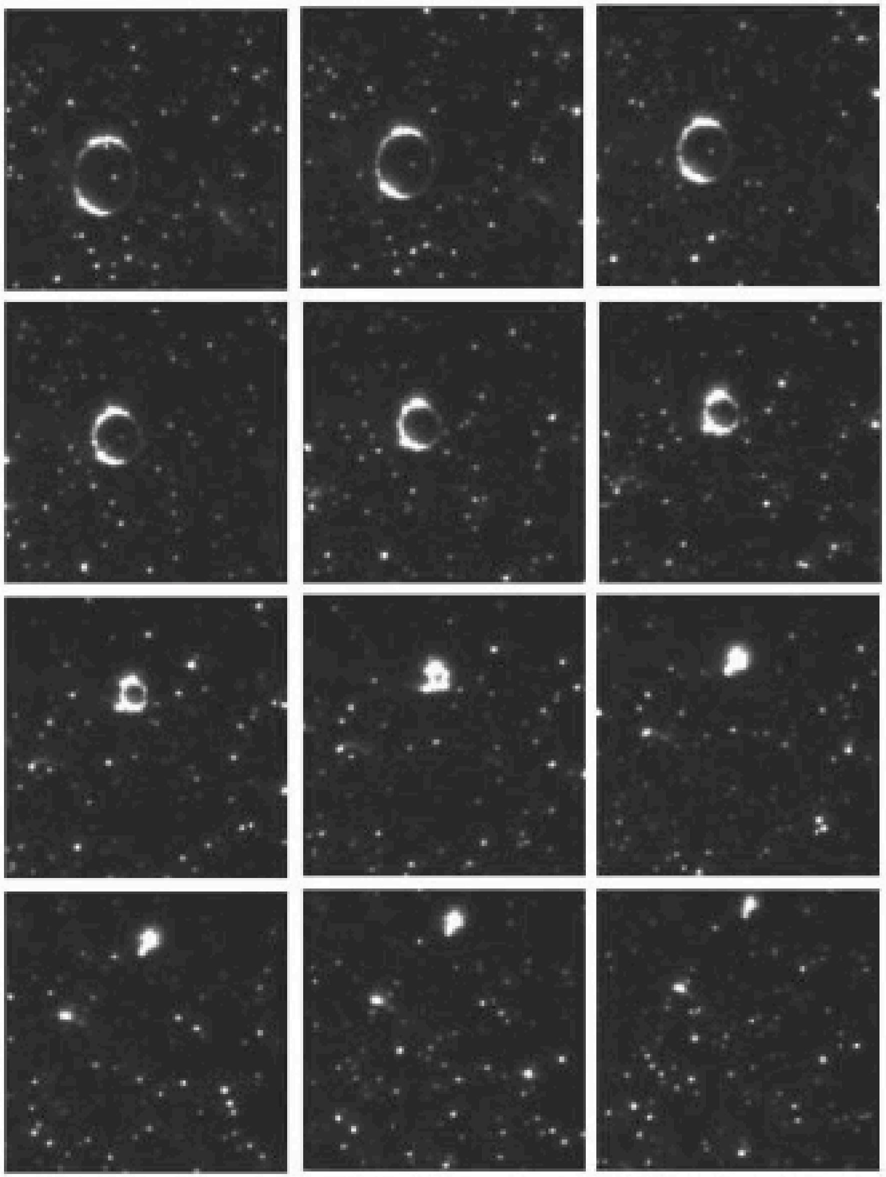
Each series of frames in (a), (b) and (c) are images of hydrogen particles suspended in liquid helium, taken at 50 *ms* intervals. Some of the particles are trapped on quantized vortex cores, while others are randomly distributed in the fluid. Before reconnection, particles drift collectively with the background flow in a configuration similar to that shown in the first frames of (a), (b) and (c). Subsequent frames show reconnection as the sudden motion of a group of particles. In (a), both vortices participating in the reconnection have several particles along their cores. In projection, the approaching vortices in the first frame appear crossed. In (b), particles make only one vortex visible, the other vortex probably has not yet trapped any particles. In (c), we infer the existence of a pair of reconnecting vortices from the sudden motion of pairs of particles recoiling from each other.

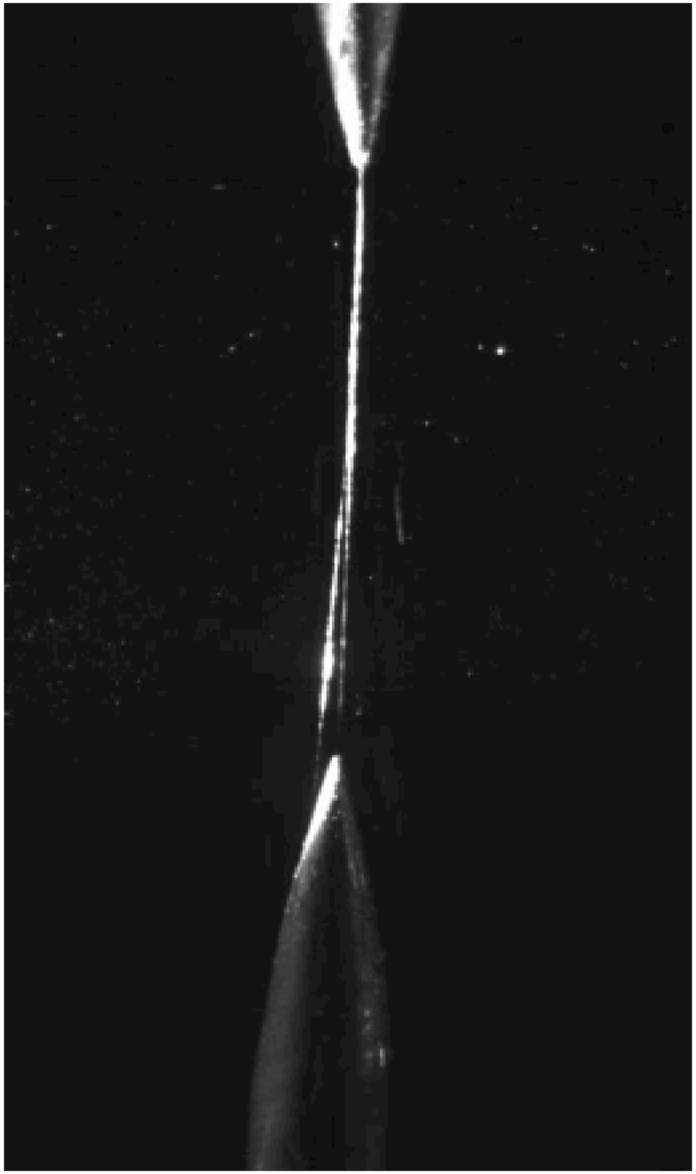


The lines are the measurements, $\sigma'(t)/a(t\_t_0)$– for 52 candidate reconnections, where *a* and $t_0$ were found by fitting the measurements to a power law with arbitrary scaling exponent. If the data were governed by power laws, each line would be straight. If the scaling exponents were all _, the data would fall on a horizontal line. A randomly chosen curve is plotted with a thicker line in order to show the path of a typical single trajectory.



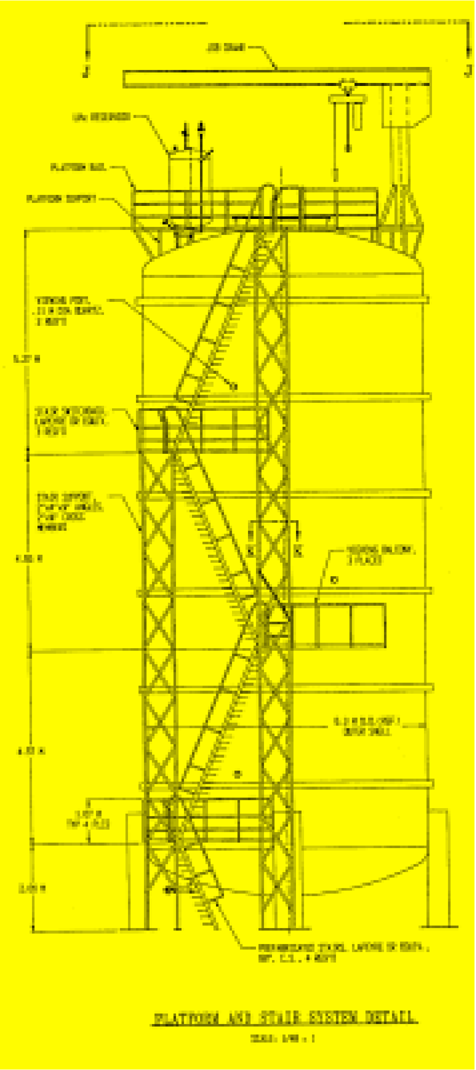
Histogram of the scaling exponents for the data in previous figure, as well as those found for randomly chosen particle trajectories. We normalized the histogram for the random-pair data to have the same area beneath it as the histogram for the experimental data. The mean value of the scaling exponents for the candidate reconnections is 0.45, but the peak value is _, as indicated with a vertical dotted line. For quantum mechanical effects near the reconnection point, see J. Koplik and H. Levine, Phys. Rev. Lett., **71**, 1375 (1993).





# What next?

1. Superfluidity of He-3 and turbulence
2. Turbulence of condensates



### Cell dimensions

D = 5m, L = 10m,
Max volume ~ 25,000 gallons of liquid equivalent (minimum ~1 gallon)

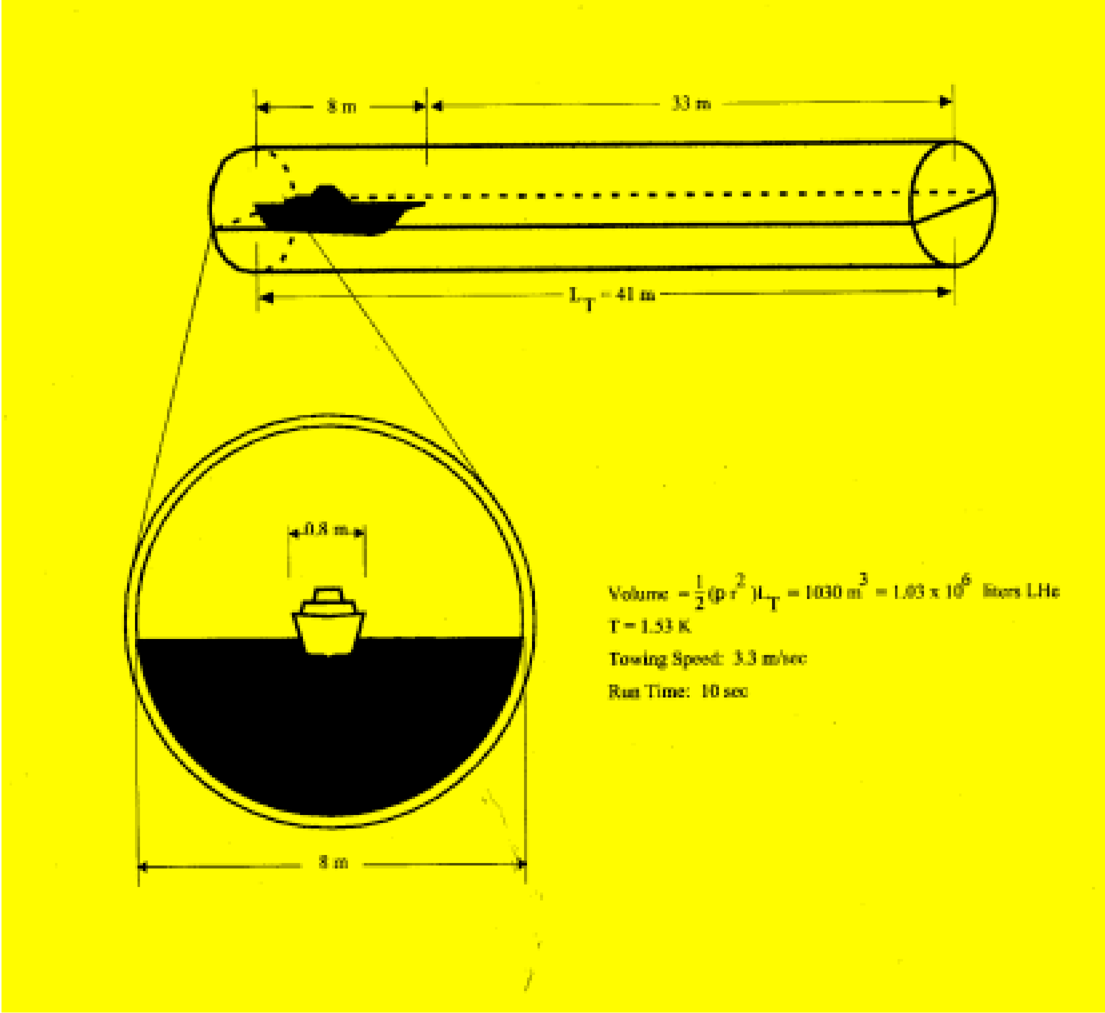
### Outside dimensions

~7 m dia and ~20 m high

### Refrigeration needed

< 200 W at extreme conditions, most of the time much smaller

**Proposed 10m high convection cell in which Rayleigh numbers of the order $10^{20}$ (probably somewhat higher) can be reached**



Volume $= \frac{1}{2}(p\,r^2)L_T = 1030\ m^3 = 1.03 \times 10^6$ liters LHe

T = 1.53 K

Towing Speed: 3.3 m/sec

Run Time: 10 sec

**Sketch of a liquid helium tow tank, which could simultaneously match Froude and Reynolds numbers**

# Other groups and people that could be mentioned (some past, some current)

Tabeling's group: Karman flow

Castaing's group: convection, jets, instrumentation

Murakami, Van Sciver, Ihas: instrumentation, particles, PIV

Groups in Helsinki, Prague, Birmingham, Manchester, Lancaster: superfluid helium (experiments in He II and He-3, some instrumentation, some theory)

Barenghi, Tsubota, Fauve, Schwarz: Numerical simulations, some theory