Appendix A

Lunar Gravity Assists

This appendix discusses gravity assists in the context of a restricted three-body problem (Earth-Moon-test particle), and explores an interesting phenomenon of possibly great utility to astrodynamics. I will not rigorously explore the phenomenon, but instead will describe it and present some simple arguments which provide some intuitive insight. This appendix assumes that the reader is familiar with the material presented in Sec. 3.2.1.

A.1 Motivation

At the start of the heliocentric stage of the lunar simulations with $v_{ej} = 2.4$ km/s, a small clump of particles (see Fig. 3.6) with common orbital elements ($a \simeq 1.05$ AU, $e \simeq 0.045$) was noticed. Such a clump is surprising since particles are typically liberated from the Earth-Moon system via gravitational scatterings by the Moon; random encounter geometries occurring at random times should evenly scatter particles both interior and exterior to the Earth's orbit. The particles in this clump were identified and their orbital histories tracked. Very surprisingly, these particles did not complete a single geocentric orbit, but escaped directly to heliocentric orbit even though they were launched only barely faster than the lunar escape velocity! What is happening here?

A.2 Energy arguments

Consider a small object launched from an isolated massive body. Based on elementary principles, the escape velocity $v_{\rm esc}$ from a body of mass M and surface radius R_s is

$$v_{\rm esc} = \sqrt{\frac{2GM}{R_s}}\ , \tag{A.1}$$

where G is the gravitational constant. For the Moon, $v_{\rm esc} = 2.38$ km/s. A body launched with velocity $v_l < v_{\rm esc}$ will only reach a height $R_{\rm max}$ which satisfies

$$\frac{1}{R_s} - \frac{1}{R_{\rm max}} = \frac{v_l^2}{2GM}\ , \tag{A.2}$$

before falling back to the surface.

The Moon is, of course, not isolated. Once a particle recedes a sufficient distance it is clear that the gravitational pull of the Earth will become more important than that of the Moon. There are various estimates of this distance (Greenberg *et al.* 1991); I adopt that of the Hill sphere

$$R_H = a_{\rm orb}\left(\frac{\mu}{3}\right)^{1/3} \simeq 35R_s \simeq 9.6R_\oplus\ , \tag{A.3}$$

where $a_{\rm orb}$ is the Moon's orbital radius ($\simeq 60R_\oplus$), and $\mu \simeq 1/81$ is the ratio of the Moon's mass to that of the Earth. Therefore a quantity of greater interest than the Moon's 'escape' velocity is the launch speed v_* necessary for a particle to reach the lunar Hill sphere. Since, due to the $1/r$ potential, most of the gravitational work is done near the Moon, v_* is only slightly smaller than $v_{\rm esc}$; in fact $v_* = 2.35$ km/s. This is confirmed in an additional set of launch simulations (Table A.1) identical to those performed in Sec. 3.2.1, except that the launch velocities were all near the lunar escape speed. For $v_l < v_*$, all the particles fall back to the lunar surface without escaping to geocentric orbit. Above v_*, an increasing fraction of particles escape the Moon's gravitational pull, although many of these subsequently re-impact the parent object. Eventually more than 80% of the particles escape to heliocentric orbit (in concordance with the earlier results of Chap. 3), mostly via repeated gravity assists by the Moon.

A.3 Gravity assists

This section will develop the background of gravity-assist trajectories in the context of a patched-conic approach. I will use one of the computed trajectories from the simulations just discussed for illustrative purposes.

Fig. A.1 shows the orbital path taken by the particle escaping to the Earth's Hill sphere in the shortest time (about 8 months) from the 2.35 km/s simulation. The particle follows the lunar orbit closely for the first month. Close approaches appear as cusps in the trajectory. In fact, the particle is in a temporary capture orbit around the Moon, as shown in Fig. A.2. Almost exactly one month after launch (*i.e.,* near the same geocentric coordinates where the launch took place) a close encounter occurs which ejects the particle from lunar orbit and alters the path to one with orbital semi-major axis considerably smaller than the Moon's, which the particle follows for seven revolutions (Fig. A.1). The Moon then draws

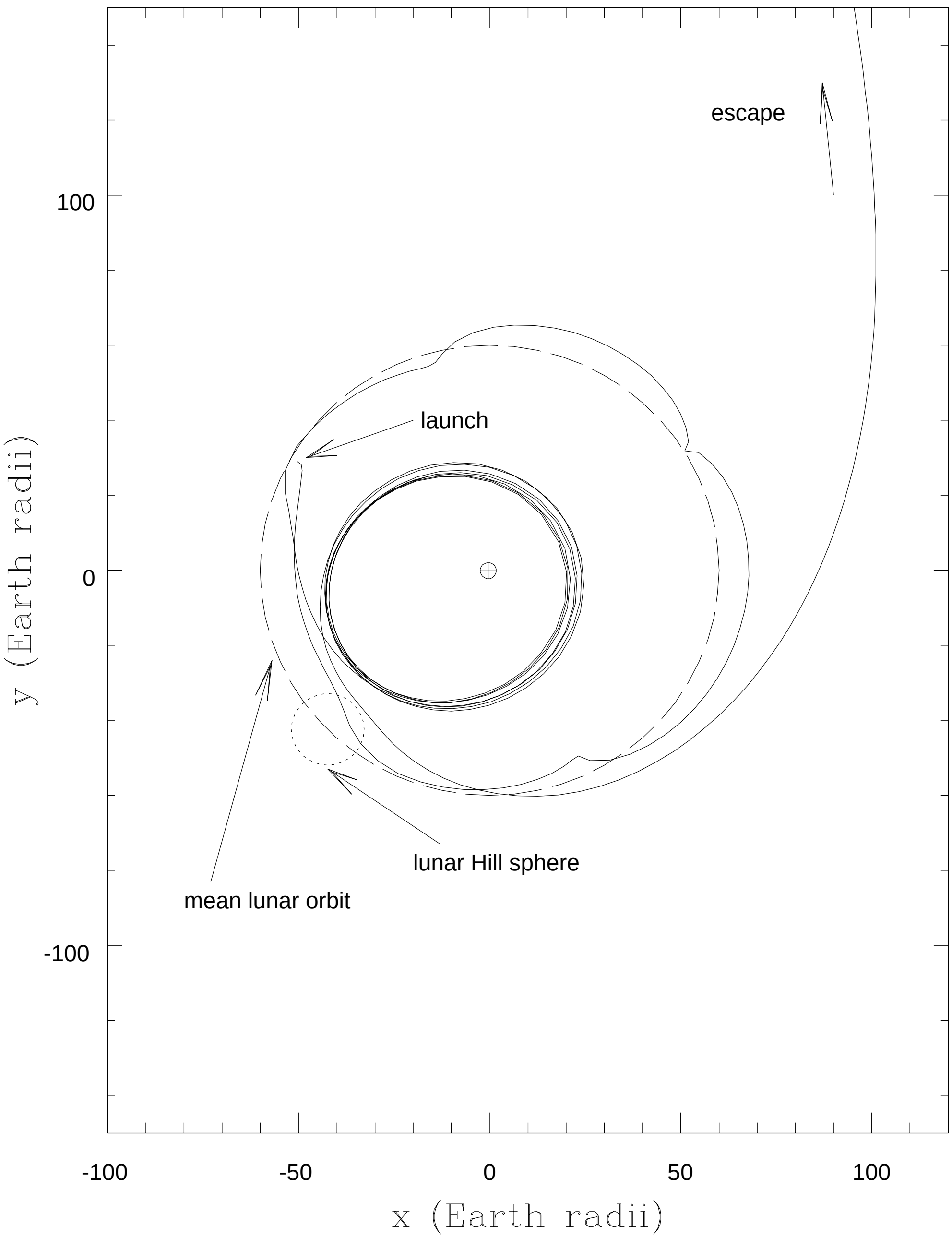

Figure A.1 The path taken by a particle escaping the Earth-Moon system via a series of gravity assists. The relative geocentric position is shown, in an non-rotating frame centered on the planet and projected onto the ecliptic plane. The dashed line shows the mean lunar orbit (at $60R_\oplus$). Recall that the Moon's Hill sphere (shown) is $\approx$10 $R_\oplus$. The launch does not occur at the distance of the mean lunar orbit due to the Moon's eccentricity. See text for description.

Table A.1 For each geocentric-stage simulation the table lists the total number N of particles launched (18 particles per impact site), the percentage Esc of particles that escape to heliocentric space, and the percentages that impact the Earth ($\oplus$) and Moon (M), as a function of the lunar launch speed v_l. To reach the lunar Hill sphere requires v_l=2.35 km/s.

Geocentric Stage Results				
v_l (km/s)	N	% Esc	% $\oplus$	% M
2.33	360	0.0	0.0	100.0
2.34	360	0.0	0.0	100.0
2.35	360	2.8	0.0	97.2
2.36	360	31.7	1.9	66.4
2.37	360	65.8	2.2	32.0
2.38	360	81.9	2.5	15.6
2.39	360	84.7	3.1	12.2

the particle off this orbit and flings it away at sufficient speed that it escapes the Earth-Moon system entirely. This second encounter is also seen in Fig. A.2 (marked 'second entry' and 'second escape').

It is clear that the route to escape in this case is that discussed in Sec. 3.2.1, namely, during close approaches to the Moon, the particle's geocentric velocity vector is modified. The most common way to conceptualize this is by a 'patched conic' approach (see D'Amario *et al.* 1992 for example). This method views the orbits before and after the encounter as unperturbed Kepler orbits about the Earth, and during the encounter as a hyperbolic orbit past the Moon. However, *this is an idealization*: Fig. A.2 illustrates that, during its second entry into the lunar Hill sphere, the particle's orbit never seems to be particularly well represented by a lunocentric hyperbola. This is because the particle is never much closer to the Moon than $R_H/2$ and therefore a two-body problem centered on the Moon is not a particularly good representation of the dynamics. However, we can still use the usual patched-conic vector-addition picture (Fig. A.3) to understand how the final close encounter resulted in a gravity-assist that produced escape velocity. From Figs. A.1 and A.2, it can be seen that the Moon catches up to the test particle just before the particle enters the lunar Hill sphere (the Moon is moving in the positive x direction during the fly-by). Therefore, as shown schematically in Fig. A.3, while inbound to the encounter the particle's geocentric velocity is smaller than the Moon's, and in the lunar frame $\vec{\boldsymbol{v}}_{\boldsymbol{in}}$ is moving opposite to the apex of the lunar motion. The close encounter reverses the relative velocity vector to $\vec{\boldsymbol{v}}_{\boldsymbol{out}}$ and so, when the vector addition is performed, the particle has rather substantially increased the magnitude of its geocentric speed (from $\vec{\boldsymbol{V}}_{\boldsymbol{in}}$ to

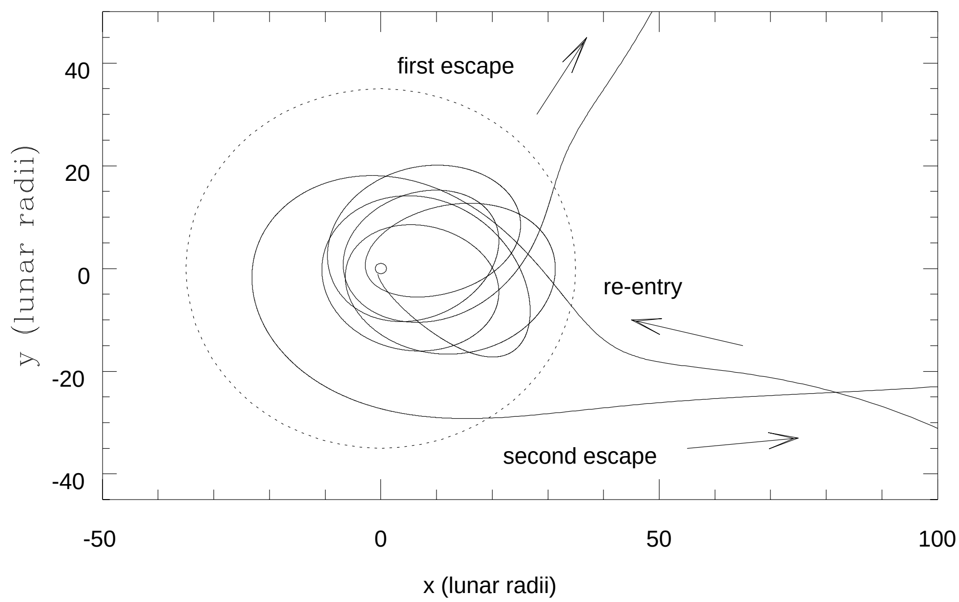

Figure A.2 The lunocentric trajectory of the particle shown in Fig. A.1, in the same non-rotating frame projected onto the ecliptic plane. The Moon (to scale) is indicated, as well as the approximate location of its Hill sphere (dotted circle). After launch the particle remains in temporary orbit until escape. It returns later for a hyperbolic encounter which ejects it from the geocentric orbit. See text for discussion.

$\vec{V}_{out}$). In this case the magnitude of the increase is enough to make the particle unbound with respect to the Earth.

A.4 The three- and four-body problems

The type of gravity assist just discussed was the picture in the author's mind of how all escapes from the Earth-Moon system occur, at least when particles are launched off the Moon with a residual velocity (Fig. 3.3) below geocentric escape speed. That is, after escaping the Moon the particle's geocentric orbit will be modified repeatedly by close approaches until one of these fortuitously flings the particle 'off to infinity'. (It turns out that the example just discussed is extremely atypical in that only a single re-encounter with the Moon was necessary; this particle was selected as being the fastest escaping meteoroid.) However, it turns out that a gravity assist is *not* the method of escape that produces the group of particles that make up the above-mentioned clump in Fig. 3.6 in the original $v_l = 2.4$ km/s simulation.

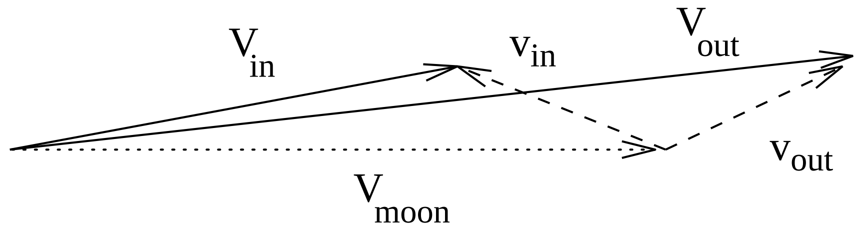

Figure A.3 A vector-addition diagram representing a patched-conic gravity-assist. This is a schematic representation of the final encounter pictured in Figs. A.1 and A.2. Geocentric velocities are indicated by uppercase letters and lunocentric velocities by lower case letters.

Snapshots of the full geocentric simulation over the first 3 months of time are shown in Figs. A.4 and A.5. Since the simulation consists of 20 launch events ejecting 18 particles each in a cone, the Moon appears to eject particles in an expanding shell (all launch directions being equally likely). After a few days this shell acquires a noticeable asymmetry, which becomes more pronounced after 0.2 months. This asymmetry occurs because some particles have had their orbital energies reduced by the Moon's gravity; thus they have smaller semimajor axes and correspondingly shorter orbital periods. Such particles therefore lead the Moon. Many of these same particles have acquired substantial geocentric eccentricities, producing perigees of $\simeq$20 $R_\oplus$, as seen in Fig. A.4c and d. In contrast, other particles have gained orbital energy from the Moon, acquiring larger semimajor axes, and therefore lag the Moon in longitude. However, the most interesting aspect of these figures is the devlopment of a 'lobe' or 'petal' of these lagging particles (containing about 10–15% of the total), seen clearly in the lower right quadrant of panels (e)–(h), which simply expands away from the Earth, never to return. This cluster becomes the clump of Fig. 3.6; all of these particles exit the Earth's heliocentric Hill sphere (of about 230 $R_\oplus$) in short order (about a month). The remaining particles circle the planet, continuing in Earth orbit until they hit a primary or are ejected by subsequent lunar gravity assists.

In the remainder of this appendix I will attempt to develop an intuitive understanding of the dynamics in terms of simple energy arguments and 2-body reasoning. These discussions will not be rigorously correct, because the real dynamics are closer to 4-body problem (or at least, two sequential restricted 3-body problems). A complete understading of this phenomenon is probably equivalent to another thesis.

Let us examine two of these quickly escaping trajectories more carefully (Fig. A.6). The particles simply leave the Moon on hyperbolic trajectories, and continue smoothly out to the Earth's Hill sphere, with no temporary capture

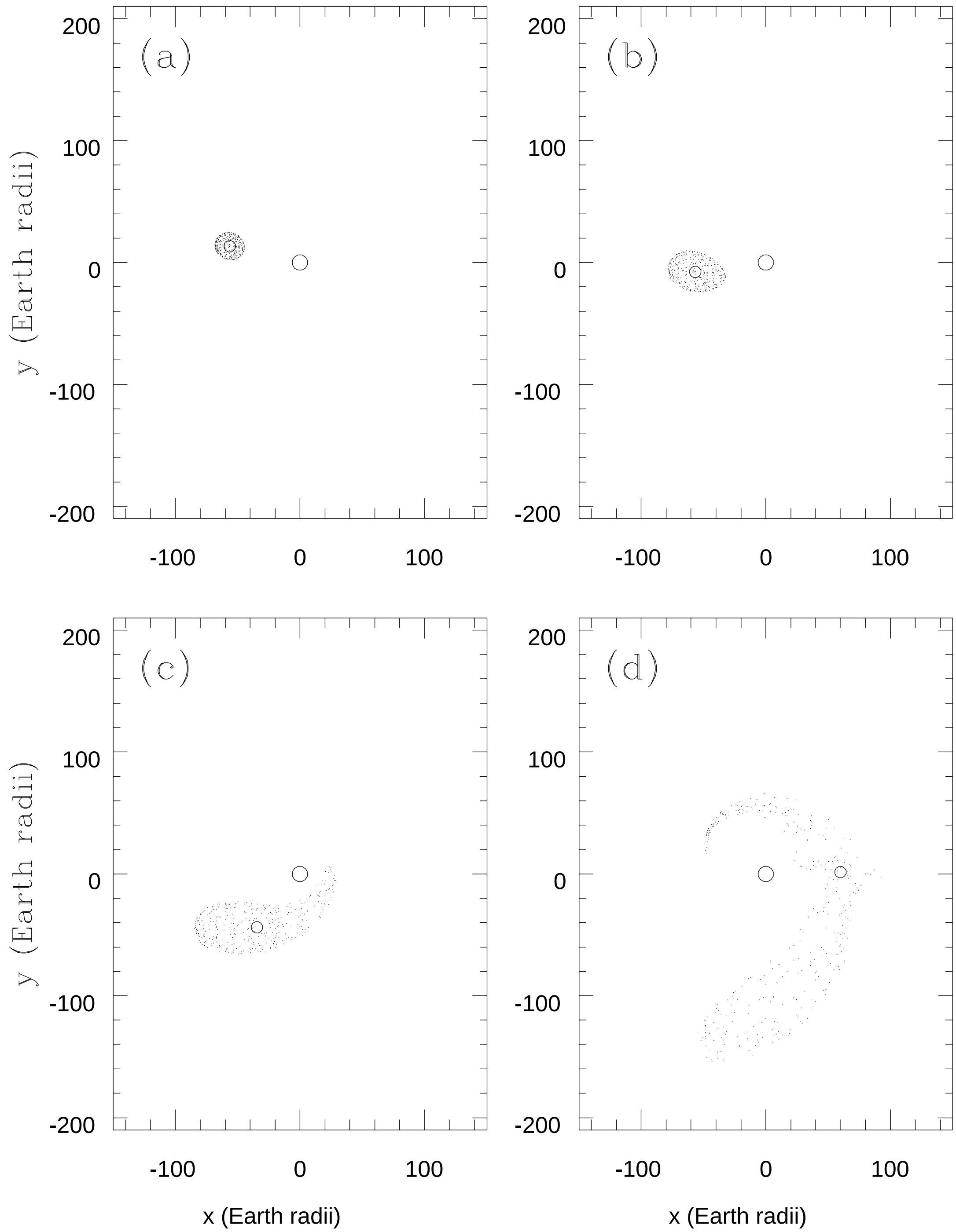

Figure A.4 Geocentric snapshots of the evolution of the swarm of lunar ejecta in a v_l=2.4 km/s simulation. The positions of all surviving test particles are projected onto the non-rotating ecliptic plane. The Earth (at 0,0) and Moon are indicated by the open circles (not to scale). The snapshots are taken at elapsed times of approximately (a) 0.05 months, (b) 0.1 months, (c) 0.2 months, and (d) 0.5 months.

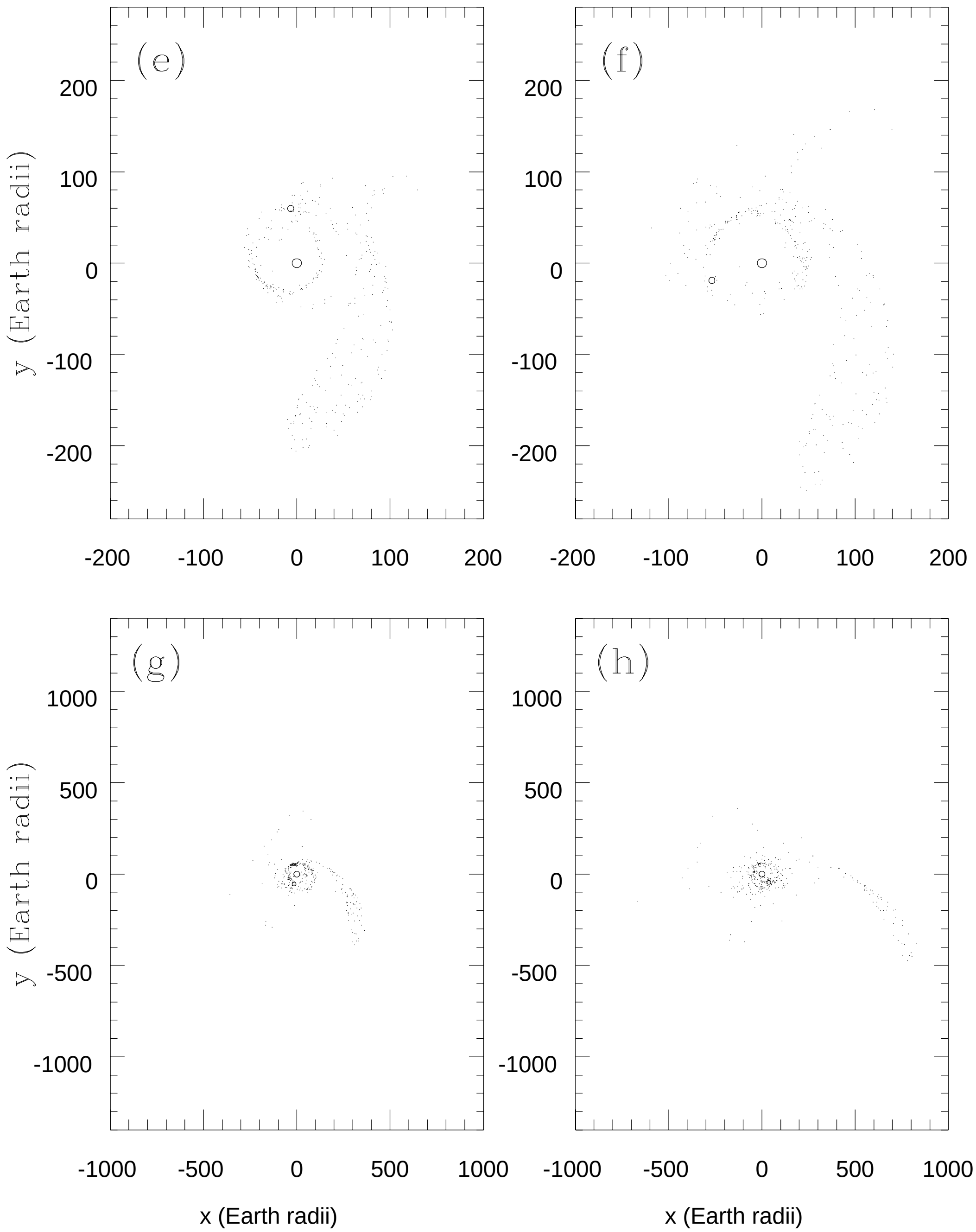

Figure A.5 Geocentric snapshots of the evolution of the swarm of lunar ejecta in a v_l=2.4 km/s simulation. The positions of all surviving test particles are projected onto the ecliptic plane. The Earth (at 0,0) and Moon are indicated by the open circles (not to scale). Note the scale of the figures change. The snapshots are shown at elapsed times of approximately (e) 0.75 months, (f) 1 months, (g) 2 months, and (h) 3 months.

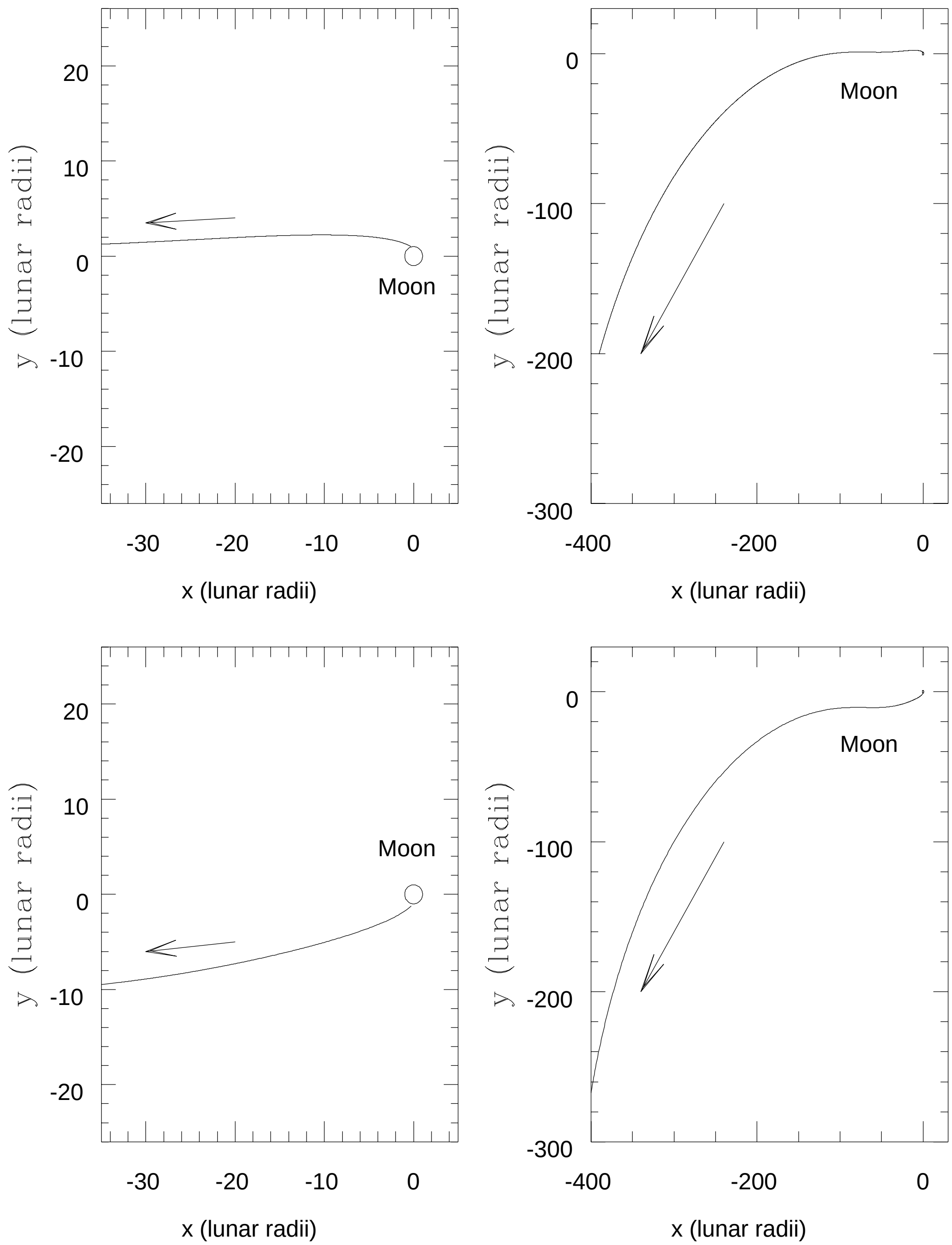

Figure A.6 The escape trajectories of two fast escapers, in lunocentric coordinates (non-rotating frame). The Moon is shown (to scale) in each case. Left panels: the hyperbolic ejection out to the lunar Hill sphere (requiring about a day). Right panels: The trajectory out to about one-half of the Earth's Hill sphere.

or second lunar encounter involved. In terms of a patched-conic picture of the dynamics, the motion could be explained as follows. In the lunar frame, the particle's specific orbital energy (*i.e.,* per unit mass) is

$$E_L = \frac{1}{2}v_L^2 - \frac{GM_m}{r_L} \tag{A.4}$$

where the L subscripts indicate quantities referred to the Moon, and so r_L is the lunar distance of the particle. If the particle is launched at v_l=2.4 km/s, then by the time the particle reaches the lunar Hill sphere (at $\simeq$ 35 lunar radii), its residual speed should be

$$\begin{aligned} v_r^2 &= v_l^2 + 2\frac{GM_m}{R_m}(\frac{1}{35} - 1) \\ &\simeq 500\text{m/s} . \end{aligned}$$

If this residual velocity were fortuitously added to the lunar velocity vector ($V_m \simeq$ 1 km/s) as discussed in conjunction with Fig. 3.3), then temporarily the particle would have the escape speed necessary to depart geocentric orbit (*i.e.,* about $\sqrt{2} \times 1$ km/s at the lunar distance). However, just reaching the lunar Hill sphere is really not sufficient since the particle will still have to do work against the Moon's gravitational field to get to 'infinity'. At infinity the residual velocity is only $\simeq$300 m/s (Eq. 3.2), and thus the particle should not have enough energy to escape geocentric orbit.

The discussion just presented is wrong because the physics cannot be represented as a superposition of two 2-body problems (*i.e.,* escape from the Moon followed by escape from Earth). Instead, we need to consider a 4-body problem. Since there is no relevant theory for the dynamics of these particles a 4-body context, I shall continue in the previous vein.

A better way to discuss the problem is to examine a particle's orbital energy in the *geocentric* frame, viewing the Moon and Sun as objects which do work on the particle. That is, in order to be on a hyperbolic geocentric orbit, the particle must reach a positive specific orbital energy in the expression

$$E_g = \frac{1}{2}v_g^2 - \frac{GM_\oplus}{R_g} , \tag{A.5}$$

where the g subscripts refer to geocentric quantities. Of course, this is still using 2-body reasoning, but it should be valid within the Earth's Hill sphere as long as the object is more than several lunar Hill-sphere radii from the Moon (so that the latter's influence on the particle's geocentric velocity is negligible). Expression (A.5) allows us to easily calculate that the Moon has a specific orbital energy of -0.538 MJ/kg (bound orbits have $E_g < 0$). Thus this much work per unit masss must be performed in the geocentric frame on an escaping particle in order to unbind it from Earth orbit.

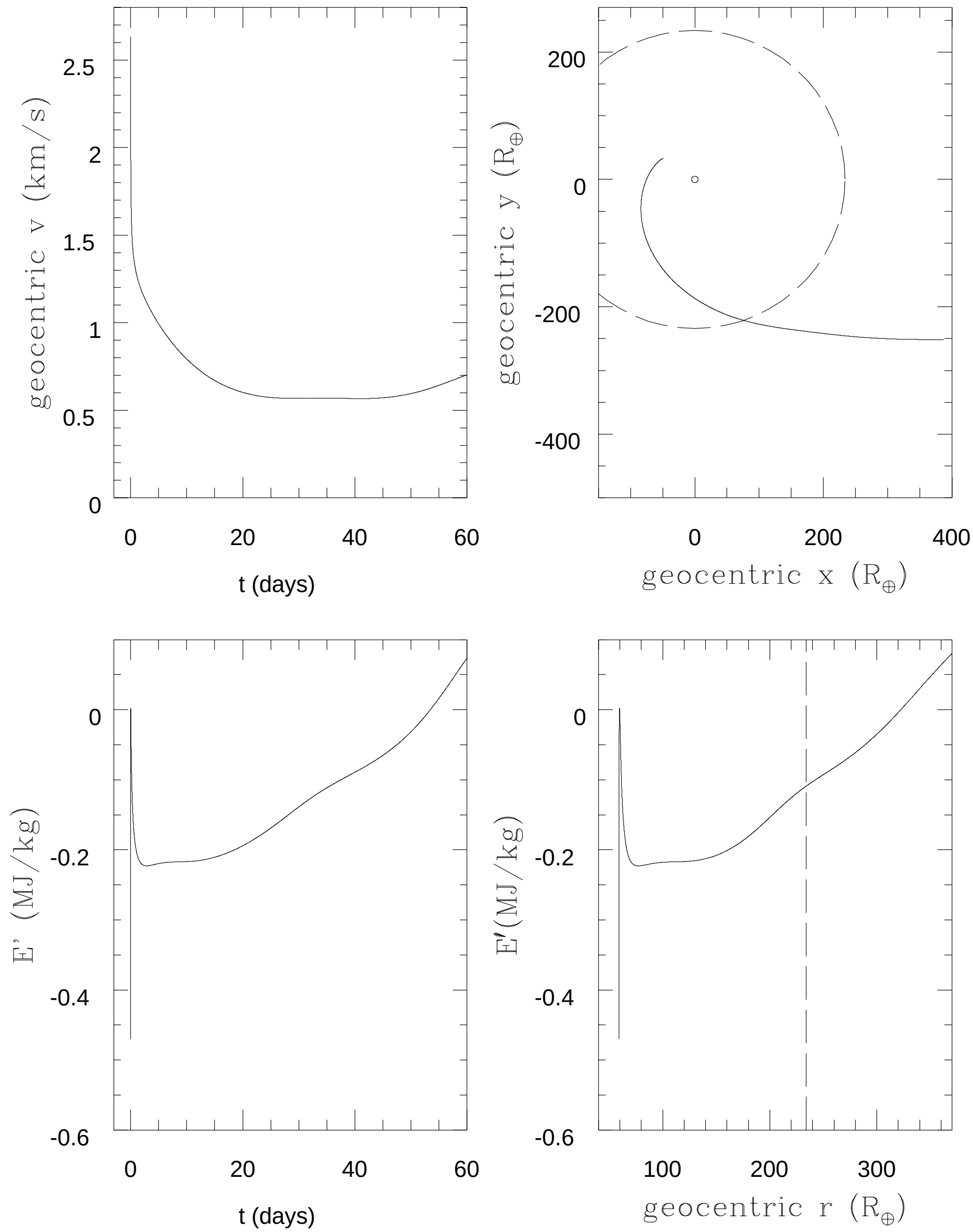

Figure A.7 The history of a fast escaper. The dashed lines mark the Earth's Hill sphere. In the upper right panel the orbit is shown projected onto a non-rotating coordinate system in the ecliptic plane with the Earth at the origin; during this particular escape the Sun occupies a point near the positive y-axis (at $y \simeq 23,000 R_{\oplus}$). See the text for discussion.

The energy history for one of the rapidly escaping particles is shown in Fig. A.7, where instead of plotting E_g, I have shown

$$E' = \frac{1}{2}v_g^2 - \frac{GM_\oplus}{R_g} - \frac{GM_m}{R_l} \ . \tag{A.6}$$

E' is more meaningful since, at the instant of launch, the geocentric velocity is very large; this is because $v_l = 2.4$ km/s, which translates into a geocentric velocity of $v_g \simeq 2.6$ km/s for this particle (upper left panel). The particle would thus be immediately unbound if the Moon had no mass. Of course, most of this kinetic energy is lost escaping the Moon's gravity-well; this is heuristically corrected for by the presence of the lunar term in Eq. A.6. Far from the Moon $E' \to E_g$.

The lower left panel of Fig. A.7 shows the energy history. In the few days it takes the particle to leave the Moon's vicinity, E' first increases rapidly, and then drops again. The peak, where the particle appears temporarily unbound ($E' > 0$), occurs near the lunar Hill sphere. However, by the time the particle escapes to a few lunar Hill-sphere radii the Moon's influence is negligible, and the geocentric energy levels off to a constant (bound) value of $\simeq -0.2$ MJ/kg. The apogee of this Keplerian orbit would be at $\simeq 313R_\oplus$, which is outside the Earth's Hill sphere of $\simeq 234R_\oplus$. Unsurprisingly, as this particle approaches the Earth's Hill sphere (upper right plot), the Sun's influence manifests itself; orbital energy increases starting at 2/3 R_H. After leaving the Earth's Hill sphere, the Sun's continuing influence unbinds the particle from Earth orbit, and the particle escapes to an independent heliocentric orbit.

A.5 Implications

The conclusion that can be drawn from this study is rather dramatic: a judiciously chosen launch at the lunar escape velocity can not only escape the Moon, but geocentric orbit as well! This can be done in a single stage; no second lunar encounter is necessary to take advantage of a gravity assist. Escape to the Earth's Hill sphere required approximately one month.

This method is more energy efficient and faster than might be expected. Using 2-body reasoning, one might think that enough fuel is needed to apply a Δv of 2.38 km/s to escape the lunar gravity field, and then another $(\sqrt{2} - 1)$ times the lunar orbital velocity (400 m/s) to achieve the transition from a circular geocentric orbit to a hyperbolic escape trajectory. In fact, the extra 400 m/s of Δv is not necessary. The trajectory is also the fastest, since escape cannot occur in less than a month if a second lunar close encounter is required to obtain a gravity assist.

The dynamics discussed here is similar to the 4-body dynamics described by Belbruno and Miller (1993), who constructed energy-optimal transfer trajectories.

I have not attempted to discover how to specify the initial conditions which belong to the escaping set; I suspect that this will be a complex problem. Such a calculation would be of interest since it would provide a low-fuel method to launch spacecraft to heliocentric space from the lunar surface.

Appendix B

The Numerical Method

The heliocentric simulations of this thesis were carried out with a Regularized Mixed Variable Symplectic (RMVS) algorithm. This method allows for the efficient numerical integration of a large number of test particles that suffer recurring close approaches with planets. The code was not written as part of this thesis research, although some minor modifications were made and the code was rigorously tested. The RMVS algorithm was developed as part of a software package called SWIFT (Levison and Duncan 1994) that implements and improves the mixed-variable symplectic method of Wisdom and Holman (1991).

B.1 Mixed-variable symplectic (MVS) integrators

These codes are outgrowths of the intense work that has gone into the study of efficient methods for integrating the equations of motion of Hamiltonian systems in the last two decades (Ruth 1983, Forest and Ruth 1990, Channell and Scovel 1990, Yoshida 1993). The differential equations of motion derived for a Hamiltonian system have special properties. The goal is to take advantage of this 'symplectic' structure in the phase space of these systems; for example, as particles move they must remain on a hypersurface of constant energy. Most numerical integration algorithms (*e.g.,* Runge-Kutta) do not make allowances for this special structure, and thus the trajectories drift away from surfaces representing integrals of the motion (for example, the energy or angular momenta may change). Symplectic integrators are designed to conserve these invariants; much more complete discussions are given in Gladman *et al.* (1991) and Yoshida (1993).

The basic algorithm is to break the problem into exactly soluble pieces, and then to solve them alternately in such a fashion as to match as closely as possible the original system. Until 1991 the two pieces of the Hamiltonian used were the kinetic and potential energy functions, either of which by itself describes a trivially integrable system (see Gladman *et al.* 1991).

A further improvement was provided by Wisdom and Holman (1991), who created a symplectic integrator that took explicit advantage of the fact that the Kepler problem is exactly soluble. These authors alternate between solving the Kepler problem and the interaction Hamiltonian of the system. That is, each test particle moves along a Kepler ellipse, which is then perturbed by the remaining, non-Keplerian forces. The Kepler problem is solved efficiently and exactly by using the Gauss f and g functions (Danby 1988). The interaction Hamiltonian contains the forces from all the interactions between the particles (excepting the Sun), and is trivially integrable if expressed in the Jacobi coordinates for the N-body problem (Wisdom and Holman 1991). Because two sets of coordinates are simultaneously used in the solution, the method is referred to as 'Mixed-Variable Symplectic' (MVS).

The code is structured in two pieces. The N_p planets are integrated in an N-body fashion; that is, each planet feels the gravitional perturbations of all the others. The number of force calculations necessary per time step scales as N_p^2. Simultaneously, the forces exerted by these planets on a large number of test particles is calculated and the test particle orbits are also evolved. The n test particles do not interact with each other or with the planets, and thus the number of force calculations scales only linearly with n. Therefore the CPU time t required to perform a simulation for a simulated duration T scales as $t \propto TN_p^2 n$.

In this algorithm the non-Keplerian forces must be small in order for the integrator to remain efficient. If the test particles were to undergo a close approach to a planet, the truncation error of the algorithm will increase. Decreasing the time step of a numerical integrator is a common way of dealing with such a problem. Unfortunately, changing the time step in a symplectic integrator degrades the integral conservation properties (Gladman *et al.* 1991), and thus the MVS algorithm is best applied to systems that do not undergo close approaches (*e.g.*, studies of dynamical stability for orbits that are not planet-crossing).

B.2 The regularized MVS (RMVS) integrator

Since many interesting problems involve planet-crossing orbits, it was important to attempt to extend the MVS algorithm to allow for test particles to have repeated close encounters. RMVS (Levison and Duncan 1994) uses an adaptive Hamiltonian that in some sense 'regularizes' close approaches (producing the 'R' in the acronym). The idea is simple: when a particle approaches a planet, instead of treating the problem as a Kepler orbit about the Sun perturbed by the planet, the orbit is considered to be around the planet and perturbed by the Sun. In the intermediate regime (near the planetary Hill-sphere) the time step is simply lowered. Although this destroys the exact symplectic nature of the algorithm in a rigorous sense, extensive testing has shown (Levison and Duncan 1994, Michel and Valsecchi 1996) that the statistical results of code are correct.

The term 'regularized' should be applied to this integrator only in an intuitive sense. Regularization is a mathematical process that removes a singularity from a differential equation. Regularizing the Kepler problem (or the N-body problem) has been a method of dealing with the force singularity at $r = 0$ (see Stiefel and Schiefele 1971). In this process the Kepler problem is turned into the harmonic oscillator by coordinate transformations; the singularity is thus eliminated from the equations. In the RMVS algorithm, the singularity is effectively removed from the problem by switching to a Kepler problem centered on the planet. Using the exact solution to the problem provided by the Gauss f and g functions then allows arbitrarily-close approaches to be computed efficiently.

The RMVS code is thus able to deal with large numbers of test particles moving in a system with multiple planets. Objects not close to a planet are integrated with the MVS algorithm. Particles within 3 Hill-sphere radii of a planet are temporarily treated differently than the rest of the bodies; after their close encounter is over they are returned to the swarm of other test particles. Only particles that enter a planetary Hill-sphere have their Hamiltonian recast so that the planet is the central object. Upon emerging from the Hill sphere, the particles are carefully re-synchronized with the rest of the integration.

This algorithm is more efficient computationally (by an order of magnitude) than all other methods that this author is aware of. Several hundred planet-crossing particles can be integrated in the field of 7 planets at a rate of order 500,000 years per day of CPU time on a very fast dedicated workstation. All told, the lunar simulations required about 5 months of actual computer time on a Hewlett Packard 715 with a 75 MHz clock speed. The martian simulations required somewhat more time, and were performed on a variety of computer platforms.

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