

## DYNAMICS OF THE GIANT PLANETS OF THE SOLAR SYSTEM IN THE GASEOUS PROTOPLANETARY DISK AND THEIR RELATIONSHIP TO THE CURRENT ORBITAL ARCHITECTURE

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### ABSTRACT

We study the orbital evolution of the four giant planets of our solar system in a gas disk. Our investigation extends the previous works by Masset & Snellgrove and Morbidelli & Crida, which focused on the dynamics of the Jupiter-Saturn system. The only systems we found to reach a steady state are those in which the planets are locked in a quadruple mean-motion resonance (i.e., each planet is in resonance with its neighbor). In total, we found six such configurations. For the gas-disk parameters found in Morbidelli & Crida, these configurations are characterized by a negligible migration rate. After the disappearance of the gas, and in the absence of planetesimals, only two of these six configurations (the least compact ones) are stable for a time of hundreds of millions of years or more. The others become unstable on a timescale of a few Myr. Our preliminary simulations show that, when a planetesimal disk is added beyond the orbit of the outermost planet, the planets can evolve from the most stable of these configurations to their current orbits in a fashion qualitatively similar to that described in Tsiganis et al.

*Key words:* planets and satellites: formation — solar system: formation

### 1. INTRODUCTION

Of all the planetary systems known to date, our solar system undoubtedly remains the one for which we have the largest number of accurate observational constraints to use for modeling the evolution of the giant planets up to their current orbital configuration. Reconstructing this evolution as far back in time as possible is very important, because it can constrain the orbits on which the planets formed and, in turn, shed new light on their formation mechanism. In particular, knowledge of these aspects may allow us to understand why our solar system looks so different from the extrasolar systems discovered so far. Two differences are particularly striking. First, our giant planets are all far from the Sun, whereas giant planets in the near vicinity of their host stars are numerous in extrasolar systems. Observational biases favor the discovery of these planets, but the very fact that they exist in other systems and not in our own is real and remarkable. Thus, we need to understand in which conditions planets can avoid large-range radial migration toward the central star. Second, the orbital eccentricities of extrasolar planets, including those at distances of several AU from the central star, are generally much larger than the eccentricities of the giant planets of our system. The latter, equal to several percent, are nevertheless nonnegligible. Planet eccentricities are believed to be the result of mutual perturbations (Rasio & Ford 1996; Marzari & Weidenschilling 2002). It is important to understand which perturbations are responsible for the moderate eccentricities of the giant planets of the solar system and why the orbital excitation could be much stronger in most extrasolar cases.

Our group has recently proposed two models that aim to reconstruct two different phases of the evolution of the solar system: one that was dominated by the gas disk and one that occurred after the disappearance of the gas.

The first model specifically addresses the migration of Jupiter and Saturn in the protoplanetary gas disk. If considered individually, these planets should have evolved toward the Sun as the result of type II migration. However, Masset & Snellgrove (2001, hereafter MS01) showed that Saturn tends to get locked in a 2:3 mean-motion resonance (MMR) with Jupiter. In this configuration, the gaps opened in the disk by the two planets can overlap with each other. This can lead to a reversal of the migration direction. This mechanism has more recently been studied in greater detail by Morbidelli & Crida (2007, hereafter MC07). They showed that, in the gas-disk parameter space represented by viscosity and scale height, there is a one-parameter family of solutions such that, once locked in the 2:3 MMR, Jupiter and Saturn do not migrate. Sets of parameters close to this family lead to inward or outward migration but with rates that are much slower than the theoretically predicted type II migration rates for a single planet. MC07 also showed that this kind of nonmigrating or slowly migrating evolution is possible only if the planets involved have a mass ratio close to that of Jupiter and Saturn. Planets of similar masses, or—worse—systems where the outer planet is the more massive, inevitably lead to a fast inward migration. Therefore, they argued that the absence of a hot/warm Jupiter in our solar system is due to the specific mass hierarchy of our giant planets and to their formation on initially close-by orbits. To support this claim they pointed out that none of the known

extrasolar planetary systems with two bodies close to their parent stars fulfill the conditions necessary to avoid type II migration: either the planets have comparable masses, the outermost one is the most massive, or they are too separated to have sculpted overlapping gaps in their primordial gas disks.

The second model (Tsiganis et al. 2005; Gomes et al. 2005), which actually was developed first, is often called the “Nice model” because it was developed by an international collaboration at the Nice Observatory in France. This model argued that, if the giant planets had a more compact configuration at the end of the gas-disk phase, their subsequent migration driven by interaction with a planetesimal disk could have forced them to cross some mutual MMR, thereby triggering a global instability; the current orbital configuration could then be achieved from the gravitational interaction between the planets and the disk particles. More precisely, the Nice model postulated that the ratio of the orbital periods of Saturn and Jupiter was initially slightly less than 2, so that the planets were close to their mutual 1:2 MMR; Uranus and Neptune were supposedly orbiting the Sun a few AU beyond the gas giants, and a massive planetesimal disk extended from about 1.5 AU beyond the last planet up to 30–35 AU. As a consequence of the interaction of the planets with the planetesimal disk, the giant planets suffered orbital migration, which slowly increased their orbital separation. As shown in their  $N$ -body simulations, after a long quiescent phase (with a duration varying from 300 Myr to 1 Gyr, depending on the exact initial conditions), Jupiter and Saturn were forced to cross their mutual 1:2 MMR. This event excited their orbital eccentricities to values similar to those currently observed. The acquisition of eccentricity by both gas giants destabilized Uranus and Neptune. Their orbits became very eccentric, so that they penetrated deep into the planetesimal disk. Thus, the planetesimal disk was dispersed, and the interaction between planets and planetesimals finally parked all four planets on orbits with separations, eccentricities, and inclinations similar to what we currently observe. This model has a long list of successes. As already stated, it explains the current orbital architecture of the giant planets (Tsiganis et al. 2005). It also explains the origin of the so-called late heavy bombardment (LHB), a spike in the cratering history of the terrestrial planets that occurred  $\sim 650$  Myr after planet formation. In the Nice model, the LHB is triggered by the dispersion of the planetesimal disk; the timing, the duration, and the intensity of the LHB deduced from lunar constraints are well reproduced by the model (Gomes et al. 2005). Furthermore, the Nice model also explains the capture of planetesimals around the Lagrangian points of Jupiter, with a total mass and orbital distribution consistent with the observed Jupiter Trojans (Morbidelli et al. 2005). More recently, it has been shown to provide a framework for understanding the capture and orbital distribution of the irregular satellites of Saturn, Uranus, and Neptune (Nesvorný et al. 2007). The main properties of the Kuiper Belt (the relic of the primitive transplanetary planetesimal disk) have also been explained in the context of the Nice model (Levison et al. 2007; see Morbidelli et al. 2007 for a review).

The problem we are now facing is that a 2:3 MMR configuration of Jupiter and Saturn, advocated to explain why the planets did not migrate toward the Sun, is different from the initial conditions of the Nice model, in which Jupiter and Saturn have initially nonresonant orbits  $\sim 0.5$  AU interior to the 1:2 MMR. MC07 proposed a few mechanisms to extract Saturn from the 2:3 MMR with Jupiter and bring it close to the 1:2 MMR at the end of the gas-disk phase. However, we believe that a more complete system (i.e., one with Uranus and Neptune) would probably become immediately unstable when this happens, in contrast with the Nice model.

This paper is the first in a series of two in which we explore this problem more thoroughly. In particular, we try to bridge the results of MS01 and MC07 with the fundamental aspects of the Nice model. In § 2 we start from a configuration found in MC07 where Jupiter and Saturn are not migrating, and we progressively add Uranus and Neptune into the problem. Following this procedure, we find six fully resonant relative configurations of the four planets, which are stable and avoid migration toward the Sun. In § 3 we study the stability of these configurations on longer timescales, after the gas disk has disappeared and in the absence of a planetesimal disk. We find that two of the six configurations are stable over a very long time ( $10^8$ – $10^9$  yr). Although a detailed analysis of the evolution of these configurations under the influence of a planetesimal disk is left to the forthcoming paper, as proof of concept we present a couple of simulations in § 4, showing that the planets could eventually achieve an orbital architecture similar to their current one. The details of the proposed evolution are different from those of the Nice model (different resonances are involved in triggering the planetary instability), but the basic mechanism and evolution are the same. The conclusions of this paper are given in § 5.

## 2. FOUR-PLANET DYNAMICS IN THE GAS DISK

We use the hydrodynamic code developed by Crida et al. (2007) on the basis of the FARGO code by Masset (2000a, 2000b) to simulate the dynamics of the planets in the protoplanetary gas disk. In the Crida et al. scheme, the disk is represented using a combination of two-dimensional and one-dimensional grids. The main part of the disk, in which the planets evolve, is sampled by a two-dimensional grid in polar barycentric coordinates. This grid extends from  $r = 0.3$  to 5 in radius (in units of the initial orbital radius of Jupiter) and has a resolution of 282 in radius and 325 in azimuth. The planets also evolve on coplanar orbits. The inner part of the disk (ranging from  $r = 0.016$  to the innermost boundary of the two-dimensional grid) and the outer part of the disk (ranging from the outermost boundary of the two-dimensional grid to  $r = 40$ ) are sampled by a one-dimensional grid. These one-dimensional grids have open outflow boundaries at  $r = 0.016$  and 40, while they exchange information with the two-dimensional grid at their common boundaries in order to supply realistic, time-dependent boundary conditions for the latter. The algorithm for this interface between the one-dimensional and two-dimensional grids explicitly requires that the angular momentum of the global system (the disk in the two-dimensional section, plus the disk in the one-dimensional section, plus the system of planets and star) be conserved. With this approach the global viscous evolution of the disk and the local planet-disk interactions are both described well, and the feedback of one on the other is properly taken into account. Because the migration of the giant planets depends on the global evolution of the disk, this code provides more realistic results than the usual algorithms in which the evolution of the considered two-dimensional portion of the disk depends crucially on the adopted (arbitrary) boundary conditions. For more information and accuracy tests we refer the reader to Crida et al. (2007).

We adopt a set of disk parameters from MC07 in which Jupiter and Saturn did not migrate after they became locked in the 2:3 MMR. The scale height of this disk is 5%, and its viscosity (constant over radius) is  $\nu = 3.2 \times 10^{-6}$ , assuming that the Sun’s mass and the initial semimajor axis of Jupiter are the units of mass and distance, respectively. In the usual  $\alpha$ -prescription (Shakura & Sunyaev 1973), this viscosity corresponds to  $\alpha = 1.2 \times 10^{-3}$  at  $r = 1$ . The initial surface density of the disk is  $\Sigma(r) = 3 \times 10^{-4} \exp(-r^2/53)$ . This  $\Sigma$  was inspired by the results of Guillot

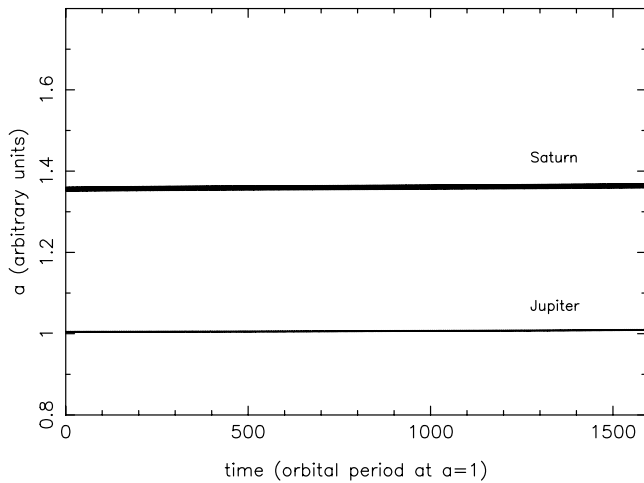


FIG. 1.—Evolution of Jupiter and Saturn in the gas disk ( $H/r = 0.05$ ,  $\nu = 3.2 \times 10^{-6}$ ) after they became locked in their mutual 2:3 MMR (MC07).

& Hueso (2006), who studied the structure of a disk that viscously evolved under the effects of the collapse of fresh matter from the protostellar cloud, the viscous spreading of gas, and photoevaporation by the central and neighboring stars. The exact choice of  $\Sigma(r)$  should not be crucial for our analysis, although some issues should be kept in mind. Here  $\Sigma$  is a multiplicative factor in the equations of motion, so it simply governs the evolution timescale. In the case of differential migration of multiple planets, the value of  $\Sigma$ —here close to the minimal-mass solar nebula (Hayashi 1981)—determines the relative migration rates, and hence the probability of capture in various mutual resonances. The radial profile of  $\Sigma(r)$  also affects the relative migration rates of planets at different locations. However, given that in our simulations (see below) all the planets are within a factor of 2.5 in heliocentric distance, the sensitivity of their relative evolution to the radial profile of  $\Sigma$  should be moderate. We will come back to these issues when discussing our results.

Figure 1, reproduced from MC07, shows the evolution of the semimajor axes of Jupiter and Saturn, after they became locked in the 2:3 MMR, over 1500 Jovian orbital periods. A slight parallel outward migration is visible, which could be annealed with a slight increase in the disk’s scale height or viscosity. The eccentricities have only small-amplitude oscillations around a small constant value (0.015 for Saturn and 0.004 for Jupiter). We refer to MS01 and MC07 for an explanation of why type II migration is prevented in this configuration. We just stress here that this mechanism is robust. For a given (reasonable) viscosity, the coupled Jupiter-Saturn pair migrates outward in a thin disk, while it moves inward in a thick disk. Thus, it is always possible to find a disk scale height for which the migration vanishes. If some simulation parameters are changed (e.g., the prescription of the boundary conditions, the radial dependence of the viscosity, or the scale over which the potential of each planet is smoothed—here set to  $0.7H$ , where  $H$  is the local height of the disk at the planet’s position), the exact value of the disk scale height that allows for a nonmigrating solution may change, but the very existence of such a solution is not at risk.

The fact that Jupiter did not migrate closer to the Sun argues that the inward migration of the Jupiter-Saturn pair was, for the most part, inhibited. On the other hand, it is difficult to believe that Jupiter and Saturn migrated outward because the asteroid belt would have been completely decimated if Jupiter had been closer to the Sun. (Note that models that assume that Jupiter was, more or less, at its current location adequately reproduce the ob-

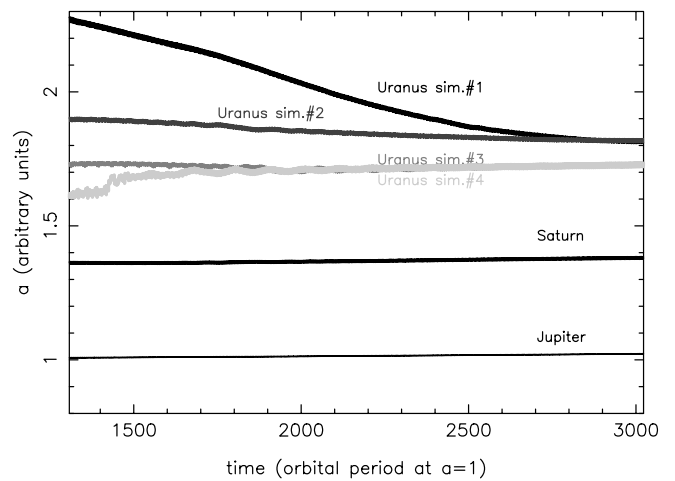


FIG. 2.—Evolution of Uranus, after it was added to the simulation presented in Fig. 1. Four simulations are presented. In the first (*black curve*) Uranus is started beyond the 1:2 MMR with Saturn. During its inward migration it passes across the 1:2 MMR and eventually gets trapped into the 2:3 MMR. In the second simulation (*dark-gray curve*) Uranus is introduced between the 2:3 and 1:2 MMRs with Saturn. Again, Uranus gets captured into the 2:3 MMR. In the third simulation (*medium-gray curve*) Uranus is started between the 3:4 and the 2:3 MMRs with Saturn and gets captured in the former. In the fourth simulation (*light-gray curve*) Uranus is started between the 4:5 and 3:4 MMRs and evolves outward until it is captured again in the 3:4 MMR. The evolution of Jupiter and Saturn is essentially the same in all the simulations, so we only present one example.

served structure of the asteroid belt; see Petit et al. 2002 for a review). Thus, the philosophy of MC07, as well as of this work, is to construct models where the structure of the protoplanetary disk is such that the planets do not migrate significantly. Given that this happens in our numerical scheme when, e.g.,  $H/r = 0.05$  and  $\nu = 3.2 \times 10^{-6}$ , we simply adopt these parameters with the understanding that the real disk may have been different.

We now extend the work of MC07 by adding Uranus and Neptune to the calculation. Given that the heliocentric order of the ice giants changed during  $\sim 50\%$  of the successful simulations of the original Nice model, so that we do not know which one formed closer to the Sun, we assume that the two planets have the same mass:  $15 M_{\oplus}$ . For simplicity, we nevertheless call the innermost ice giant “Uranus” and the outermost one “Neptune.”

The goal of this section is to find stable configurations for the four planets. To accomplish this we employ the following procedures. We pick up the MC07 simulation shown in Figure 1 at  $t = 1300$ , at which point we introduce Uranus into the calculation. In contrast to the procedures used by MC07 for Jupiter and Saturn, we let Uranus migrate freely from the moment that it is introduced into the simulation. We think that this is a legitimate change because Uranus’s effect on the disk’s surface density profile is minimal and occurs on a timescale that is short compared to Uranus’s migration timescale.

In the first simulation, Uranus is initially placed at  $r = 2.55$ , which is beyond the 1:2 resonance with Saturn. Uranus moves inward relatively rapidly due to type I migration. It jumps across the 1:2 resonance with Saturn (the resonance is too weak to capture it given the migration speed generated by our assumed disk; see also MS01) and is eventually trapped in the 2:3 resonance with Saturn (Fig. 2, *black curve*). In the second simulation, we start Uranus at  $r = 1.90$ , which is between the 2:3 and 1:2 resonances with Saturn. Again, the planet gets trapped in the 2:3 resonance (Fig. 2, *dark-gray curve*). In both simulations, the capture into resonance increases the eccentricity of Uranus from  $\sim 0$  to  $\sim 0.025$ . After this capture, the three planets evolve in

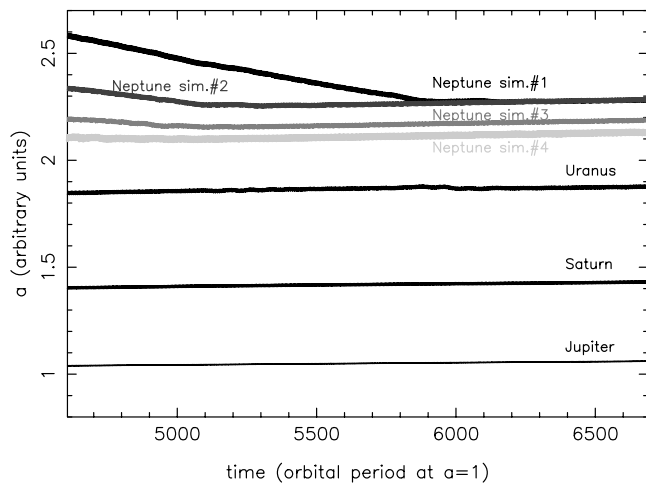


FIG. 3.— Evolution of Neptune, after it was added to the simulation in which Uranus and Saturn are in the 2:3 MMR. Four simulations are presented. In the first simulation (*black curve*) Neptune is started beyond the Uranus 2:3 MMR but jumps over it and gets captured in the Uranus 3:4 MMR. In the second simulation (*dark-gray curve*) Neptune is started between the 3:4 and 2:3 MMRs with Uranus and gets captured in the former. In the third simulation (*medium-gray curve*) Neptune is started between the 4:5 and 3:4 MMRs and is captured in the former. In the fourth simulation (*light-gray curve*) Neptune is started between the 5:6 and 4:5 MMRs and is captured in the former. The evolution of Jupiter, Saturn, and Uranus is essentially the same in the four simulations, so we only present one example.

parallel, showing that they have reached a stable relative configuration in a three-body 2:3 + 2:3 resonance.

Note that after this configuration is reached, Jupiter's and Saturn's outward migration is accelerated somewhat. We might expect the opposite, since Uranus feels a negative torque from the disk that should be transmitted to Jupiter and Saturn through the resonances. The slow outward migration is due to the fact that Uranus slightly depletes the disk outside of Saturn's orbit. As a consequence, the balance of the torques exerted on Jupiter and Saturn by the disk is broken, and the positive torque felt by Jupiter dominates. However, as we said above, this does not invalidate the general MS01/MC07 scenario because we could restore the torque equilibrium if we were to slightly increase the scale height of the disk.

In a third simulation, we started Uranus at  $r = 1.73$ , which is between the 2:3 and 3:4 resonances with Saturn. Again, we observe an inward drift due to type I migration until the planet is trapped in Saturn's 3:4 MMR. After this, the relative configuration of the three planets does not change (see Fig. 2, *medium-gray curve*), although, as described above, the whole system migrates outward. The eccentricity of Uranus does not exceed 0.01. The evolutions of Jupiter and Saturn are indistinguishable in all three simulations, so we just plot those of the first simulation for clarity.

Furthermore, we perform a final simulation where we place Uranus initially at  $r = 1.61$ , which is between the 3:4 and 4:5 MMRs with Saturn. In this case, the evolution is different from those described above. In particular, the motion of Uranus is unstable due to its proximity to the gas giants. Hence, Uranus is pushed outward until it again finds a stable relative configuration in the 3:4 MMR with Saturn (see Fig. 2, *light-gray curve*). Its subsequent evolution is indistinguishable from the previous one.

From the above four experiments we deduce that, for our assumed disk, there are two stable and invariant configurations of the three-planet system: Uranus is in either the 2:3 or the 3:4 MMR with Saturn.

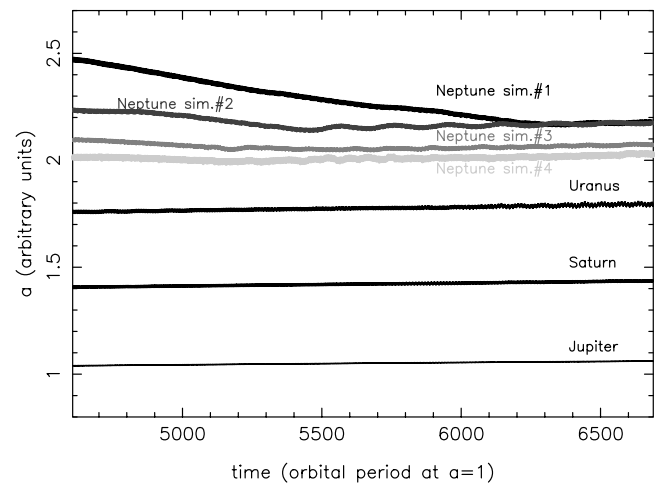


FIG. 4.— Same as Fig. 3, but for the case where Neptune is added to the simulation in which Uranus is in the 3:4 MMR with Saturn.

Next, we introduce Neptune into the problem. We start by considering the first of the aforementioned simulations, where Uranus was trapped in the 2:3 MMR with Saturn. We continue this simulation after placing Neptune at  $r = 2.58$ , i.e., between the 2:3 and 1:2 MMRs with Uranus. As expected, it drifts inward due to type I migration (Fig. 3, *black curve*). However, Neptune is not trapped in Uranus's 2:3 MMR, but crosses it, because the resonance is too weak to trap a body at Neptune's migration speed. Neptune is, however, subsequently trapped in Uranus's 3:4 MMR. Note that the 3:4 MMR with Uranus is also the 1:2 MMR with Saturn, since Uranus and Saturn are in the 2:3 MMR. The capture of Neptune into resonance pumps the eccentricity of Uranus up to about 0.05, whereas the eccentricity of Neptune increases only to  $\sim 0.01$ .

Repeating the simulation with Neptune starting from  $r = 2.34$  (between the 3:4 and 2:3 MMRs with Uranus) also leads to capture in the 3:4 MMR with Uranus. After the trapping, the evolutions of the two systems are indistinguishable (Fig. 3, *dark-gray curve*). Conversely, starting the simulation with Neptune at  $r = 2.19$  or  $2.11$  leads to its capture into the 4:5 or 5:6 MMRs with Uranus, respectively (Fig. 3, *medium-gray and light-gray curves*). The eccentricities of the ice giants are progressively smaller with increasing  $m$ , for an  $m:m+1$  resonance. For the 5:6 MMR, the eccentricities of Uranus and Neptune become  $\sim 0.04$  and  $\sim 0.007$ , respectively.

We repeat the same exercise, but this time in the system where Uranus was trapped in the 3:4 MMR with Saturn, and find similar results (see Fig. 4). The evolution of the eccentricities (not shown in the figure) is also similar to what is observed in the runs of Figure 4. Thus, we conclude that for each of the two stable Jupiter-Saturn-Uranus configurations, there are three stable locations for Neptune: in Uranus's 3:4, 4:5, or 5:6 MMRs. Thus, we find six planetary configurations in total. In all cases, the four planets form a fully resonant system. In addition, all these configurations are characterized by an almost complete absence of radial migration, which is required to explain the absence of a hot/warm Jupiter in our solar system.

It is important to keep in mind that we cannot be sure that the six configurations found here are the only possible final states for the problem at hand. For example, we might reach a different configuration if we were to introduce Uranus and Neptune at the same time and on mutually scattering orbits in the vicinity of Jupiter or Saturn. Similarly, if we were to use a lower mass disk, the ice giants might be captured into weaker resonances, e.g., the

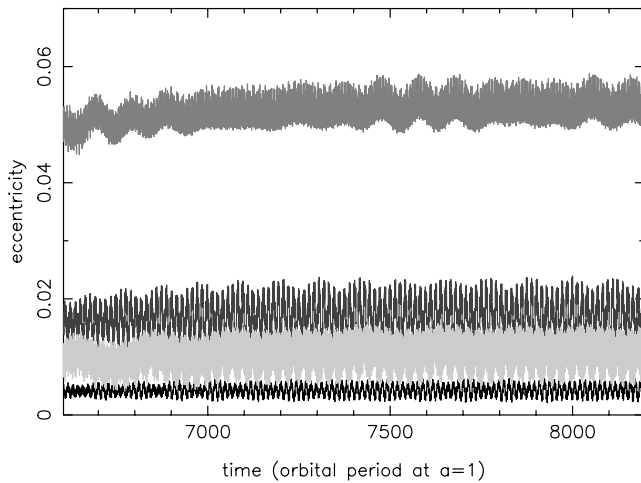


FIG. 5.—Evolution of the eccentricities of Uranus (*first curve*), Saturn (*second curve*), Neptune (*third curve*), and Jupiter (*fourth curve*) during the simulation in which Uranus is in the 2:3 MMR with Saturn and Neptune is in the 3:4 MMR with Uranus. The surface density of the disk is now halved every 160 orbital periods at  $r = 1$ .

1:2 resonance with Saturn (for Uranus) or the 2:3 resonance with Uranus (for Neptune). However, these systems will be less compact than our six, and, given the results in § 4, we believe that they are unlikely to evolve into systems resembling the real giant planets. Thus, for the remainder of this paper we restrict our analysis to the six configurations found in this section.

### 3. EVOLUTION OF THE PLANETS AFTER THE DISAPPEARANCE OF THE GAS DISK

In order to test the long-term stability of the planetary configurations constructed in the last section, we first have to transition from a gaseous to a gas-free environment. Gas disks are typically dispersed on a timescale of  $10^5$ – $10^6$  yr (Haisch et al. 2001). Unfortunately, we do not yet know how this dispersal takes place. Photoevaporation is probably the key, but exactly how it proceeds (i.e., from the inside first, as argued in Alexander et al. [2006], or from the outside first, as found by Adams et al. [2004]) is still debated. Thus, we decided to implement a very simple transition in our hydrodynamic code, where we do not change the shape of the disk’s surface density profile but decrease the total amount of gas exponentially in time. We chose a decay rate such that the gas mass was halved in 160 “Jovian” orbital periods (i.e., at  $r = 1$ ). These hydrodynamic simulations are performed for 1500 Jovian orbital periods, implying that the disk is reduced by a factor of 670 at the end of the simulation; i.e., there is effectively no gas left.

Note that we are not claiming that the disappearance of the gas disk actually followed this simple recipe. Our aim is only to change the disk potential as smoothly and slowly as possible (given the available computing time) in order to give the planets enough time to adapt to the evolving situation. During these simulations we do not observe any significant evolution in the semimajor axes of the planets, and so the resonant structure is preserved. The eccentricities of some of the planets (particularly Uranus and Saturn) increase slightly but attain new equilibrium values. The evolution of the eccentricities for the system with Saturn and Uranus in the 2:3 MMR and Uranus and Neptune in the 3:4 MMR is shown in Figure 5.

Once the gas is removed, we can continue following the evolution of the systems with an  $N$ -body code, accounting only for the Sun and the four planets. The simulations are done with the

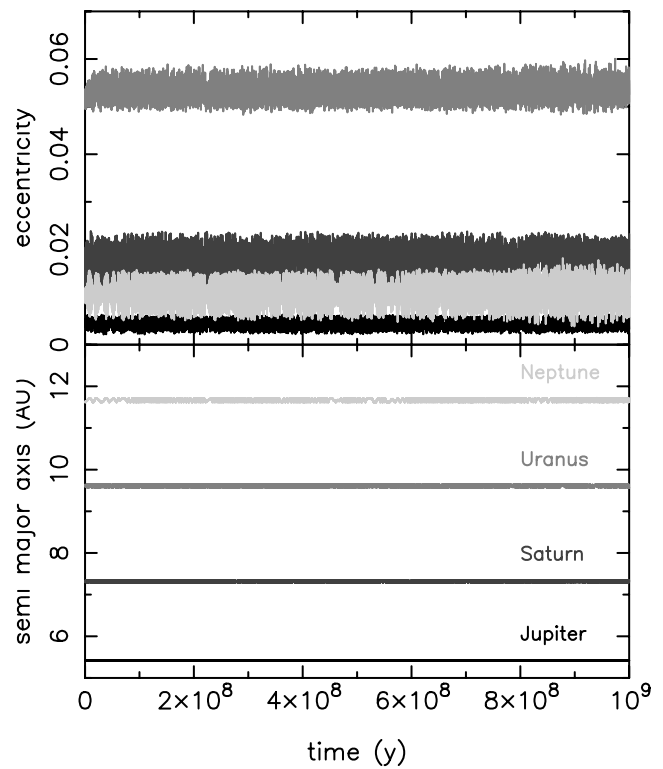


FIG. 6.—Evolution of the giant planets over 1 Gyr, according to an  $N$ -body simulation, starting from the final output of Fig. 5. The top panel shows the evolution of the eccentricities with the same color code as Fig. 5. The bottom panel shows the evolution of the semimajor axes. The evolution looks perfectly regular and stable. No planetesimal disk is considered, and hence no migration of the planets relative to each other is observed.

symplectic integrator SyMBA (Duncan et al. 1998) and cover a timescale of 1 Gyr (assuming that  $r = 1$  corresponds to 5.1 AU so that the orbital period there is 11.5 yr). The time step is 0.2 yr.

Note that because the hydrodynamic simulations were carried out in two dimensions, the planetary orbits are strictly coplanar in our  $N$ -body simulations. We do not think that this is a significant limitation because it is well known that gas disks very effectively damp planetary inclinations (Lubow & Ogilvie 2001; Tanaka & Ward 2004), and we see no way to effectively excite them again. Planetary inclinations can only be excited by close encounters (which do not occur in our resonant configurations) or if the planets were trapped in inclination MMRs. Inclination resonances, however, are much weaker than the eccentricity resonances that occur at the same location (they act as second-order resonances because the inclination has to appear with an even power in the equations of motion for d’Alembert rules; see Morbidelli 2002), so trapping in inclination resonances is highly unlikely. Thus, we expect that the real planets have small inclinations when they emerge from the gas disk, so that the study of the long-term stability of the multiresonant configuration can be done effectively in two dimensions.

We find that the configuration with Saturn and Uranus in the 2:3 MMR and Uranus and Neptune in the 3:4 MMR remains stable for the full integration time, with no visible changes in semimajor axes or eccentricities (Fig. 6). Remember, however, that this simulation does not take into account the effects of a remnant planetesimal disk, which was used in the original Nice model to make the planetary system unstable. We study this situation in § 4.

The configuration with Saturn and Uranus in the 2:3 MMR and Uranus and Neptune in the 4:5 MMR remains stable for

400 Myr. This time is long enough, however, that this configuration might be a reasonable starting point for a Nice-model-like evolution, because it is consistent with the 650 Myr delay between planet formation and the onset of the LHB.

All other configurations become unstable very quickly, at times ranging from a fraction of a Myr (for the most compact configuration) to 27 Myr (in the case with both Saturn and Uranus and Uranus and Neptune in the 3:4 MMR). In these cases, however, we became concerned that we were making these systems unstable because our disk dispersal time was too short. To test this possibility, we again did the later simulation with a gas-halving time of 1100 orbital periods at  $r = 1$ . The simulation was run for 10,000 orbital periods, at the end of which the gas surface density had been reduced by a factor of  $\sim 500$ . Even during the hydrodynamic simulation we saw signs of instability, and the system became unstable in less than 1 Myr in a subsequent  $N$ -body simulation.

Another possibility is that the gravitational effects of a distant planetesimal disk could stabilize planetary configurations that were otherwise unstable. Thus, we redid each of the aforementioned  $N$ -body simulations twice: once adding a disk of  $50 M_{\bullet}$  and once adding an  $80 M_{\bullet}$  disk. In both cases, the disk was represented by a collection of 2000 massive particles that ranged in heliocentric distance from just beyond the 2:3 MMR with the outermost planet to  $\sim 30$  AU. We placed the inner edge of the disk this far from the Sun because we did not want a significant number of particles to leak out of the disk and trigger planetary migration. Such a migration would drastically change the structure of the system, making a comparison with the disk-free simulations impossible. Such a distant disk could still irreversibly damp the planets' eccentricities through a secular exchange of angular momentum and mixing of the planetesimals' secular phases. Nevertheless, in all our simulations we found that the planetary configurations became unstable in very short periods of time ( $\sim 10$  Myr).

Given the above results, we believe that four out of the six relative configurations that we found are so unstable that they could not have lasted long enough to explain the 650 Myr delay between the formation of the planets and the LHB. This, however, does not preclude the idea that other planetary systems might have passed through similar configurations, becoming unstable soon after the disappearance of the gas disk. Quite interestingly, we find that the instabilities that characterize these systems can often be much more violent than the one we can tolerate for the LHB and involve close encounters between Jupiter and Saturn. As such, they can leave Jupiter on an orbit at about 4–5 AU with an eccentricity comparable to that of some extrasolar planets (see Fig. 7; also see Rasio & Ford 1996; Marzari & Weidenschilling 2002). This creates the possibility that those extrasolar planets that have been found on eccentric orbits beyond  $\sim 3$  AU from their host stars might have followed an evolution similar to those we found in our hydrodynamic simulations, but that we rejected for our solar system, based on the LHB constraint.

Finally, we thought it would be instructive to investigate whether it was possible to add an additional  $15 M_{\bullet}$  ice giant to the system and still produce stable configurations. In order to have the best chance at successfully constructing such a system, we start with the most stable configuration produced above, namely, the one in which Uranus is in the 2:3 MMR with Saturn and Neptune is in the 3:4 MMR with Uranus. We place a fifth planet, of equal mass to that of the other ice giants, between the 3:4 and 2:3 MMRs with Neptune at  $r = 2.87$ . We then let the system evolve under the effects of the gas disk. As expected, after a short period of type I migration toward the Sun, the fifth planet is captured in

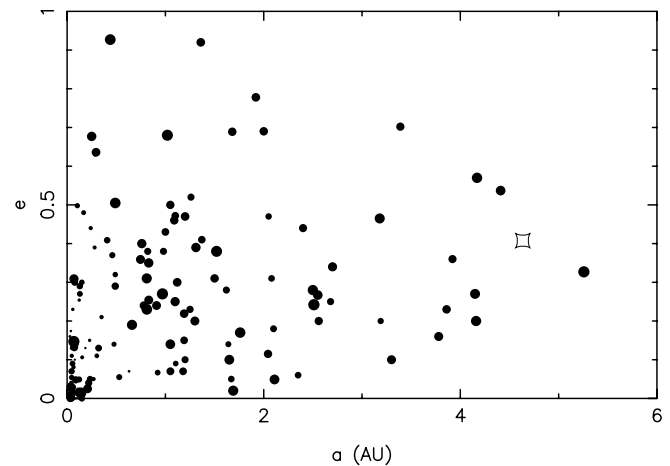


Fig. 7.— Semimajor axis vs. eccentricity distribution of the extrasolar planets discovered by radial velocity technique. The size of each circle is proportional to the planet's radius (simply estimated from the cubic root of its  $M \sin i$ ). The rhombus shows the final orbit of Jupiter, achieved in the  $N$ -body simulation starting from the configuration with Uranus and Neptune, both in the 3:4 MMR with the immediately interior planet. The excitation of Jupiter's orbit is due to a strong encounter which ejects Saturn into a very elongated orbit.

the 3:4 MMR with Neptune. The capture into resonance excites the eccentricities of Uranus and Neptune, which stabilize at about 0.08 and 0.03, respectively. These values are about twice as large as those achieved in the simulations with only four planets, reported in Figures 3 and 4. This is due to the fact that Uranus and Neptune have to restrain the fifth planet from migrating through their MMRs. Thus, they suffer an additional negative torque. Because they are themselves locked in resonances and therefore cannot migrate, this translates into a stronger excitation of their orbital eccentricities. On the other hand, the eccentricity of the fifth planet stays below 0.02.

After 2000 Jovian orbital periods, we start gradually depleting the gas disk using the procedure explained in the previous section. Unlike the previous cases, however, this system already shows signs of being unstable during this phase of its evolution. In particular, we see a secular increase in the amplitude of oscillation of both Jupiter's and Saturn's eccentricities. In addition, the variations in the eccentricities of all three ice giants become noticeably erratic. All this is probably a consequence of the enhanced eccentricities of Uranus and Neptune, relative to the four-planet simulations.

Once the gas is gone, we move the system to our  $N$ -body code. We find that the system becomes violently unstable on a time-scale of  $\sim 10$  Myr. To test this result, we have performed three additional  $N$ -body simulations starting from the output of the hydrodynamic code at slightly different times, separated by 100 orbital periods at  $r = 1$ . In all cases the planets become unstable in less than 20 Myr. We have also performed runs where we have added either a  $50$  or an  $80 M_{\bullet}$  planetesimal disk (as described above). Again, the results are essentially the same. We only manage to slightly delay the onset of the instability to  $\sim 30$  Myr.

Of course, we cannot rule out that there was once a fifth fully grown planet in the outer solar system based on these simulations alone. After all, we only studied one configuration, and the fifth planet might have had a different mass, or Uranus and Neptune might have been in different resonances than we assumed. Furthermore, as noted in the previous section, it is possible that other resonant configurations may have been reached if we changed the structure of the gas disk. Finally, there is a chance that the system was more unstable than it might have been because we

dispersed the gas disk too quickly. Nevertheless, the striking difference between the behavior of our five-planet system and the four-planet case that has Uranus and Neptune in the same MMRs suggests that it is probably much more difficult to find a five-planet configuration that is stable for a long period of time.

#### 4. FROM A FULLY RESONANT EVOLUTION TO THE CURRENT ORBITAL ARCHITECTURE

We conclude this paper by presenting a couple of proof-of-concept  $N$ -body simulations in which the four giant planets interact with a transplanetary disk of planetesimals that is close enough to the planets to cause them to migrate. The results demonstrate that the multiresonant planetary systems described above can indeed evolve into an orbital configuration similar to that of the real giant planets.

We start with the most stable of our multiresonant systems, namely, the one in which Jupiter and Saturn are in the 2:3 MMR, Saturn and Uranus are in the 2:3 MMR, and Uranus and Neptune are in the 3:4 MMR. As we have seen in Figure 6, this system is stable for at least a billion years in the absence of external perturbations. We now add a transplanetary disk of planetesimals. As in Tsiganis et al. (2005), we place the inner edge of the disk close to the outermost planet (0.5 AU beyond it), so that the planets migrate very quickly. This is a purely practical decision that allows us to save a significant amount of computing time. As in Tsiganis et al. (2005), we model the disk with 1000 equal-mass planetesimals, with a surface density profile that is inversely proportional to heliocentric distance,  $\Sigma(r) \sim 1/r$ . The outer edge of the disk is placed at 30 AU, and its total mass is set to  $50 M_{\oplus}$ . All particles are initially on nearly circular and coplanar orbits with  $e \sim \sin i \sim 10^{-3}$ .

The initial conditions of the four planets are based on the output of the hydrodynamic simulation with a decreasing gas disk. However, since our hydrodynamic simulations were performed in two dimensions, they output only a coplanar configuration. This limitation is acceptable as long as the orbits of the planets do not cross one another. However, if they do cross (as we expect in these simulations), the two-dimensional assumption artificially increases the chances of a collision to an unacceptable level. To avoid this technical problem, we add a small ( $\sim 10^{-3}$  AU yr $^{-1}$ )  $z$ -component to the velocity vector of each planet at the beginning of these  $N$ -body simulations. We follow the evolution of each system for 100 Myr using SyMBA.

The result for our first run, shown in Figure 8, has many of the characteristics seen in Figure 1 of Tsiganis et al. (2005). In particular, the planets undergo a short period of smooth migration, during which they are on circular orbits. This is followed by an abrupt increase in the eccentricities of the gas giants, which destabilizes the orbits of the ice giants and leads to a short but violent period of repeated encounters between the planets. Then there is a period when the planets migrate very quickly through the remaining disk, while their eccentricities slowly decay due to dynamical friction. The planets reach their final orbits in  $\sim 100$  Myr, when the planetesimal disk has been dispersed.

The essential ingredients of the Nice model are preserved in these new simulations. The initial orbits of the planets are stable, and thus the instability does not occur until the planets are forced to migrate across a MMR. In the original Nice model the instability was caused by Jupiter and Saturn crossing the 1:2 MMR. Here, since Saturn starts off much closer to the Sun, the first important resonance that the planets encounter is the 3:5 MMR between Jupiter and Saturn. As Figure 8 shows, this crossing causes the instability.

Using a perturbation theory similar to that in the supplementary material of Tsiganis et al. (2005), one can show that the 3:5 MMR

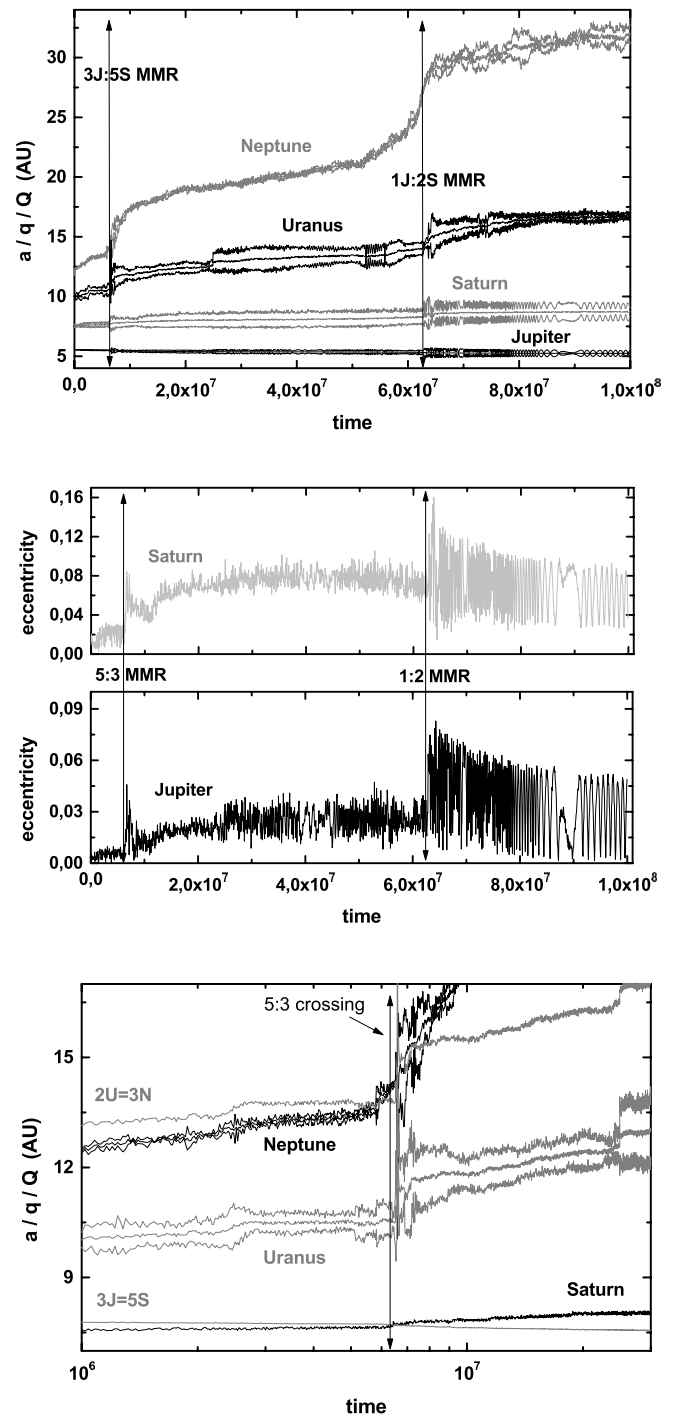


FIG. 8.—*Top*: Evolution of  $a$ ,  $q$ , and  $Q$  for the outer planets under the effects of a  $50 M_{\oplus}$  planetesimal disk. For illustrative purposes, the unit of distance used in the hydrodynamic simulations is scaled such that Jupiter initially has a semi-major axis of 5.42 AU. The unit of time is years. The resonance-crossing events are marked by vertical lines. The 3:5 MMR between Jupiter and Saturn is crossed at  $t \approx 6.5$  Myr, while the 1:2 MMR is crossed much later (at  $t \approx 63$  Myr). In this run Uranus falls  $\sim 2$  AU short of its true location. *Middle*: Eccentricity evolution for Jupiter and Saturn. The two excitation episodes are clearly seen, as is the slow damping due to dynamical friction. *Bottom*: Closer look at the instability-onset phase. The evolution of the semimajor axes of Saturn, Uranus, and Neptune are shown in the interval 1–30 Myr. Also, the evolution of the locations of the 3:5 MMR between Jupiter and Saturn and the 2:3 MMR between Uranus and Neptune are shown (time in log scale). Small variations in Uranus's  $e$  are visible, produced by the crossing of high-order resonances with Neptune. However, not even their mutual 2:3 crossing destabilizes their orbits. This occurs exactly after the crossing of the 3:5 MMR between Jupiter and Saturn, at  $t \approx 6.5$  Myr. After this event, Uranus and Neptune have repeated encounters, and planetary migration is accelerated.

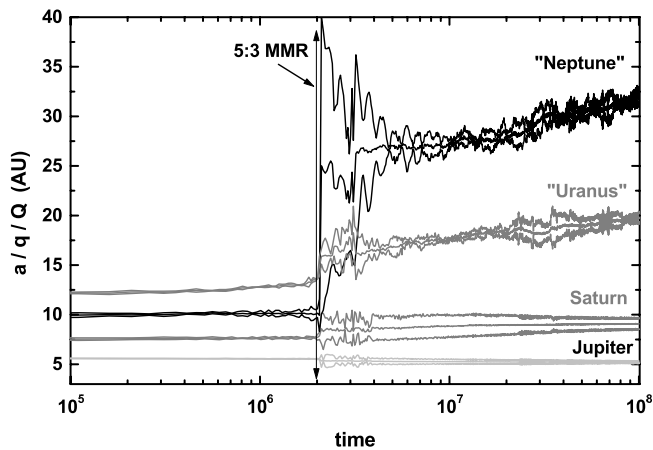


FIG. 9.— Same as Fig. 8 (*bottom*), but for the run with a disk of  $65 M_{\odot}$  (time in log scale). The crossing of the 3:5 MMR between Jupiter and Saturn occurs at  $t \approx 2$  Myr. Then the innermost ice giant suffers close encounters with both Saturn and Jupiter, which receive a “kick” in  $a$  that forces them to cross their mutual 1:2 MMR. The eccentricity of the ice giant grows to  $\sim 0.6$ . Repeated encounters between the two ice giants follow, resulting in an exchange of the heliocentric ordering of their orbits. The planets are stabilized at 5.2, 9.2, 20, and 32 AU. The final eccentricities of Jupiter and Saturn are 0.03 and 0.07, respectively.

is less effective than the 1:2 MMR at increasing the eccentricities of the gas giants. However, since the planetary configuration we used here is more compact than that in the original Nice model, a mild eccentricity “jump” is enough to destabilize the orbits of the ice giants. Thus, as we suggested in § 2, we can conclude that in order to reproduce a Nice-model-like instability in the orbits of the ice giants of a system that initially has Jupiter and Saturn locked in the 2:3 MMR, Uranus and Neptune probably need to be closer to the Sun than the original Nice model postulated.

In the run shown in Figure 8, Uranus and Neptune suffer a few encounters with each other before the latter is scattered into the disk (at about 15 AU). Dynamical friction from the disk decouples Neptune from Uranus, after which it migrates smoothly outward on a nearly circular orbit. Neptune stops migrating when it hits the outer edge of the disk. Note that Uranus does not migrate far enough, since its final semimajor axis is  $\sim 17$  AU instead of 19.2 AU. This behavior is reminiscent of the subset of simulations from Tsiganis et al. (2005), where the ice giants do not encounter Saturn. Indeed, the location of Uranus was one reason why Tsiganis et al. concluded that such encounters must have happened.

However, at the instability time, a number of different behaviors are possible due to the chaotic nature of the dynamics. For example, in a second simulation that is similar to the one in Figure 8 but with a  $65 M_{\odot}$  disk (Fig. 9), Saturn is involved in gravitational encounters with the ice giants. As a result, Neptune is thrown much farther into the disk, landing at  $a \sim 25$  AU on a very eccentric orbit. Its orbit is subsequently circularized by dynamical friction, and it comes to rest in a nearly circular orbit near 30 AU. In addition, Uranus’s final semimajor axis is very close to its observed value. This result is also consistent with the findings of Tsiganis et al. (2005). Note that, in this run, the ice giant that formed closer to the Sun became the most distant planet in the final system.

As described above, in the original Nice model the orbital instability was caused by Jupiter and Saturn crossing the 1:2 MMR, while in these simulations it is caused by the 3:5 MMR. The 3:5 MMR is closer to the Sun than the 1:2 MMR, and, since Saturn is currently found beyond Jupiter’s 1:2 MMR, it eventually had to cross it. This resonance crossing once more excites the gas giants’ eccentricities and thus helps them maintain nonzero values

against dynamical friction. We note that by the time Jupiter and Saturn cross their 1:2 MMR, the mass of the remaining disk in our runs is roughly  $25 M_{\odot}$ , a value that is well within the range needed to explain the capture of Jupiter Trojans during the 1:2 MMR crossing, according to the model in Morbidelli et al. (2005).

Of course, much more work is needed to build a successful “Nice model II” that starts from an initial multiresonant configuration of the giant planets. It is crucially important, for example, to determine whether it is possible to delay the instability for 650 Myr in order to be consistent with the LHB chronology. Recall that in the above simulations, we purposely set the disk’s initial distribution so that the resonance crossing occurs early in order to save CPU time. However, as discussed in Gomes et al. (2005), a more realistic distribution of the planetesimal disk should contain only particles whose dynamic lifetime is of the order of the lifetime of the gas disk (a few Myr) or longer. Assuming this disk distribution, Gomes et al. showed that, at least for the initial planetary configuration assumed in the original Nice model, the migration of the planets is slow enough that the instability is achieved only after hundreds of millions of years, consistent with the LHB timing. The same would hopefully happen for our new initial planetary configuration. Moreover, a large number of simulations need to be performed in order to quantify the probability that the final orbits achieved by the planets from our new initial configuration are consistent with observations. This study, which will require many time-consuming simulations, is currently ongoing and will be the subject of a forthcoming paper.

## 5. CONCLUDING REMARKS

There are two important characteristics of our solar system that any model must be able to explain. First, the hot and warm Jupiters that are seen around some other stars are not present in our system. Second, the Moon and the planets most likely carry the scars of a spike in the impact flux that occurred  $\sim 650$  Myr (the late heavy bombardment [LHB]) after the planets formed. The LHB strongly suggests that the planets suddenly became unstable at that time, destabilizing a massive reservoir of small bodies (Levison et al. 2001).

The so-called Nice model (Tsiganis et al. 2005; Gomes et al. 2005) has been proposed, in part, to explain the LHB. This model has a long list of successes in reproducing many observational characteristics of the solar system. These include the number and orbital distribution of the Jovian Trojans (Morbidelli et al. 2005), the irregular satellites of Saturn, Uranus, and Neptune (Nesvorný et al. 2007), and the structure of the Kuiper Belt (Levison et al. 2007). The main weakness of the Nice model is that the initial conditions of the planets were chosen without concern for the previous phase of planetary evolution when the protoplanetary gas disk was still in existence.

Recent hydrodynamic simulations of Jupiter and Saturn embedded in a gas disk have supplied an important clue about the initial stable configuration of the planets. In particular, MS01 and MC07 showed that the absence of a hot/warm Jupiter in our system could be explained if these two planets had been locked in their mutual 2:3 MMR. The main goal of this paper, therefore, is to extend these results and find a four-planet configuration that is both nonmigrating while the gas disk is present and dynamically stable long after the gas disk disperses.

To accomplish this goal, we have taken a system from MC07 consisting of Jupiter, Saturn, and a gaseous disk and progressively added Uranus and Neptune, one at a time, into the simulations. We find that the interaction with the gas disk drives these planets into a configuration where each is in a MMR with its immediate neighbor(s). We have found six such fully resonant



configurations, all of which are characterized by, at most, a small amount of radial migration. Four configurations, however, rapidly become unstable (on a timescale of a few Myr) after the disappearance of the gas disk. The remaining two are the least compact systems, with Saturn and Uranus in the 2:3 MMR and Uranus and Neptune in either the 4:5 or 3:4 MMRs. They were stable for 400 Myr and over a Gyr, respectively.

Furthermore, we have presented a pair of proof-of-concept simulations showing that a quadruple resonant configuration, like the stable one above, can evolve into a system with a structure similar to that observed in the real solar system, once it interacts with a suitable transplanetary planetesimal disk. The system evolves as follows. The migration of the giant planets, induced by the interaction with this disk, increases the ratios of the orbital periods between each pair of planets. Thus, the planets are extracted from their mutual quadruple resonance. Because the system is very compact, new resonances are crossed during the migration. These resonances excite the eccentricities of the planets, triggering a global instability of the system. The orbits of the planets are eventually stabilized by the dynamical friction exerted by the planetesimal disk during its dispersal. Thus, this evolution is different from the original Nice model (Tsiganis et al. 2005) in the technical details only (e.g., different resonances are involved) but not in

terms of the basic dynamic processes at work. More work is needed in order to quantify the statistical outcome of the chaotic evolution of the planets and, in particular, to prove that the onset of the planetary instability can occur late, as in Gomes et al. (2005). This will be the object of a forthcoming paper.

The long-term aim of this research is to build a bridge between our knowledge of solar system dynamics during the gas-disk era and during the planetesimal-disk era, which remained up to now totally disconnected from each other. Success in this task would represent a significant advancement of our understanding of planet formation and of the key processes that made our solar system so different from all extrasolar systems discovered so far. In this paper we have taken the first steps toward this goal.

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