A&A manuscript no.
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06(06.05.1, 06.06.4, 06.09.1)

ASTRONOMY
AND
ASTROPHYSICS
October 21, 1999

Research Note

About the time of evolution of a Solar Model

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Received date / Accepted date

Abstract. The evolution of a solar model is initialized with homogeneous models of either, pre-main sequence (P-models) or, zero-age main sequence (Z-models). The zero-age of a solar model is conventionally referenced as the time where the nuclear reactions just begin to dominate gravitation as the primary source of energy. Fixing the physics, we found that the structure of P- and Z-models computed with the same physics are almost similar soon after the exhaustion of their convective core. This similarity gives a connection between the age of the Sun $t_\odot$ and the time $t_{\rm cal}$ elapsed in the calculation of calibrated solar models. We found that a Z-model calibrated with $t_{\rm cal} = t_\odot$ and a P-model calibrated with $t_{\rm cal} = t_\odot + 25\,$My, are indistinguishable at the relative accuracy level of a few 10^{-4}.

Key words: Sun: evolution – Sun: fundamental parameters – Sun: interior

1. Introduction

The evolution sequences of a solar model are initialized, either with a chemically homogeneous pre-main sequence model (P-models) powered only by gravothermal energy, or with a fictitious homogeneous zero-age main sequence model (Z-models) powered only by thermonuclear energy. Fixing the physics, whatever they start from, calibrated solar P- or Z-models have, by definition, the same radius, luminosity and surface mass fraction of heavy elements. If one is not concerned with specific pre-main sequence physical processes, e.g. primeval mass and angular momentum losses or lithium depletion, the amount of calculations is half as large for Z-models (see e.g. Morel et al. 1997). The initial internal structure of initial P- and Z-models differs, despite the same surface parameters at solar age. It is not obvious that the internal structure of the resulting calibrated models is alike and, if so, is it just for the resulting models or also for parts of their previous evolution? It is important also to establish a relationship between the age of the Sun and the sum of the time steps elapsed in the calculation. As an example, to take the time of the pre-main sequence evolution into account, Weiss & Schlattl (1998) have evolved their calibrated solar P-models 40 My beyond the age they have assigned to the Sun.

A preliminary discussion is given in Morel et al. (1998). The paper is organized as follows: in Sec. 2 we recall how the age of a solar model is defined with respect to the age of the Sun. Section 3 is devoted to our analysis. Results are given in Sec. 4 and we conclude in Sec. 5.

2. The solar age

In a sequence of evolving P-models, the zero-age main sequence (ZAMS) model is usually defined as the first model where $\epsilon_{\rm nuc}$, the nuclear energy generation *just begin* to dominate $\epsilon_{\rm g}$, the gravothermal energy liberation (Guenther & Demarque 1997). That definition influences the various age definitions. Here, with this definition, for a sequence of solar P-models the age of the *calibrated* P-model will be defined as the evolution time i.e., the sum of the time steps, from the ZAMS model to the present day model. As we shall see that definition influences slightly the various age definitions.

Conventionally, the solar age $t_\odot$ is the time it has taken the Sun to evolve from ZAMS to present day (Guenther & Demarque 1997). According to a reasonable hypothesis, $t_\odot$ needs to be consistent with the age of the oldest meteorites $4566 \pm 5\,$My (Bahcall et al. 1995). Guenther & Demarque (1997) have argued that the radioactive clocks of these meteorites are zeroing during the last high-temperature event in the primordial solar system nebula. This has occurred, according to Guenther (1989), $40 \pm 10\,$My before the ZAMS, therefore Guenther & Demarque (*loc. cit.*) estimate the "meteoritic solar age" to be $t_{\odot\rm m} = 4530 \pm 40\,$My. The determination of $t_\odot$ has been recently revisited using new methods of investigation including statistical arguments based on helioseismological data. In their pioneering work Weiss & Schlattl (1998) investigated how the known deficits of the solar models would translate into an age, if the best-fitting model's age is assumed to be the solar age. They found a "helioseismic solar age" in the interval

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$4650\,\mathrm{My} \lesssim t_{\odot\mathrm{h}} \lesssim 5650\,\mathrm{My}$. Recently, another analysis of Dziembowski et al. (1999), based only on central helioseismic data, leads to $t_{\odot\mathrm{h}} = 4660\pm110\,\mathrm{My}$, a value marginally consistent with $t_{\odot\mathrm{m}}$.

3. Solar P- and Z-models

Hereafter we shall note t_{ev} the evolution time of any model. For a calibrated solar model we shall note $t_{\mathrm{cal}} = t_{\mathrm{ev}}$. We shall extend these notations to the Sun itself by setting $t_{\mathrm{ev}} = 0$ at the time of the last high-temperature event in the primordial solar system nebula, thus $t_{\mathrm{cal}} = t_{\odot\mathrm{m}} + 40\,\mathrm{My}$ (Guenter *loc. cit.*).

For a sequence of P-models we conventionally set $t_{\mathrm{ev}} = 0$ at the ignition of deuterium; that occurs as soon as the temperature reaches $T \approx 0.5\,\mathrm{MK}$. In such physical conditions the model is fully convective and then homogeneous. At $t_{\mathrm{ev}} \approx 0.3\,\mathrm{My}$, all the initial ^{2}H is already converted into ^{3}He via the reaction $^2\mathrm{H}(p,\gamma)^3\mathrm{He}$ and $0 < \epsilon_{\mathrm{nuc}} << \epsilon_{\mathrm{g}}$. Therefore, long before the ZAMS, the abundance of ^{3}He is enhanced by a factor of ≈ 4 (in mass) with respect to its primeval value, namely from $\approx 2.3\,10^{-5}$ to $\approx 8.6\,10^{-5}$. At $t_{\mathrm{ev}} \approx 1.2\,\mathrm{My}$, the central temperature approaches $T_{\mathrm{cal}} \sim 3\,\mathrm{MK}$, and the PP burning of ^{1}H begins in the core. Soon after, due to the sudden energy liberation, the temperature increases, the opacity decreases and the core becomes radiative. At $t_{\mathrm{ev}} \approx 13\,\mathrm{My}$ the innermost limit of the solar convection zone has receded to about its present day location $R_{\mathrm{ZC}} \sim 0.7R_\odot$; the central temperature has jumped to $T_{\mathrm{cal}} \approx 13\,\mathrm{MK}$. The zero-age main-sequence occurs at $t_{\mathrm{ev}} = t_{\mathrm{ZAMS}} \approx 25\,\mathrm{My}$ just when $\epsilon_{\mathrm{nuc}} \gtrsim \epsilon_{\mathrm{g}}$. At $t_{\mathrm{ev}} \approx 37\,\mathrm{My}$ an extra energy generation due to the CNO burning of ^{12}C into ^{14}N creates a convective core. It lasts until $t_{\mathrm{ev}} \approx 90\,\mathrm{My}$ when the CNO reactions reach their equilibrium state. Beyond $t_{\mathrm{ev}} \gtrsim 50\,\mathrm{My}$ more than 99% of the energy generated have a nuclear origin. After the exhaustion of the convective core, the model evolves quietly without fundamental modifications of its structure until present day.

Therefore, for a calibrated solar P-model $\tau_{\mathrm{p}} = t_{\mathrm{cal}}$ writes:

$$\tau_{\mathrm{p}} = t_{\mathrm{ZAMS}} + t_\odot. \tag{1}$$

For a sequence of Z-models we conventionally assign $t_{\mathrm{ev}} = 0\,\mathrm{My}$ to the first model. In such a model the energy only comes from nuclear reactions i.e., $\epsilon_{\mathrm{g}} = 0$. It is an idealized model, because such a homogeneous state cannot be issued from any previous PMS evolution. As the time goes on, the nuclear burning and the gravity relax the initial inconsistencies of chemical composition, temperature, density and pressure. At $t_{\mathrm{ev}} = 0$ a solar Z-model presents a convective core which lasts until $t_{\mathrm{ev}} \approx 65\,\mathrm{My}$, when the CNO bi-cycle begins to work at equilibrium. Then, as for the P-models, after the exhaustion of the convective core, the model changes quietly until present day.

The definition of the age given in Sec. 2 does not apply to Z-models, since the gravothermal energy generation *never* overcomes the nuclear one. Thus, the time of the ZAMS is undefined for a sequence of Z-models. To connect $t_\odot$ and $\tau_{\mathrm{z}} = t_{\mathrm{ev}}$, *we infer* that a P- and a Z-model computed with the same physics are almost identical as soon as the nuclear reactions start to work at equilibrium. That occurs $\approx 10\,\mathrm{My}$ after the exhaustion of the convective cores. Beyond this epoch the models will remain alike. As illustrated in Fig. 1, calibrated P- and Z-models will have the same age if the same amount of time is elapsed between present day and the epochs τ_{cp} and τ_{cz} of convective core exhaustion: $\tau_{\mathrm{p}} - \tau_{\mathrm{cp}} = \tau_{\mathrm{z}} - \tau_{\mathrm{cz}}$. Owing to Eq. (1), τ_{z} writes:

$$\tau_{\mathrm{z}} = \tau_{\mathrm{p}} - \tau_{\mathrm{cp}} + \tau_{\mathrm{cz}} = t_{\mathrm{ZAMS}} + t_\odot - \tau_{\mathrm{cp}} + \tau_{\mathrm{cz}}. \tag{2}$$

4. Results

Basically, the physics used in the calculations is the same as in Morel et al. (1997), it uses OPAL opacities and equation of state. The microscopic diffusion of chemicals is allowed for. The models are calibrated, within a relative accuracy better than 10^{-4}, by adjusting: the ratio l/H_{p} of the mixing-length to the pressure scale height, the initial mass fraction Y_{ini} of helium and the initial mass fraction $(Z/X)_{\mathrm{ini}}$ of heavy elements to hydrogen, so that, at present day, the models have the luminosity $L_\odot = 3.846\,10^{33}\,\mathrm{erg\,s^{-1}}$, the radius $R_\odot = 6.9599\,10^{10}\,\mathrm{cm}$, and the mass fraction of heavy element to hydrogen $(Z/X)_\odot = 0.0245$. We have taken into account the most important nuclear reactions of PP+CNO cycles with the species ^{2}H, ^{7}Li, ^{7}Be at equilibrium. As ^{2}H is set at equilibrium, the protosolar ^{2}H is included into the initial ^{3}He; one has for the initial isotopic ratio $^3\mathrm{He}/^4\mathrm{He} = 4.19\,10^{-4}$ (in number) (Gautier & Morel 1997). The recently updated rates of Adelberger et al. (1998) are used. The models have been computed using the CESAM code (Morel 1997). The evolution of a Z-model necessitates about 40 time steps whereas 90 for a P-model. Around epochs of convective core exhaustion and ZAMS, the time step is refined in order to define these instants with an accuracy better than $\pm2\,\mathrm{My}$.

We have adjusted τ_{p} for the calibrated P-model Sp so that its age was $t_\odot = 4530\,\mathrm{My}$. The ZAMS occurred at $t_{\mathrm{ZAMS}} = 25 \pm 2\,\mathrm{My}$ and the convective core was exhausted at $\tau_{\mathrm{cp}} = 90 \pm 2\,\mathrm{My}$. With the calibration parameters *obtained* for Sp, we have computed the evolution of the Z-model Sz. We have found that the exhaustion of its convective core occurred at $\tau_{\mathrm{cz}} = 65 \pm 2\,\mathrm{My}$. According to our analysis, after $\tau_{\mathrm{z}} = t_\odot \pm 6\,\mathrm{My}$, given by Eq. (2), the Z-model Sz is expected to be *calibrated* and to have a structure close to the P-model Sp. That is indeed the case, as seen in Table 1 and Fig. 2, as far as the global parameters and sound speed profiles are concerned.

Figure 3 shows diagrams of effective temperature and luminosity versus time. As expected, an abscissa shift of

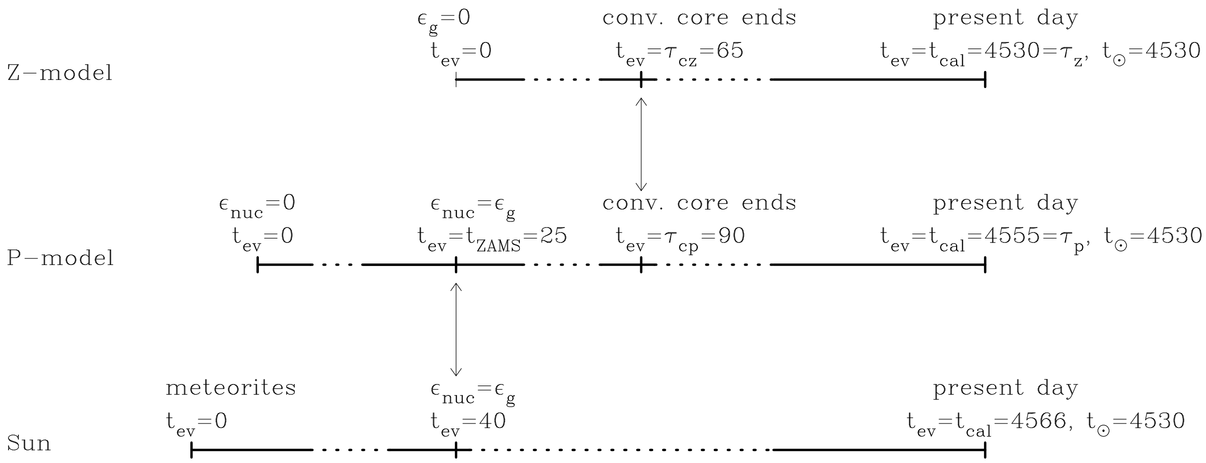

Fig. 1. Schematic illustrations of chronologies with $t_\odot = t_{\odot\mathrm{m}}$ for the Sun, P- and Z-models; the times are in My.

Table 1. Comparison of global characteristics of solar P-models (labels p and P) and Z-models (labels z and Z). The subscripts "$_s$" and "$_c$" respectively designate a surface and a center value. The data are for the time $t_{\rm ev} = 0$ (first group), for the time 10 My after the epoch $t_{\rm cv}$ of convective core exhaustion (second group) and for $t_{\rm cal}$ i.e. present day (last group). The units are My for time, 10^7K for the temperatures and g cm^{-3} for the densities.

	Sp	Sz	SP	SZ
Y_s	0.2731	0.2731	0.2720	0.2720
$(Z/X)_s$	0.0278	0.0278	0.02775	0.02775
$t_{\rm cv}$	90 ± 2	65 ± 2	90 ± 2	64 ± 2
Y_s	0.2724	0.2725		
Z_s	0.0196	0.0196		
$R_{\rm ZC}$	0.7220	0.7221		
T_c	1.347	1.349		
ρ_c	81.35	81.31		
Y_c	0.2781	0.2778		
Z_c	0.0202	0.0202		
$t_{\rm ev}$	4555	4530	4685	4660
$t_\odot$	4530	4530	4660	4660
$1 - R_s/R_\odot$	$-5\,10^{-5}$	$6\,10^{-5}$	$6\,10^{-5}$	$-7\,10^{-7}$
$1 - L_s/L_\odot$	$-2\,10^{-5}$	$4\,10^{-5}$	$5\,10^{-5}$	$-9\,10^{-5}$
Y_s	0.2446	0.2446	0.2433	0.2433
$(Z/X)_s$	0.0245	0.0245	0.0245	0.0245
$R_{\rm ZC}/R_\odot$	0.7137	0.7137	0.7121	0.7122
T_c	1.569	1.569	1.572	1.572
ρ_c	152.2	152.2	154.4	154.3
Y_c	0.6396	0.6395	0.6467	0.6465
Z_c	0.0210	0.0210	0.0210	0.0210

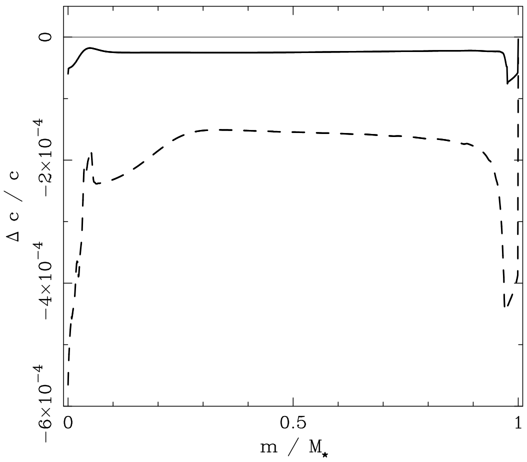

Fig. 2. Relative differences in sound speed between the P and Z-models Sp and Sz; dashed: 10 My after the convective core exhaustion, full: present day.

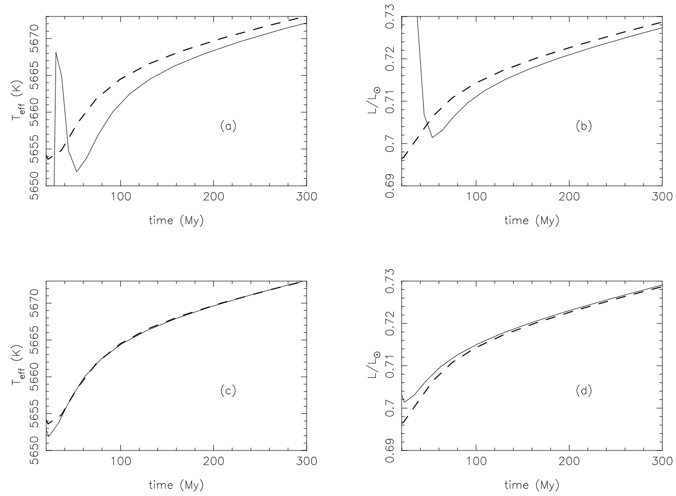

Fig. 3. (a) Effective temperature and (b) luminosity with respect to time, for the solar P- and Z-models, Sp (full) and Sz (dashed); (c) and (d) the same with a +25 My abscissa shift for Sz.

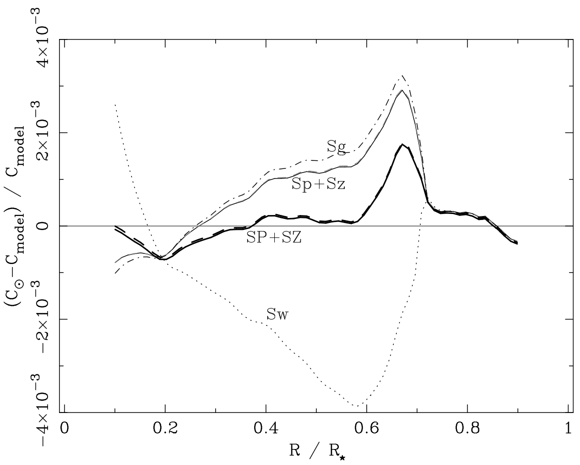

Fig. 4. Relative difference in sound speed between the Sun and models, Sp (thin, full), Sz (thin, dashed), SP (heavy, full), SZ (heavy, dashed), Sg (dash-dot) and Sw (dotted).

+25 My for the Z-model Sz allows to superimpose the loci of Sz and Sp beyond $t_{\rm ev} \approx 100$ My. We also found that the value of $\tau_{\rm cp} - \tau_{\rm cz} = 25 \pm 4$ My is almost insensitive to the required solar age, as seen in Table 1 from the comparison of the P-model Sp (*resp.* Z-model Sz) of age 4530 My with the P-model SP (*resp.* Z-model SZ) of age 4660 My. At solar age $t_\odot = 4530$ My (*resp.* $t_\odot = 4660$ My) the differences between the models Sp and Sz (*resp.* SP and SZ) are below 10^{-4} i.e. the level of accuracy of the calibration. All these results validate our analysis.

As a matter of comparisons, we have calibrated the solar Z-models Sg and Sw for ages $t_\odot = 4500$ My (Guenther & Demarque 1997) and $t_\odot = 5100$ My (Weiss & Schlattl 1998), respectively. Figure 4 shows the relative difference between the sound speed of models and the seismic sound speed experimental results of Turck-Chièze et al. (1997). The best fit appears to be for the models SP (and SZ), of age $t_\odot = t_{\odot\rm h} = 4660$ My, i.e. the helioseismic solar age of Dziembowski et al. (1999). This means that the physics used in solar models leads to deficits which are equivalent to an evolution slower than indicated by $t_{\odot\rm m}$.

5. Discussion and conclusions

Fixing the physics we have compared calibrated solar models initialized with a homogeneous zero-age main sequence model (Z-model) and with a pre-main sequence model (P-model). We have verified that the models which evolved from these different initial models merge into the same structure about 10 My after the end of the convective core phase, if the times, $t_{\rm cal}$, of evolution used in each calculation, are shifted by an amount of 25 My. That similarity allows us to connect $t_{\rm cal}$ to the solar age, i.e. the time it has taken the Sun to evolve from the time (ZAMS), where nuclear reactions *just begin* to dominate gravitation as the primary energy source, to present day. For the calibrated Z-model we found $t_{\rm cal} = t_\odot$, while for a P-model $t_{\rm cal} = t_\odot + 25$ My. We emphasized the fact that these two relations between $t_{\rm cal}$ and $t_\odot$ are only valuable when the zero-age reference corresponds to the time when the nuclear reactions *just begin* to be the primary energy source. If the epoch of the ZAMS is defined at the instant where 99%, instead of 50%, of energy comes from nuclear reactions, the shift becomes 50 My, then, for a P-model $t_{\rm cal} = t_\odot + 50$ My and $t_{\rm cal} = t_\odot + 25$ My for a Z-model. Though 25 My represent only 0.5% of the solar age, it is of the same order of accuracy as present day solar models. The basic idea of our analysis is based on the similarity between the models soon after the convective core exhaustion; it is useless for stars of masses greater than about $1.2\,M_\odot$, which exhibit a convective core on the main sequence. Another analysis is needed for these stars, that may be of importance in asteroseismology to differentiate between age and $t_{\rm cal}$ for modeling the COROT targets (Baglin 1998).

Acknowledgements. We thank the referee Dr. A. Weiss for valuable comments. This work has been performed using the computing facilities provided by the OCA program "Simulations Interactives et Visualisation en Astronomie et Mécanique (SIVAM)".

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