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The acoustic cut-off frequency of roAp stars*

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Abstract. Some of the rapidly oscillating (roAp) stars, have frequencies which are larger than the acoustic cut-off frequency determined from stellar models with atmospheres based on an Eddington or Hopf law.

As the cut-off frequency depends on the $T(\tau)$ relation in the atmosphere, we have computed models and adiabatic frequencies for pulsating Ap stars with $T(\tau)$ laws based on Kurucz model atmospheres and on the Hopf's purely radiative relation.

We compare the values of the cut-off frequency derived from expressions for the potential from Vorontsov & Zarkhov (1989), from Gough (1986), and from the approximation of an isothermal atmosphere. These models predict a different reflexion efficiency for waves and hence marginally different values for the cut-off frequency. The frequency-dependent treatment of radiative transfer as well as an improved calculation of the radiative pressure in Kurucz model atmospheres increase the theoretical acoustic cut-off frequency for roAp stars by about $200\,\mu$Hz, which is closer to the observations.

Since evolutionary effects significantly influence the acoustic cut-off frequency we restrict the comparison of our computations with observations to those two 'pathological' roAp stars for which more reliable astrophysical parameters are available, HD 24712 and α Cir, and comment briefly on a third one, HD 134214. For α Cir we find models with Kurucz atmospheres which have indeed a cut-off frequency beyond the largest observed frequency and which are well within the $T_{\rm eff} - L/L_\odot$ error box. For HD 24712 only models which are hotter by about 100 K and less luminous by nearly 10% than what is actually the most probable value would have an acoustic cut-off frequency large enough. HD 134214 fits our models best, however, the error box for $T_{\rm eff} - L/L_\odot$ is the largest of all three stars.

One may thus speculate that the old controversy about a mismatch between observed largest frequencies and theoretical cut-off frequencies of roAp star models is resolved. However, the observational errors for the astrophysical fundamental parameters have to be reduced further and the model atmospheres refined by investigating NLTE effects, among others, before a definite conclusion can be drawn.

Key words: Stars: atmospheres - chemically peculiar - oscillations - individual: HD 24712, HD 128898, HD 134214 - variables: roAp

1. Introduction

For 5 out of 28 known rapidly oscillating magnetic chemically peculiar (CP2, Preston 1974) stars, the so-called roAp stars, the largest observed frequency exceeds the theoretical acoustic cut-off frequency, which is determined by the outermost stellar regions.

Waves with frequencies larger than the cut-off frequency are not well reflected towards the resonant cavity and decay in the atmosphere with a decreasing amplitude. It has been argued by Shibahashi and Saio (1985) that the cut-off frequency is largely influenced by the $T(\tau)$ relation which requires a careful modelling of these layers. Extensive studies of the external layers and of the cut-off frequency have been carried out for the Sun (e.g., Gough 1986, Balmforth & Gough 1990). Investigations of other types of stars have been neglected until the discovery of roAp stars.

Frequently, atmospheres in stellar models are based on an Eddington or Hopf law (e.g. Mihalas 1978, called here-

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after standard models). In such a case the $T(\tau)$ relation is given by

$$T^4 = (3/4) \cdot T_{\rm eff}^4 (\tau + q(\tau)),$$

where T is the temperature, $T_{\rm eff}$ the effective temperature, τ the Rosseland optical depth and $q(\tau) \equiv 2/3$ for the Eddington law. These $T(\tau)$ relations suffer from two major approximations: convection is not included and radiative transfer is considered to be frequency independent (i.e. the grey case is assumed). CP2 stars with masses between 1.6 to 2.2 $M_\odot$ have two very thin convection zones below the surface. The outer zone extends into the atmosphere and it is therefore necessary to include convection in the model atmospheres.

The ATLAS9-code developed by Kurucz (1991) takes into account the effect of frequency-dependent opacities from both continuum sources and from about 58 million lines, hence including blanketing effects. It also explicitly accounts for convection and radiative pressure.

In this paper we describe our computations of stellar models where atmospheres have been implemented which were derived from the Hopf law as well as from the more consistent Kurucz model atmospheres.

2. Stellar models

In stellar structure calculations one separates the inner part, where the diffusion approximation for radiative transfer is valid, from an outer part, where the atmosphere is interpolated using a $T(\tau, T_{\rm eff}, g)$ law, with the gravity g. When convection is included in a model atmosphere, the transition between the inner part of a stellar model and the atmosphere must be located at an optical depth $\tau = \tau_{\rm b} \gtrsim 10$, where the diffusion approximation is still valid (Morel et al. 1994).

Effects from a magnetic field are neglected in our stellar models. To discuss the influence of convection and opacities on a depth dependence of the critical frequency we compare our results obtained with advanced model atmospheres (based on ATLAS9) with a much simpler grey atmosphere (Hopf) which is a very limited description of a stellar atmosphere, but which is frequently assumed for stellar models.

2.1. Atmosphere

In order to describe the atmosphere we used the LTE Kurucz ATLAS9 code without the "overshooting option" (Castelli 1996) to calculate an interpolation table for $T(\tau, T_{\rm eff}, g)$. Model atmospheres with solar composition were computed for $\log g = 4.2$ and for $T_{\rm eff}$ ranging from 7400 to 10000 K, with steps of 100 K, and no additional contribution to line opacity by microturbulence has been assumed. The gravity varies in our 1.8 $M_\odot$ model (Tab. 1) from $\log g = 4.313$ at the ZAMS to $\log g = 4.196$ at an age of 500 Myr. However, the approximation of a constant $\log g$ *only for the atmosphere!* in evolutionary calculations is justified as the $T(\tau, T_{\rm eff}, \log g)$ law is rather insensitive to small variations of $\log g$. Gravity effects on the cut-off frequency along evolutionary tracks have been studied by, e.g., Shibahashi (1991) and shall be discussed in subsection 3.4.

Table 1. Fundamental astrophysical parameters for models of 1.8 $M_\odot$ at different evolutionary stages, with an atmosphere derived from Kurucz ATLAS9 model atmospheres.

age (10^6 yrs)	$\log(T_{\rm eff})$	$\log(L/L_\odot)$	$R/R_\odot$	$\log g$
40	3.9209	1.0162	1.550	4.312
100	3.9198	1.0264	1.574	4.299
225	3.9171	1.0426	1.624	4.272
325	3.9140	1.0561	1.673	4.246
400	3.9113	1.0645	1.711	4.227
500	3.9067	1.0772	1.773	4.196

Generally it is adequate to compute Kurucz models with 72 layers, but the numerical accuracy is insufficient when derivatives are important, such as the temperature gradient. This problem was also mentioned by D. Katz and C. van't Veer (priv. comm.) and we have therefore computed all our atmosphere models with 288 layers.

For stellar models with a simpler atmosphere based on the Hopf law where convection is not treated it is sufficient to restrict the computations for the atmosphere down to $\tau_{\rm b} = 2$. The values of $q(\tau)$ at each optical depth are interpolated from the values given in Table (3.2) of Mihalas (1978).

The outer boundary of solar model atmospheres is usually set to the bottom of the chromosphere, at $\tau_{\rm ext} \sim 10^{-4}$, where the minimum of temperature occurs for the Sun. Since it is not known whether CP2 stars have a corona or not, we have tentatively defined the outer boundary of our atmospheres at $\tau_{\rm ext} = 10^{-6}$ and will justify this choice later. The model atmospheres, however, were always computed up to $\tau = 10^{-6.875}$, as in Kurucz (1993) model grids. The density at the stellar model boundary was fixed to $\rho_{\rm ext} = 1.895 \cdot 10^{-11} {\rm g} \cdot {\rm cm}^{-3}$, which is the value for the Kurucz model atmosphere with $T_{\rm eff} = 8080$ K at the given optical depth.

2.2. Internal structure

We have computed representative models for CP2 stars of 1.8 $M_\odot$ with the CESAM code (Morel 1993 and 1997). These models have about 1600 mesh points and include a ZAMS sequence. The EFF equation of state (Eggleton et al. 1973) was used, an initial hydrogen content $X = 0.7$, and a heavy-element abundance $Z = 0.02$. The most recent OPAL95 opacities (Iglesias & Rogers 1996) with a bi-rational spline interpolation (Houdek & Rogl 1996) were incorporated. Convection is described by the standard mixing-length theory with a mixing length $\lambda = \alpha H_{\rm p}$ (where $\alpha \equiv 1.4$ is the mixing-length parameter and $H_{\rm p}$ is the pressure scale height). Because the two external convection zones are very thin, the general properties of the models are insensitive to the value of α (Gough & Novotny 1993, Audard & Provost 1994).

We shall call a "Hopf model" a full stellar model constructed with an atmosphere derived from Hopf's law, and a "Kurucz model" a model with a Kurucz model atmosphere implemented.

The evolution of a $1.8\,M_\odot$ Kurucz model is summarized in Tab. 1, and the main characteristics of Hopf and Kurucz models with an age of $500 \cdot 10^6$ years are given in Tab. 2. As expected, they have similar effective temperatures of about 8066 K, and their radii and therefore luminosities are also very similar. The position of the convection zones, which move very little during the evolution, are also given.

Table 2. Characteristics of our $1.8\,{\rm M}_\odot$ models with an age of $500 \cdot 10^6$ yrs. The Hopf model has an atmosphere derived from the Hopf law, while the atmosphere of the Kurucz model is derived from Kurucz's model atmospheres. The first lines give the effective temperature, luminosity and radius in solar units, and the characteristic frequency $\Omega_g = (GM/R_\star^3)^{1/2}$. The second lines indicate the positions of the two outer convection zones in units of the stellar radius ${\rm R}_\star$. The deeper one is located between the radii r_{c1} and r_{c2}, the outermost convection zone lies between r_{c3} and r_{c4}. The positions of these limits are expressed in logarithm of the pressure in the third lines.

model	$T_{\rm eff}$ (K) $r_{c1}/{\rm R}_\star$ $\log P_1$	$L/L_\odot$ $r_{c2}/{\rm R}_\star$ $\log P_2$	$R/R_\odot$ $r_{c3}/{\rm R}_\star$ $\log P_3$	$\Omega_g(10^{-6}{\rm rad.s}^{-1})$ $r_{c4}/{\rm R}_\star$ $\log P_4$
Hopf	8067	11.94	1.772	356.9
	0.9915	0.9946	0.9988	1.000
	5.96	5.53	4.595	3.97
Kurucz	8066	11.95	1.773	356.6
	0.9914	0.9945	0.9988	0.9999
	5.96	5.53	4.57	3.93

3. Critical and cut-off frequencies

In this section we comment on the definitions of the critical ($\nu_{\rm c}$) and cut-off ($\nu_{\rm M}$) frequencies, report on our calculations of these quantities and compare them with observations.

3.1. Theoretical background

Our oscillation code integrates the 4^{th} order system of equations governing the stellar nonradial adiabatic oscillations and takes into account the gravitational potential perturbations (Unno et al. 1989). These equations are obtained by perturbing the equations of momentum and continuity. They do not include the effects of a magnetic field.

Because the light received from stars other than the Sun is integrated over the visible hemisphere, essentially only modes of low degree ℓ can be observed. We have therefore restricted our calculations to $\ell = 0$ to 3, and we consider high radial orders n, which justifies a Richardson extrapolation between frequencies derived from models with about 1600 and 800 points (Shibahashi & Osaki 1981).

To calculate the critical and the cut-off frequencies we make the following assumptions. For low-degree modes, the displacement is essentially vertical, so that the horizontal component can be neglected. Consequently, we consider only radial modes and, because we investigate modes of high radial orders, we adopt the Cowling approximation, i.e. neglect the perturbation of the gravitational potential.

The equation of motion for adiabatic oscillations can be written (Unno et al, 1989) as:

$$\Phi'' + \frac{\omega^2 - V^2}{c^2}\Phi = 0, \tag{1}$$

where $\Phi = \rho^{1/2} c^2 {\rm div}\xi$, and ξ is the fluid displacement, $c = (\Gamma_1 P/\rho)^{1/2}$ is the sound speed, $\omega = 2\pi\nu$ is the angular velocity (with the pulsation frequency ν), V is the acoustic potential, and the derivative is with respect to the radius. Eq. 1 shows that acoustic waves are reflected towards the interior and are well trapped, if ω^2 is smaller than V^2, whereas if ω^2 is larger, the mode propagates into the atmosphere dissipating mechanical energy which decreases the mode amplitude. The *critical frequency,* ν_c, is defined as the frequency above which modes propagate outwards. The *cut-off frequency,* $\nu_{\rm M}$, is the maximum value of the critical frequency encountered in the outermost stellar layers.

According to Vorontsov & Zarkhov (1989), the potential for radial modes can be written as:

$$V_1^2 = N^2 - \frac{c}{2}\frac{\rm d}{{\rm d}r}\left[c\left(\frac{2}{r} + \frac{N^2}{g} - \frac{g}{c^2} - \frac{1}{2c^2}\frac{{\rm d}c^2}{{\rm d}r}\right)\right] +$$

$$\frac{c^2}{4}\left(\frac{2}{r} + \frac{N^2}{g} - \frac{g}{c^2} - \frac{1}{2c^2}\frac{{\rm d}c^2}{{\rm d}r}\right)^2, \tag{2}$$

where N is the Brunt-Väisälä frequency. Another formulation is proposed by Gough (1986), assuming the characteristic length of the eigenfunctions $({\rm d}\ln(\delta r/r)/{\rm d}r)^{-1}$ being short compared to r, so that the problem corresponds to a plane-parallel layer in constant gravity. Under these conditions, the potential reduces to:

$$V_2^2 = \omega_{\rm c}^2, \tag{3}$$

where $\omega_c^2 = c^2/(4H_\rho^2)\,(1 - 2\mathrm{d}H_\rho/\mathrm{d}r)$ is the critical angular velocity and $H_\rho = -(\mathrm{d}\ln\rho/\mathrm{d}r)^{-1}$ is the density scale height. Vorontsov & Zarkhov (1989) use the acoustical depth as the dependent variable, while Gough (1986) uses the radius. The approximation of an isothermal atmosphere is also often adopted for calculating ν_c, and $H_\rho = H_p$ is then constant. In this case Eq. (2) and (3) reduce to:

$$V_3^2 = \omega_c^2 = \frac{c^2}{4H_\rho^2}. \tag{4}$$

Rapid density variations in the medium, such as occurring at the transition between the bottom of the radiative zone and the top of the convective zone, lead to an abrupt rise in the potential which constitutes a reflecting barrier to waves. But to some extent, waves penetrate and may be even tunnelling outwards (see e.g., Christensen-Dalsgaard and Frandsen 1983, Balmforth & Gough 1990). The thickness of the potential barrier, which determines the reflexion efficiency, is a function of the outer boundary conditions and of the potential V. We shall see in the next section that the different expressions for V predict a different thickness of this barrier and therefore a different efficiency of the wave reflexion. In the solar case, a potential barrier occurs at $\tau \sim 10^{-4}$, because the temperature rises above these layers and the potential behaves as $T^{-1/2}$. This temperature minimum is related to the boundary between the solar photosphere and chromosphere.

As there is no clear observational evidence for a corona around CP2 stars, the existence of a temperature minimum and therefore of a maximum of the potential is not known either (Balmforth & Gough 1990). Note however, that a temperature minimum could be caused also by other phenomena than an outside corona. The concept of the acoustic cut-off frequency as the largest critical frequency, however, requires a decrease of the critical frequency after a local maximum is reached in the same atmospheric layer where a temperature minimum is observed. First evidence of a chromosphere in Ap stars has perhaps been recently discovered by Simon & Landsman (1997).

In the next section we shall speculate on the acoustic cut-off frequency, provided that a temperature minimum at an optical depth of either $\tau_{\mathrm{ext}} = 10^{-4}$ (as for the Sun) or 10^{-6} exists. The latter depth was chosen, because layers at $\tau = 10^{-6}$ are optically transparent for *all* wavelenghts (Castelli et al. 1997). Higher layers are even more sensitive to NLTE effects which are not included in our models, while deeper layers become opaque in various wavelength ranges. Thus 10^{-6} is a reasonable compromise.

We denote $\nu_c^{(1)}$, $\nu_c^{(2)}$, and $\nu_c^{(3)}$ as the critical frequencies $V/2\pi$ derived from the equations (2), (3) and (4), respectively, and similarly $\nu_M^{(1)}$, $\nu_M^{(2)}$, and $\nu_M^{(3)}$ as the cut-off frequencies, i.e the maximum values of the critical frequencies encountered in the upper atmosphere.

3.2. Kurucz versus Hopf models

Figure 1 gives the temperature profile as a function of the logarithm of the optical depth for stellar models of 1.8 $M_\odot$ with an atmosphere based on the Hopf law and on Kurucz models, as well as the specific Kurucz ATLAS9 model atmosphere for $T_{\mathrm{eff}} = 8080$ K. We see that the Kurucz model reproduces very well the specific Kurucz model atmosphere of the same T_{eff}. In the atmosphere, the frequency-dependent treatment of radiative transfer increases the local temperature at a given radius, except in the very outer layers where the temperature of the Kurucz model decreases below the limiting value of the Hopf law (Fig. 1b).

The depth dependence of the temperature has a direct consequence on the gradient, the ionization of H and He I, on Γ_1, and on c. It directly affects therefore the critical frequency (Eq. 1). Fig. 2 shows the temperature gradient $\nabla = \mathrm{d}\log T/\mathrm{d}\log P$ and the adiabatic temperature gradient ∇_{ad} as a function of the logarithm of pressure, for the same models as in Fig. 1 (only the outermost convection zone is shown; the deeper one occurs at $r/R \sim 0.995$, i.e. at $\log P \sim 5.5$, see Tab. 2). The superadiabatic temperature gradient exhibits two close peaks, which are related to the transitions between radiation and convection and back to radiation at the borders of the thin convection zone. For $\tau \geq 0.03$, the local temperature of the Kurucz model is larger than that of the Hopf model (Fig. 1) and the peaks of the temperature gradient of the Kurucz model are shifted therefore towards the surface compared to the Hopf model. Similarly, the ionization zones of H and He I and the peaks of the adiabatic exponent Γ_1 are also closer to the surface (Fig. 3).

At $\tau \leq 0.03$, ($\log P \leq 3.9$), the temperature gradient is radiative (Fig. 2) and the temperature in the outermost layers of the Kurucz model is smaller by about 1220 K than that of the Hopf model (Fig. 1b). On the other hand, the adiabatic gradient and Γ_1 are larger for the Kurucz model (Fig. 3). While Γ_1 reaches a maximum at $\log P \sim 3.7$ for the Hopf model and then decreases monotonically, it slightly increases till $\log P \sim 2$ for the Kurucz model before decreasing. This behaviour has a direct influence on the acoustic cut-off frequency, as will be shown below. Γ_1 decreases below 4/3 for the Hopf model and tends to 1. The Eddington law has the same result and furthermore a similar tendency is found (G. Houdek 1997, private communication) for an atmosphere derived from the semi-empirical solar model 'C' from Vernazza et al. (1981). The tendency of Γ_1 to decrease below 4/3 reveals the failure of $T(\tau)$ relations based on the Hopf law, on Vernazza et al.'s model, or similar, to describe properly the upper layers of stars which are more massive than the Sun.

The potential barrier formed by V_2 (Eq. 3) at the transition between the radiative and the convective zones at $\log\tau$ between 1.5 and -1 is larger than for V_1 (Eq. 2, Fig. 4).

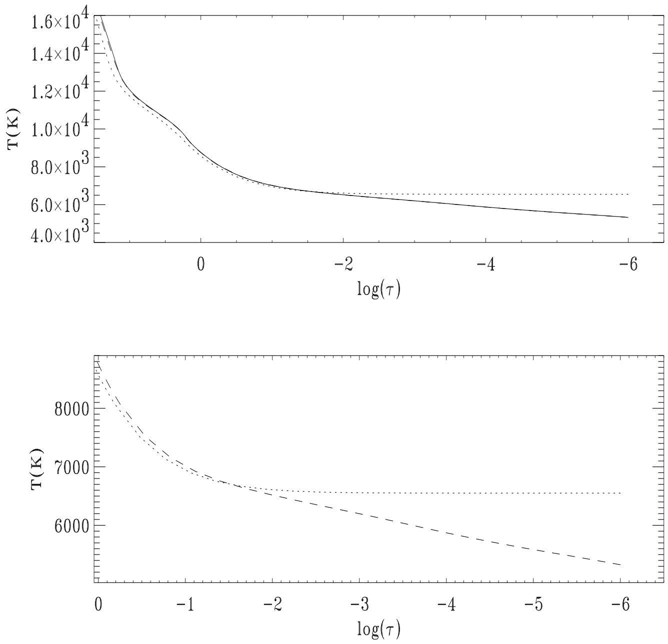

Fig. 1. Temperature profile of models with an atmosphere derived from the Hopf law (dotted) and from Kurucz model atmospheres (dashed). Both have an effective temperature of about 8080 K. In the upper panel **(a)**, the solid line corresponds to the specific Kurucz model atmosphere with $T_{\rm eff} = 8080\,$K and $\log g = 4.2$, and, as expected, coincide with the dashed line. The lower panel **(b)** shows the outermost layers.

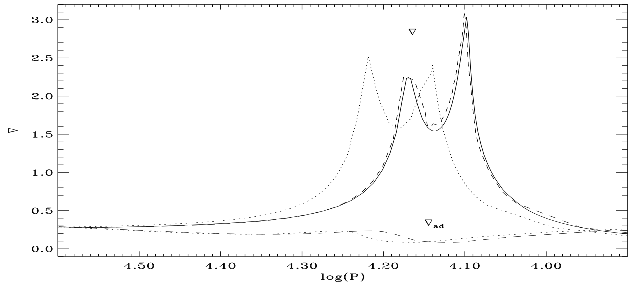

Fig. 2. Temperature gradient $\nabla = {\rm d}\log T/{\rm d}\log P$ for the models with an atmosphere derived from Hopf law (dotted) and from Kurucz model atmospheres (dashed). Both have an effective temperature of about 8080 K. The adiabatic gradient $\nabla_{\rm ad}$ is also presented. The solid line corresponds to the specific Kurucz model atmosphere with $T_{\rm eff} = 8080\,$K and $\log g = 4.2$.

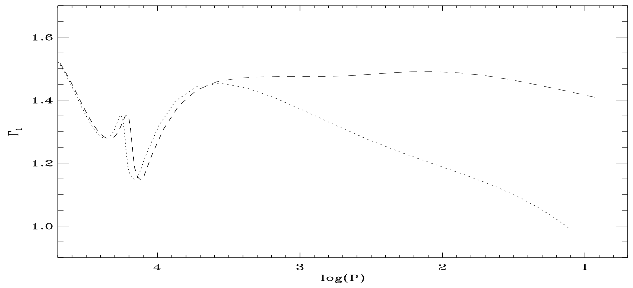

Fig. 3. Adiabatic exponent $\Gamma_1 = (\partial {\rm ln}P/\partial {\rm ln}\rho)_{\rm ad}$ for stellar models with an atmosphere derived from the Hopf law (dotted), and from Kurucz models (dashed).

The consequences of an efficiently reflecting boundary (represented by V_2) and a running wave boundary (approximated by V_1) have been investigated in the solar case by Gabriel (1992) and result in different mode amplitudes. In the upper layers the agreement between the three expressions for the potential is better for the Kurucz model (Fig. 4b) than for the Hopf model (Fig. 4a). The discrepancies at $\tau \le 10^{-4}$ are probably due to NLTE and nonadiabatic effects where the main assumptions for the formal expressions of the potentials break down.

The behaviour of the critical frequencies $\nu_{\rm c}^{(1)}$, $\nu_{\rm c}^{(2)}$, and $\nu_{\rm c}^{(3)}$ for the Hopf and Kurucz models are compared in Fig. 5. In all cases, the potential in the upper layers is larger for the Kurucz model (at $\tau \le 0.01$). Note that the maximum value of $\nu_{\rm c}^{(3)}$, the acoustic cut-off frequency $\nu_{\rm M}^{(3)}$, does not occur at the smallest optical depth, but slightly deeper in the atmosphere.

Table 3 gives the values of the cut-off frequencies $\nu_{\rm M}^{(1)}$, $\nu_{\rm M}^{(2)}$ and $\nu_{\rm M}^{(3)}$ for the Hopf and Kurucz models, computed for the potentials $\rm V_1$, $\rm V_2$, and $\rm V_3$, respectively, and with the assumption that the atmospheric boundary is located either at $\tau_{\rm ext} =\ 10^{-4}$ or at 10^{-6}. The differences of the frequencies computed for the Hopf models relative to the Kurucz models are substantial and amount to up to 10%

Table 3. Acoustic cut-off frequencies $\nu_{\rm M}^{(1)}$, $\nu_{\rm M}^{(2)}$ and $\nu_{\rm M}^{(3)}$ in μHz (see Eqs. 2, 3 and 4), for the Hopf and Kurucz models. The boundary of the model atmospheres is assumed either at $\tau_{\rm ext} =\ 10^{-4}$ or at 10^{-6}. The three last lines indicate the difference Δ between the respective Kurucz and Hopf models. For $\tau_{\rm ext} =\ 10^{-6}$, we also give the cut-off frequencies for models with an artificially suppressed convection in the outer layers (see the next subsection).

Model		$\tau_{\rm ext}$: 10^{-4}	10^{-6}	(no convection)
Hopf	$\nu_{\rm M}^{(1)}$	2139	2324	(2061)
	$\nu_{\rm M}^{(2)}$	2159	2354	(2345)
	$\nu_{\rm M}^{(3)}$	2231	2232	(2224)
Kurucz	$\nu_{\rm M}^{(1)}$	2453	2528	(2568)
	$\nu_{\rm M}^{(2)}$	2495	2743	(2748)
	$\nu_{\rm M}^{(3)}$	2463	2465	(2470)
	$\Delta\nu_{\rm M}^{1}$	314	204	
	$\Delta\nu_{\rm M}^{2}$	336	389	
	$\Delta\nu_{\rm M}^{3}$	232	233	

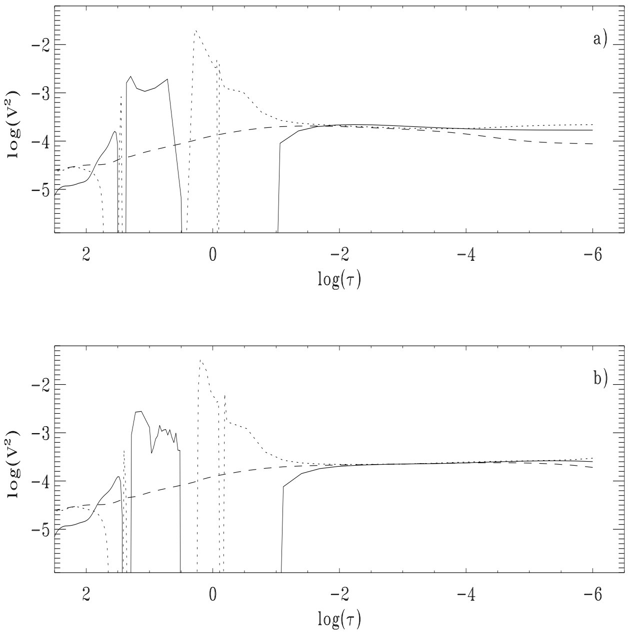

Fig. 4. a: (top) Logarithm of the potentials V_1, V_2 and V_3 (Eq. 2, 3 and 4, full, dotted and dashed lines, respectively), in the upper layers of the Hopf model; **b:** (bottom) same for the Kurucz model.

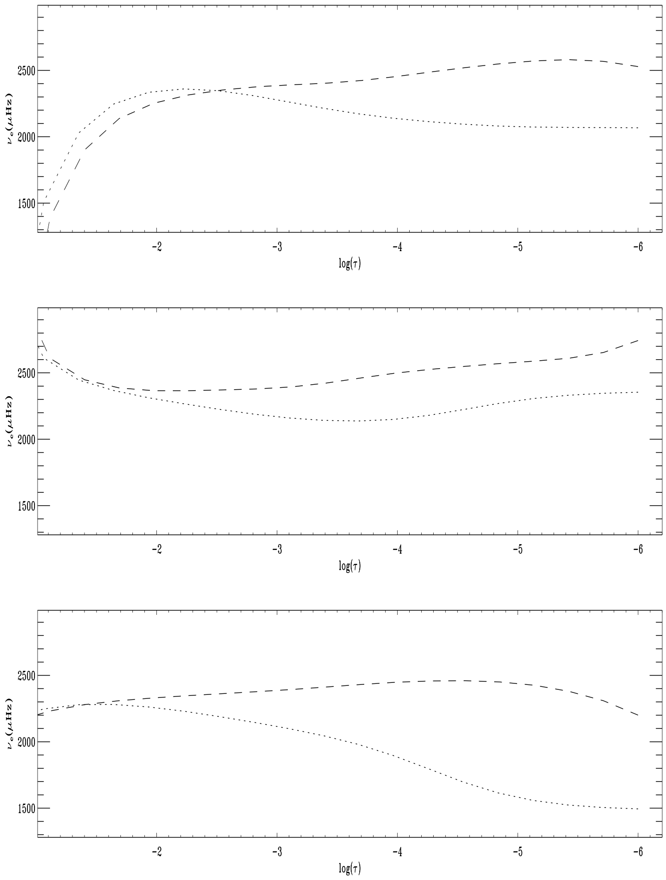

Fig. 5. a: (top) Critical frequency $\nu_c^{(1)}$ (Eq. 2) for the Hopf (dotted) and Kurucz models (dashed); **b:** (center) same for $\nu_c^{(2)}$ (Eq. 3); **c:** (bottom) same for $\nu_c^{(3)}$ (Eq. 4).

of ν_M. We must, however, keep in mind that the results derived from the Hopf law must be considered with much caution in optically thin regions, i.e. at $\tau \lesssim 10^{-3}$.

We have also compared the frequencies of acoustic modes of low and high radial order derived from the Hopf and Kurucz models. The latter have frequencies larger by about $1\,\mu$Hz compared to their Hopf counterparts. Although it represents less than 0.01% of the frequency, this difference is larger than the measurement error of ground-based observations and for the asteroseismic space mission COROT (Catala et al. 1995).

3.3. Effects of convection

As we have demonstrated in the previous subsection, Kurucz model atmospheres, taking convection and frequency-dependent treatment of radiative transfer into account, considerably improve stellar oscillation models in the upper layers. To specify which of these effects has the largest influence on the acoustic cut-off frequency, we investigate Kurucz models with a mixing-length parameter $\alpha = 0$, hence turning off convection, and compare them with Hopf models. For the internal structure models we have imposed the gradient to be radiative throughout the envelope, and the convective core, of course, is not affected. Specific Kurucz ATLAS9-model atmospheres were also computed with $\alpha = 0$. A comparison should therefore isolate the importance of the radiative transfer relative to convection.

In the models without convection ionization of H and He I occurs closer to the surface. As a result, the variations of Γ_1 and of the temperature gradient also occur closer to the surface (Fig. 6). Since the outermost stellar layers are radiative the physical quantities are unchanged for models without convection (Fig. 6) compared to models with convection (Fig. 2). Consequently, the cut-off frequencies have almost the same values (see Tab. 3 for the case of $\tau_{\rm ext} = 10^{-6}$).

These results clearly indicate that the major factor influencing the acoustic cut-off frequency is the inclusion of a frequency-dependent treatment of radiative transfer and a better calculation of the radiative pressure rather than the treatment of convection in the atmosphere. This result was indeed expected since the outermost layers, which are relevant for the cut-off frequency, are radiative.

3.4. Effects of evolution

The cut-off frequency also depends on the evolution, as was shown for example by Shibahashi (1991). From the ZAMS to the end of the main sequence and for models

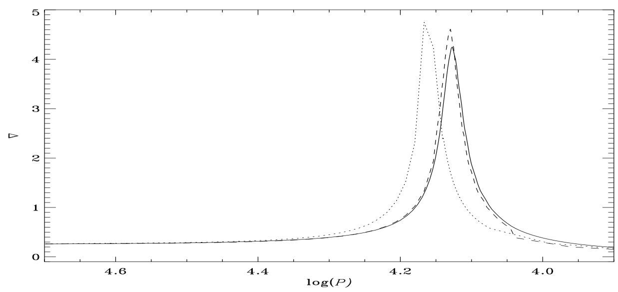

Fig. 6. Temperature gradient of models with an atmosphere computed with the Hopf $T(\tau)$ law (dotted), and with Kurucz models (dashed). The mixing-length parameter α was set to 0 in oder to suppress convection. The solid line represents the gradient of the Kurucz ATLAS9-model atmosphere with $T_{\rm eff} = 8080$ K.

of 1.8 M$_\odot$ this quantity decreases from 2855 to 1212 μHz for the Hopf models and from 3116 to 1328 μHz for the Kurucz models. However, the relative difference between the cut-off frequencies from the Kurucz and Hopf models remains almost unchanged along the main sequence and is about 8.5% (see Tab. 4). The cut-off frequency also scales as the characteristic frequency $\Omega_g = ({\rm G}M/R^3)^{1/2}$ (or as the frequency spacing, see Shibahashi 1991). We obtain similar quantitative results for all masses typical for roAp stars. When investigating a large parameter space in mass, age and metallicity, it appears to first order to be sufficient to compute models with the simple Hopf law and to approximate the effects of a better treatment of the atmosphere on the acoustic cut-off frequency by increasing this frequency by about 7% to 9%.

Table 4. Acoustic cut-off frequencies $\nu_{\rm M}^{(1)}$ (Eq. 2) as a function of age for Hopf and Kurucz models with 1.8 $M_\odot$ and Z = 0.02. The relative difference Δ is Kurucz-Hopf.

age (Myr)	Hopf (μ Hz)	Kurucz (μ Hz)	Δ in %
100	2855	3116	8.37
200	2732	2985	8.48
300	2610	2864	8.86
400	2481	2726	8.99
500	2360	2581	8.56
600	2221	2426	8.45
800	1914	2073	7.66
1000	1510	1650	8.49
1100	1212	1328	8.76

Since the uncertainty in the age determination can introduce a significant error in the computed acoustic cut-off frequency we will focus our discussion in the following sections on those two roAp stars, HD 24712 and HD 128898, for which we have more reliable mass estimates due to the availability of HIPPARCOS parallaxes.

3.5. Comparison with observations

For about 5 out of 28 known roAp stars, the largest published frequency (see Tab. 5 and references therein) exceeds the expected theoretical acoustic cut-off frequency determined from standard stellar models. We do not consider here roAp stars for which the highest observed frequency probably is a harmonic of their nonlinear oscillation (HD 83368 (Kurtz et al. 1993), HD 101065 (Martinez & Kurtz 1990), HD 137949 (Kurtz et al. 1991), and HD 161459 (Martinez et al. 1991)).

As we have demonstrated in the previous sections, stellar models with a frequency dependent treatment of the radiative transfer in the atmosphere (Kurucz models) do have a higher cut-off frequency than models with a grey atmosphere (Hopf models) which used to be the baseline for most of the investigations in this field. This result is in agreement with speculations of Shibahashi & Saio (1985) and of Matthews et al. (1990, 1996) that a steeper than solar temperature gradient would increase the cut-off frequency and hence bring theoretical results closer to the observations.

Table 5. List of roAp stars for which the largest published frequency is larger than the cut-off frequency determined from standard stellar models.

HD	Observed ν_{max} (μHz)	Ampl. mmag	Comment
6532	2 402	0.77	Kurtz et al. (1996)
24712	2 807	0.20	Kurtz et al. (1989)
128898	2 566	0.12	Kurtz et al. (1994)
134214	2 950	3.40	Kurtz et al. (1991)
203932	2 838	0.17	Martinez et al. (1990)

The photometric and spectroscopic properties of the roAp star **HD 24712** with $T_{\rm eff} = 7250 \pm 150$ K (Ryabchikova et al. 1997), $\log(L/L_\odot) = 0.91 \pm 0.04$ (based on $\pi_{\rm HIPPARCOS} = 0\farcs02041 \pm 0\farcs00084$, a bolometric correction of - 0.085 (Schmidt-Kaler 1982) and neglecting interstellar extinction), can be reproduced with a model of 1.63 $M_\odot$, $Z = 0.02$ and an age of about 900 Myr. At $\tau_{\rm ext} = 10^{-6}$, the Hopf model gives a cut-off frequency $\nu_{\rm M}^{(1)} = 2280\,\mu$Hz whereas the Kurucz model gives 2480 μHz. A stellar model with a Kurucz atmosphere hotter by about 100 K, less luminous by nearly 10%, and less evolved by 100 Myr would have a cut-off frequency in agreement with the largest observed frequency. However, such a model is compatible only with the lower left corner of the error box (see Fig. 7).

We have also computed Hopf and Kurucz models with appropriate age for **HD 128898** (α Cir). For the first time, a Kurucz model atmosphere was calculated with an opacity distribution function specific to the composition of α Cir (Piskunov & Kupka 1997). Stellar models with $1.93\,M_\odot$, $Z = 0.03$ and an age of 400 Myr, fit the observed values (Kupka et al. 1996) of $T_{\rm eff} = (7900 \pm 200)$ K and $\log(L/L_\odot) = 1.11 + 0.01/-0.02$ (based on $\pi_{\rm HIPPARCOS} = 0\farcs06097 \pm 0\farcs00058$, a bolometric correction of - 0.12 (Schmidt-Kaler 1982) and neglecting interstellar extinction). The cut-off frequencies $\nu_{\rm M}^{(1)}$ for the Hopf and Kurucz models are 2346 and $2600\,\mu$Hz, respectively, at $\tau_{\rm ext} = 10^{-6}$. The acoustic cut-off frequency computed for the Kurucz model is compatible to the largest observed frequency (see Table 5) and we can therefore conclude, that no discrepancy may exist for this roAp star between theoretical and observed cut-off frequencies.

However, the cut-off frequency depends on the model input parameters and one must therefore account for uncertainties inherent to observations and modelling. For example, there are problems with the photometric calibration of fundamental parameters of CP stars. Models of different mass and age can fit the same star in the H-R diagram within the error bars. For HD 24712, e.g., the cut-off frequency varies from $2725\,\mu$Hz (Kurucz model with $1.60\,M_\odot$ and 800 Myr) which is only $80\,\mu$Hz short of the largest observed frequency, to an even lower value of $2294\,\mu$Hz ($1.63\,M_\odot$, 1100 Myr). For α Cir, a model with $1.90\,M_\odot$ and 600 Myr gives $\nu_{\rm M}^{(1)} = 2329\,\mu$Hz which would be clearly smaller than the largest observed frequency.

A similar situation exists for **HD 134214** which, unfortunately, has a considerably larger error box due to the HIPPARCOS parallax of only (10.92±0.89) marcsec and a $T_{\rm eff}$ which could be estimated only from photometric indices.

Evolutionary tracks and lines of constant cut-off frequency for Kurucz models are plotted in Fig. 7 together with the observational error boxes. Models with $Z = 0.02$ were investigated for HD 24712 and HD 134214, and models with $Z = 0.03$ for HD 128898. The same physics as for our previous $1.8\,M_\odot$ models was used (see Sec. 2). Errors on the mass and age determination are about $0.02\,M_\odot$ and 200 Myr for HD 24712, and $0.02\,M_\odot$ and 100 Myr for HD 128898.

Since the cut-off frequency of Kurucz models is larger than for Hopf models of same age, lines corresponding to the Hopf models are shifted to higher effective temperature and lower luminosity. For $Z = 0.02$ the Hopf lines for $\nu_{\rm M}^{(1)} = 2300, 2500$ and $2800\,\mu$Hz almost coincide with the Kurucz lines for $\nu_{\rm M}^{(1)} = 2500, 2800$ and $3000\,\mu$Hz, respectively, and for $Z = 0.03$ the Hopf lines for $\nu_{\rm M}^{(1)} = 2300$ and $2500\,\mu$Hz coincide with the Kurucz lines for $\nu_{\rm M}^{(1)} = 2500$ and $2600\,\mu$Hz, respectively.

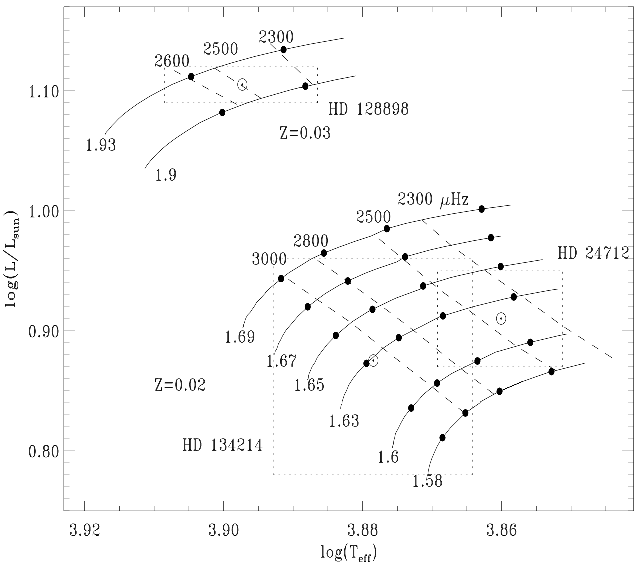

Fig. 7. HR diagramme for stars with 1.58 $M_\odot$ to 1.69 $M_\odot$ ($Z = 0.02$), and with 1.90 and 1.93 $M_\odot$ ($Z = 0.03$). Dots indicate ages of 400, 600, 800 and 1000 Myr for $Z = 0.02$, and 400 and 600 Myr for $Z = 0.03$. The roAp stars HD 24712, HD 134214 and HD 128898 are indicated by circles and error boxes. Lines of constant cut-off frequency $\nu_{\rm M}^{(1)}$ for the Kurucz models are drawn for 2300, 2500, 2800 and $3000\,\mu$Hz for $Z = 0.02$, and for 2300, 2500 and $2600\,\mu$Hz for $Z = 0.03$ (dashed lines).

We stress also that the frequency difference between models with atmospheres computed either with the Hopf law or with Kurucz models is comparable to the changes which are obtained when introducing physical processes such as convective core overshooting (see, e.g., Audard et al. 1995).

4. Conclusion

We have shown that the major impact for modelling pulsation frequencies close to the cut-off frequency comes from the inclusion of a frequency-dependent treatment of radiative transfer and of blanketing effects, as well as from a better calculation of the radiative pressure in the model atmospheres, rather than the inclusion of convection. At the main sequence, Kurucz model atmospheres merged with stellar models derived from the CESAM code increase the cut-off frequency by about 8.5 % relative to the value derived from the Hopf $T(\tau)$ relation.

For two roAp stars with probably the best available mass and luminosity estimates, HD 24712 and α Cir, we find models with Kurucz atmospheres and with parameters in agreement with the observational error box which have a theoretical cut-off frequency larger than the largest observed frequency and hence are in agreement with observations. One may thus speculate that the old controversy about a mismatch between observed largest frequencies and theoretical cut-off frequencies of roAp star models is resolved.

We have assumed solar abundances for our models except for α Cir, which, however, has an abundance pattern that does not significantly change the model atmosphere from one with solar abundances. HD 24712, on average, is even less peculiar than α Cir. Assuming that $Z = 0.03$ reflects only a surface composition for α Cir and $Z = 0.02$ would be the better choice for a stellar model, one can expect a smaller mass to fit α Cir and a smaller cut-off frequency as deduced from Fig. 7. For other roAp stars with a more peculiar abundance pattern, the deviation from model atmospheres with solar abundances or models with a scaled heavy element abundance will be larger (Gelbmann 1997). Abundant rare-earth elements, through blanketing effects, could decrease the surface temperature and thus increase the cut-off frequency. This frequency might also be affected by a chemical composition gradient (Vauclair & Dolez 1990). Finally, improving CP2 star pulsation models requires also to account for the magnetic field (Dziembowski & Goode 1996).

The low surface temperature in the outer atmospheric regions provides a potential well which reflects the acoustic waves inwards. The existence of a maximum value of this potential, which defines the cut-off frequency, requires an increase of the temperature in even higher atmospheric layers. While such an increase is observed in the solar chromosphere towards the corona, the first direct evidence of a chromosphere in Ap stars has only been recently shown by Simon & Landsman (1997). However, in the very outer atmospheric layers nonadiabatic and NLTE effects are probably relevant and should be considered in the future. Already at $\tau \lesssim 10^{-4}$ NLTE effects might lead to an increase of temperature (Mihalas 1978) which would decrease the acoustic potential, thus defining the upper limit of the critical frequency, and hence the cut-off frequency. Unfortunately, NLTE atmospheres with a treatment of line blanketing as sophisticated as in ATLAS9, for at least the main elements contributing to the opacity in the upper atmospheric regions, are not available in the foreseeable future.

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