ATMOSPHERE MODELS OF A-F STARS.

EFFECTS ON THE ACOUSTIC CUT-OFF FREQUENCY.

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We have computed stellar models and eigenmodes for δ Scuti and pulsating Ap stars. We compare models computed with atmospheres reconstructed with $T(\tau)$ laws derived from Kurucz's model atmospheres, to models with atmospheres from the radiative Hopf $T(\tau)$ law.
For some Ap stars, frequencies are observed which are larger than the theoretical acoustic cut-off frequency. The latter strongly depends on the $T(\tau)$ relation in the atmosphere. Compared to the radiative $T(\tau)$ relation, Kurucz's model atmospheres increase the cut-off frequency, and hence come closer to the observations.

1. ATMOSPHERE MODELS

δ Scuti and Ap star models with a mass between 1.7 to 2.2 $M_\odot$ have two very thin convection zones below the surface. The outer zone extends into the atmosphere and it is therefore essential for modelling pulsations to include convection in the model atmospheres. Those, however, are often computed with a radiative $T(\tau)$ law, such as Eddington or Hopf laws, which ignore convection. Kurucz's models [1] include a frequency-dependent treatment of radiative transfer, while the Hopf law assumes a grey atmosphere, and in addition take convection into account.
We have derived $T(\tau)$ laws from Kurucz's model atmospheres, without the "overshooting option" [2], for effective temperatures ranging from 7500 to 9000 K, and 4.2 for $\log g$. The approximation of a constant $\log g$ is reasonable since it varies only from 4.313 to 4.197 along our evolutionary tracks (from age 0 to 500 Myr). A solar abundance is used.
In internal structure calculations one separates the internal parts where the diffusion approximation is valid, from an outer part where the atmosphere is reconstituted using the $T(\tau, T_{\rm eff}, g)$ law. The boundary between the two parts must be located at an optical depth $\tau = \tau_b$ where the diffusion approximation is valid. For models with an atmosphere from the Hopf law, $\tau_b = 2$, while for models with an atmosphere derived from Kurucz models, τ_b is set to 10 (see [3]). The link between the internal structure model and the atmosphere is made for pressure, temperature and mass.
We used the EFF equation of state with an initial hydrogen content $X = 0.7$, a heavy-element abundance $Z = 0.02$, and the most recent OPAL95 opacities. The mixing-length theory is adopted, with a mixing-length parameter α set to 1.4 for the internal structure model, and to 1.25 for the Kurucz model atmospheres (which corresponds to 1.4 for the internal model, due to different geometry assumed for the convective elements in MLT). See [4,5] for more details about the CESAM code.

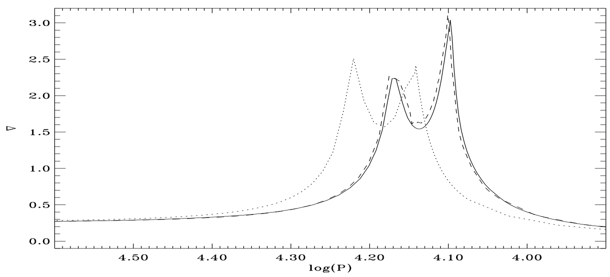

Figure 1: Temperature gradients plotted versus $\log P$ in the outer parts of models whose atmosphere is derived from the radiative Hopf $T(\tau)$ law (dotted line), and from Kurucz's code (dashed line). These models have an age of 500 million years, and $T_{eff} \sim 8080$ K. The full line corresponds to the Kurucz model atmosphere of $T_{eff} = 8080$ K, $\log g = 4.2$. The slight difference between the dashed and the full lines is due to different Rosseland opacities.

Fig. 1 compares the temperature gradient of models of 500 million years, with an atmosphere derived from the Hopf law and from Kurucz model atmospheres. We point out that we had to extend Kurucz models to 288 layers instead of the usual 72, in order to have smooth gradients. With respect to the Hopf model atmosphere, Kurucz model leads to a temperature gradient of larger amplitude, which is shifted towards the surface. As a consequence, the ionization of H and He I also occurs closer to the surface.

In the very outer layers, the adiabatic gradient of the Hopf model decreases, while it slightly increases for the Kurucz model. As a result, the same behaviour appears for the adiabatic exponent Γ_1. Figure 2 shows that it is also the case for the Brunt-Väisäla frequency N.

2. ACOUSTIC CUT-OFF FREQUENCY

Implementing Kurucz's model atmospheres is expected to influence oscillations, especially those at high frequencies, which are the most sensitive to the external layers. The acoustic cut-off frequency ν_{ac} should be even more affected. The propagation

of acoustic modes is limited in the upper frequency range by ν_{ac}. Since Γ_1 is much smaller than 5/3 in the upper layers, the approximation of a perfect gas cannot be used to calculate ν_{ac}. We have instead adopted the mechanical boundary condition corresponding to an isothermal atmosphere at the surface, for radial oscillations $\ell = 0$ (see [6], Eq.14-37). It leads to a second order differential equation for the relative displacement $\delta r/r$, whose solution depends on its discriminant $\gamma = (3-V)^2 - 4\ \left(V_g(4+C_1\omega^2)-3\right)$. $V = -\mathrm{dln}P/\mathrm{dln}r, V_g = V/\Gamma_1$ (where $\Gamma_1 = (\partial \ln P/\partial \ln \rho)_{ad}$ is the adiabatic exponent), $C_1 = (r/R)^3/(m/M)^3$, and ω is the dimensionless pulsation. With $\gamma \geq 0$ the solution is real and the oscillation mode is evanescent in the atmosphere, while with $\gamma < 0$ the mode propagates in the atmosphere. We can thus define ν_{ac} as the maximum value of ω before γ becomes negative: $\nu_{ac} = \omega\ \sqrt{(GM/R^3)}/(2\pi)$.
Table 1 compares the values of ν_{ac} for Hopf and Kurucz laws, for the same models as Fig.1. We also indicate the dimensionless quantity value ν_{ac}/ν_g, where $\nu_g = \sqrt{GM/R^3}/(2\pi)$ is the characteristic frequency of the model, and the corresponding maximum radial order n_{max}. Kurucz model increases ν_{ac} by 360 μHz, and n_{max} by 5.

Table 1: Acoustic cut-off frequency for models whose atmosphere is derived from the Hopf law, and from Kurucz model atmospheres.

model	ν_{ac} (μHz)	ν_{ac}/ν_g	n_{max}
Hopf	2103	37.1	28
Kurucz	2463	44.4	33

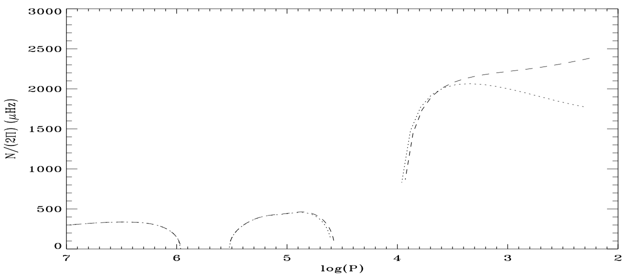

Figure 2: Brunt-Väisäla frequency of models whose atmosphere is derived from the radiative Hopf $T(\tau)$ law (dotted line), and from Kurucz's code (dashed line).

Besides the inclusion of convection, Kurucz's models also provide a frequency-dependent treatment of radiative transfer. The differences between Hopf and Kurucz models that we have seen are located in the very outer layers, which are radiative. Thus, the main factor for the determination of the acoustic cut-off frequency is the frequency-dependent treatment of radiative transfer (We have checked this by computing internal and atmosphere models where the mixing-length parameter was set to 0, i.e no convection in the upper parts, which lead to the same results as the models with convection).

3. DISCUSSION AND CONCLUSION

Models whose atmospheres are derived with frequency-dependent Kurucz model atmospheres have a temperature gradient of larger amplitude than those with an atmosphere from a radiative $T(\tau)$ law. Their frequency spectra have also a larger acoustic cut-off frequency. This result is in agreement with [7] who used an analytical $T(\tau)$ relation based on a single Kurucz's (1979) model, and with [8], [9], and [10] who argued that this increase might be caused by a steeper temperature gradient.
Since the main difference between models with an atmosphere derived from the Hopf law and from Kurucz models, is located in the very upper layers which are radiative, the main factor for the determination of the acoustic cut-off frequency is the frequency-dependent treatment of radiative transfer and not that of convection.
We must however be aware that the peculiar abundance of Ap stars might change to some extent our results which are based on a solar abundance. We also should consider rotation and magnetism effects. More details about our study can be found in [11].

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