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Research Note

Shear turbulence beneath the solar tachocline

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Abstract. Helioseismic inversions of the Sun's internal angular velocity profile show that the rotation changes from differential in latitude in the convection zone to almost uniform in the radiative region below. The transition occurs in a thin layer, the tachocline, which is the seat of strong shear in the vertical direction. In this Note we examine whether this rotation profile can lead to shear turbulence at the top of the radiation zone. By using the standard solar model, we show that such turbulence can be generated only in a narrow region $0.695\,R_\odot \lesssim r < 0.713\,R_\odot$ at the equator and even in narrower layers at higher latitudes. We conclude that the turbulence generated by this vertical shear is unlikely to play a significant role in the transport of matter and angular momentum, and that other mechanisms must be invoked to achieve this.

Key words: Convection, Turbulence, Sun: evolution, Sun: interior, Sun: rotation

1. Introduction

Helioseismology has revealed that the rotation regime in the Sun changes abruptly from latitude dependent in the convection zone to almost uniform in the radiation zone below. The top of the radiation zone is thus a region of strong vertical shear, and for this reason that layer has been called the tachocline (Spiegel & Zahn 1992). Acoustic sounding has shown also that some mild macroscopic mixing occurs in this region, to smoothen the composition gradient which, with microscopic diffusion only, would be somewhat steeper. Moreover, the Li depletion observed in solar-type stars can only be explained by some type of macroscopic transport between the convection zone and the depth where this fragile element is destroyed.

Among the possible mechanisms which may produce such mixing, a plausible one is the shear instability induced by the vertical rotation profile. In the present Note we shall verify whether this instability is able to play an effective role in the observed mixing.

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2. The solar rotation profile

Through the inversion of LOWL frequency-splitting data spanning 2 years, Charbonneau et al. (1998) have derived the following internal solar rotation rate for $r > 0.5R_\odot$:

$$\Omega(r,\theta) = \Omega_\mathrm{C} + \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{r - r_\mathrm{C}}{w}\right)\right](\Omega_\mathrm{S}(\theta) - \Omega_\mathrm{C})\,. \quad (1)$$

The observed latidinal differential rotation Ω_S is expressed as:

$$\Omega_\mathrm{S}(\theta) = \Omega_\mathrm{eq} + c_1\cos^2\theta + c_2\cos^4\theta.$$

The parameters values are: $\Omega_\mathrm{C} = 2\pi \times 432.8\,\mathrm{nHz}$, $\Omega_\mathrm{eq} = 2\pi \times 460.7\,\mathrm{nHz}$, $c_1 = -62.69\,\mathrm{nHz}$, $c_2 = -67.13\,\mathrm{nHz}$; $r_\mathrm{C} = 0.713\,R_\odot$ is the radius at the bottom of the solar convective zone and $w = 0.025\,R_\odot$ the tachocline thickness. Similar expressions can be found in the litterature e.g. Li & Wilson (1998), Corbard et al. (1998), Antia et al. (1998). The fact that $\Omega(r,\theta)$ is almost constant at greater depth is in conflict with earlier predictions of a faster rotating core (Pinsonneault et al. 1989; Zahn 1992), and it rules out that angular momemtum be transported vertically through shear-induced turbulence. But this possibility still exists in the shear flow of the tachocline.

3. Conditions for vertical shear instability

3.1. Richardson criterion

In a stellar radiation zone, the stable stratification of entropy tends to inhibit any instability arising from a vertical shear dV/dz, where V is the amplitude of the horizontal velocity and z the vertical coordinate. In the adiabatic limit, i.e. when radiative losses are negligible, the shear instability is suppressed whenever:

$$N^2 \leq \left(\frac{dV}{dz}\right)^2 \mathrm{Ri_c}. \quad (2)$$

The strength of the stratification is measured by the Brunt-Väisälä frequency N:

$$N^2 = \frac{g\delta}{H_P} \left(\nabla_{\rm ad} - \nabla\right), \tag{3}$$

with the usual notations for the gravity g, the pressure scale height H_P, the logarithmic temperature gradients $\nabla = \partial \ln T / \partial \ln P$ and $\delta = -(\partial \ln \rho / \partial \ln T)_P$, P, ρ and T being pressure, density and temperature. The critical Richardson number $\mathrm{Ri_c}$ is of order unity for typical flow profiles; in the following we shall take $\mathrm{Ri_c} = 1/4$.

However shear turbulence may still arise, and be sustained, provided that sufficient heat is lost by the turbulent eddies to lower their buyoancy. Whether this occurs is determined by the Péclet number characterizing these eddies:

$$\mathrm{P_e} = \frac{vl}{K};$$

in this expression v and l are the velocity and the size of turbulent elements and K the thermal diffusivity:

$$K = \frac{16\sigma}{3} \frac{\Gamma_1 - 1}{\Gamma_1} \frac{T^4}{\kappa \rho P},$$

σ is the Stefan constant, κ the Rosseland mean opacity and Γ_1 the first adiabatic exponent.

When $\mathrm{P_e} \ll 1$ the Richardson criterion takes the modified form (Dudis 1974; Zahn 1974):

$$N^2 < \left(\frac{dV}{dz}\right)^2 \frac{\mathrm{Ri_c}}{\mathrm{P_e}} \Longleftrightarrow vl < \left(\frac{dV}{dz}\right)^2 \frac{\mathrm{Ri_c} K}{N^2}, \tag{4}$$

from which one infers what are the largest turbulent scales that can survive in a stratified shear flow.

3.2. Critical Reynolds number

However turbulence will be maintained only if the turnover rate of the eddies is faster than their viscous decay rate:

$$\frac{v}{\ell} \lesssim \frac{\nu}{\ell^2} \Longleftrightarrow \nu \mathrm{Re_c} \leq v\ell, \tag{5}$$

ν is the viscosity. This critical Reynolds number $\mathrm{Re_c}$ is smaller than the classical one which governs the onset of instability in a shear flow whose velocity varies by U between two boundaries separated by the distance L, namely UL/ν. The reason is of course that $v < U$ and $\ell < L$.

For the value of this $\mathrm{Re_c}$ we turn to an experiment performed by Stillinger et al. (1983), who measured the size of the turbulent motions downstream a flow traversing a grid. The size of the smallest turbulent eddies was found to be $15.4\,\ell_K$, where $\ell_k = (\nu^3/\varepsilon)^{1/4}$ is the classical Kolmogorov length, with ε being the energy injection rate per unit mass. In the inertial cascade the velocities scale as $v^3 = \varepsilon \ell$, and therefore:

$$v\ell = \varepsilon^{1/3} \ell^{4/3} \geq \varepsilon^{1/3} (15.4\,\ell_K)^{4/3} = (15.4)^{4/3} \nu, \tag{6}$$

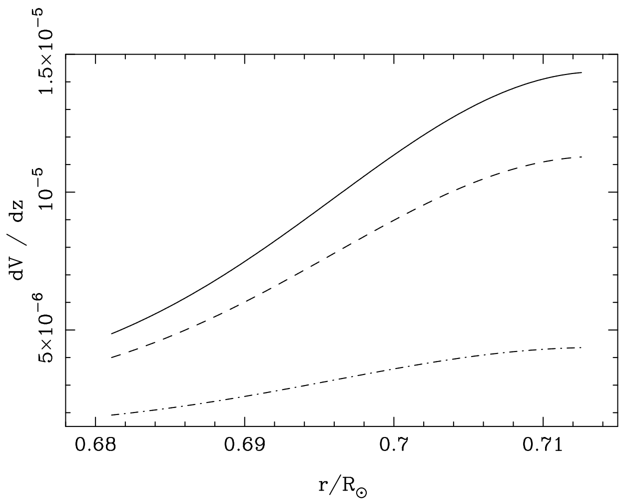

Fig. 1. The shear rate dV/dz with respect to radius beneath the convection zone for the colatitudes $\theta = 90°$ (full), $\theta = 60°$ (dashed) and $\theta = 30°$ (dash-dot-dash).

from which we draw the critical Reynolds number in that experiment: $\mathrm{Re_c} \approx 40$, a value we shall adopt for the present purpose.

The conditions (4) and (5) can be simultaneously fulfilled when:

$$\left(\frac{dV}{dz}\right)^2 > N^2 \frac{\nu}{K} \frac{\mathrm{Re_c}}{\mathrm{Ri_c}}. \tag{7}$$

4. The turbulent region in the Sun

Let us now examine where the instability condition (7) is fulfilled is the Sun. According to (1), the maximum shear rate at the top of the solar radiation zone amounts to

$$\frac{dV}{dz} \Longrightarrow r \sin\theta \frac{d\Omega}{dr} \approx 9.10^{-6}\,\mathrm{s}^{-1},$$

as illustrated in Fig. (1).

It is convenient to introduce the non-dimensioned parameter

$$\xi \equiv \left(\frac{dV}{dz}\right)^2 \frac{K}{N^2 \nu};$$

according to Eq. (7), the turbulent shear instability will occur if:

$$\xi \gtrsim \xi_{\rm limit} \equiv \frac{\mathrm{Re_c}}{\mathrm{Ri_c}} \approx 160. \tag{8}$$

Note that $\xi_{\rm limit}$, the ratio of Reynolds and Richardson critical numbers, is rather uncertain, in particular because $\mathrm{Ri_c}$ depends on the vertical flow profile.

We have calculated ξ as a function of radius and colatitude in the radiative zone beneath the tachocline of a standard solar model, in order to delimit the regions where the turbulent instability is established.

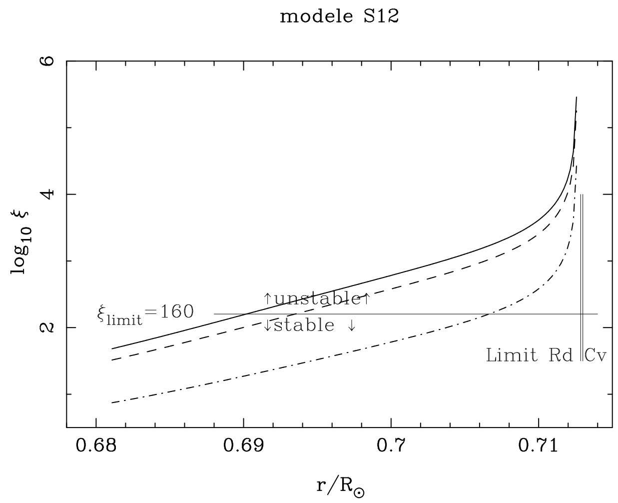

Fig. 2. For the colatitudes $\theta = 90°$ (full), $\theta = 60°$ (dashed) and $\theta = 30°$ (dash-dot-dash), stable and unstable regions beneath the tachocline located about at the limit Rd Cv between the convection zone and the radiative interior.

The standard solar model is computed with CESAM code (Morel 1997); the evolution includes the pre main-sequence phase. Basically, the physics and calibration parameters are the same as in Morel et al. (2000); it uses OPAL opacities and equation of state, the microscopic diffusion coefficients of Michaud & Proffitt (1993) and the recently updated thermonuclear reaction rates of the European compilation NACRE (Angulo et al. 1999). The angular velocity beneath the convection zone is derived from Eq. 1. The dynamical viscosity is taken as in a fully ionized hydrogen plasma.

5. Results and discussion

Figure 2 shows $\log_{10}\xi$ as a function of radius for three colatitude values. According to the Eq. (8) criterion, the turbulent instability may occur at the equator ($\theta = 90°$) in the interval $0.695\,R_\odot \lesssim r < 0.713\,R_\odot$; the turbulent layer is thinner at higher latitudes. In other words, the unstable region for the vertical shear instability, beneath the convection zone, measures barely 2 percent of the solar radius and is localized around the equator.

This instability should not be confused with the shear instability arising from the differential rotation in latitude, which is not inhibited by the vertical stratification, and which may play an important role in smoothing horizontal gradients in composition and angular velocity (Spiegel & Zahn 1992). But that instability does not contribute to the vertical transport.

We conclude that another physical process is needed to transport matter and angular momentum beneath the solar tachocline, which would operate more efficiently than the turbulence generated by the vertical Ω-gradient. A plausible mechanism is the meridional circulation which probably exists in the tachocline (Brun et al. 1999). A more powerful process is required also to establish the almost uniform angular velocity at greater depth, which could be magnetic torquing (Gough & McIntyre 1998) or transport of angular momentum by waves (Schatzman 1996; Kumar et al. 1999).

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